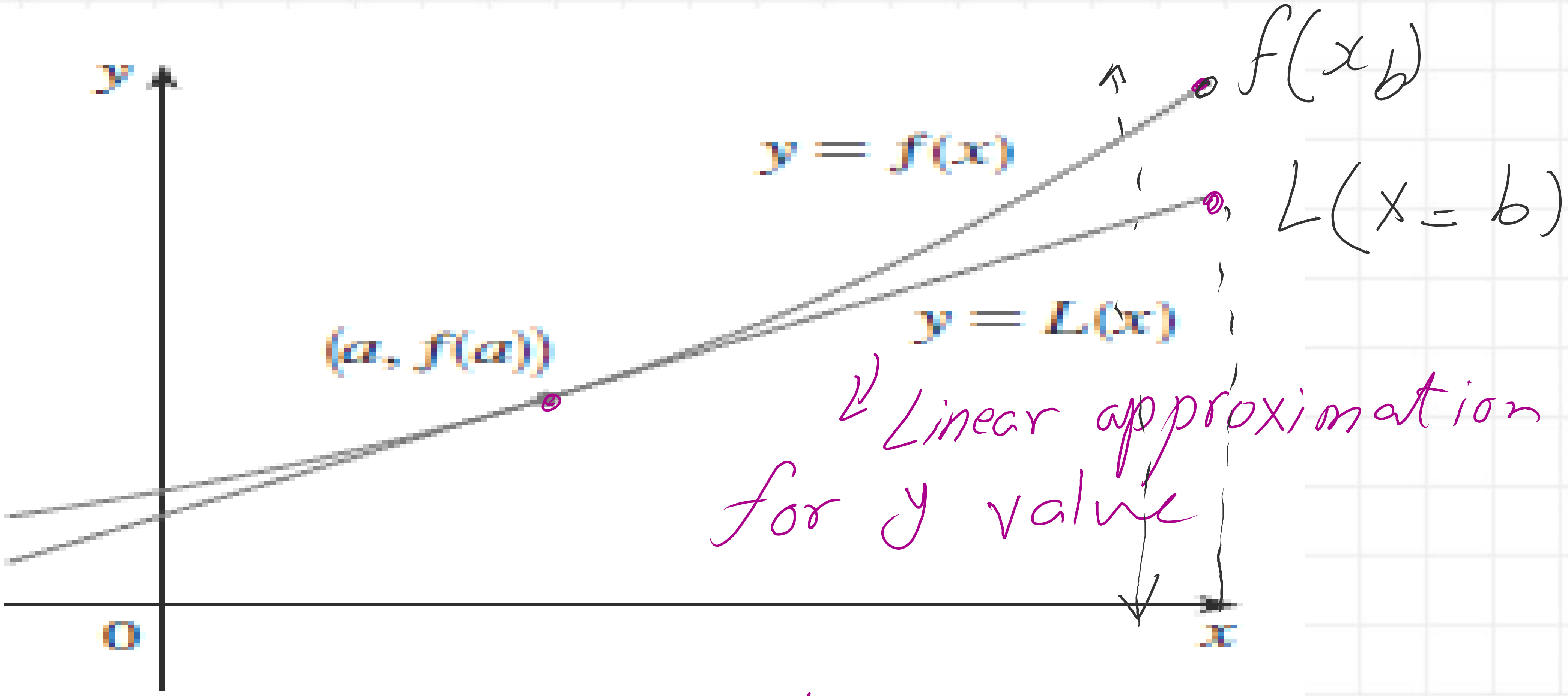
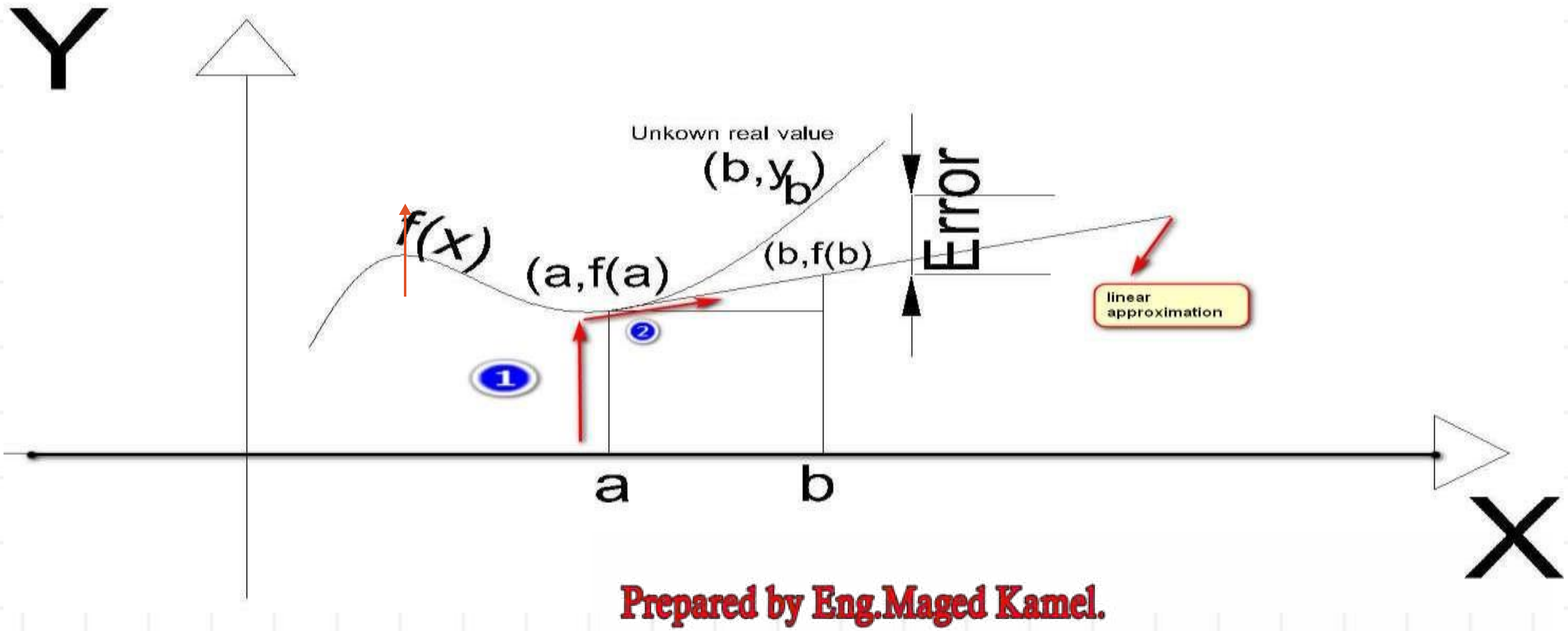


at $x = a$

You treat the function as linear one for any point needed get , the y value from that line.



Linear Approximation.

Linear approximation is the y value on a point b knowing the value x of another point called a and it's y value.

$$L(x=xb)=f(a)+f'(a)(x_b - x_a)$$

Application of the linear approximation.

The value of Y_b is equal to the value of f(a) plus the product of slope by the difference in X value between a,b

- Linear approximation of any function.
- Estimation of Power value of a number
- Estimation of Square root of number.

Prepared by Eng.Maged Kamel.

Example #1

Howard Anton
Calculus

(a) Find the local linear approximation of $f(x) = \sin x$ at $x_0 = 0$.

(b) Use the local linear approximation obtained in part (a) to approximate $\sin 2^\circ$, and compare your approximation to the result produced directly by your calculating device.

Solution $\quad f(x) = \sin(x) \Rightarrow f'(x) = +\cos(x)$

at x_0 $\quad L(x) = f(x_b) = f(x_a) + f'(a) * (x_b - x_a)$

let $x = x_b$ & $x_0 = 0 \rightarrow f(x_0) = \sin(0) = 0$

$f'(x_0) = \cos(0) = 1$

$L(x) = f(x) = 0 + 1(x) = x$

$L(x_0 = 0) = 0$

Part a

Example #1

Howard Anton Calculus

(a) Find the local linear approximation of $f(x) = \sin x$ at $x_0 = 0$.

(b) Use the local linear approximation obtained in part (a) to approximate $\sin 2^\circ$, and compare your approximation to the result produced directly by your calculating device.

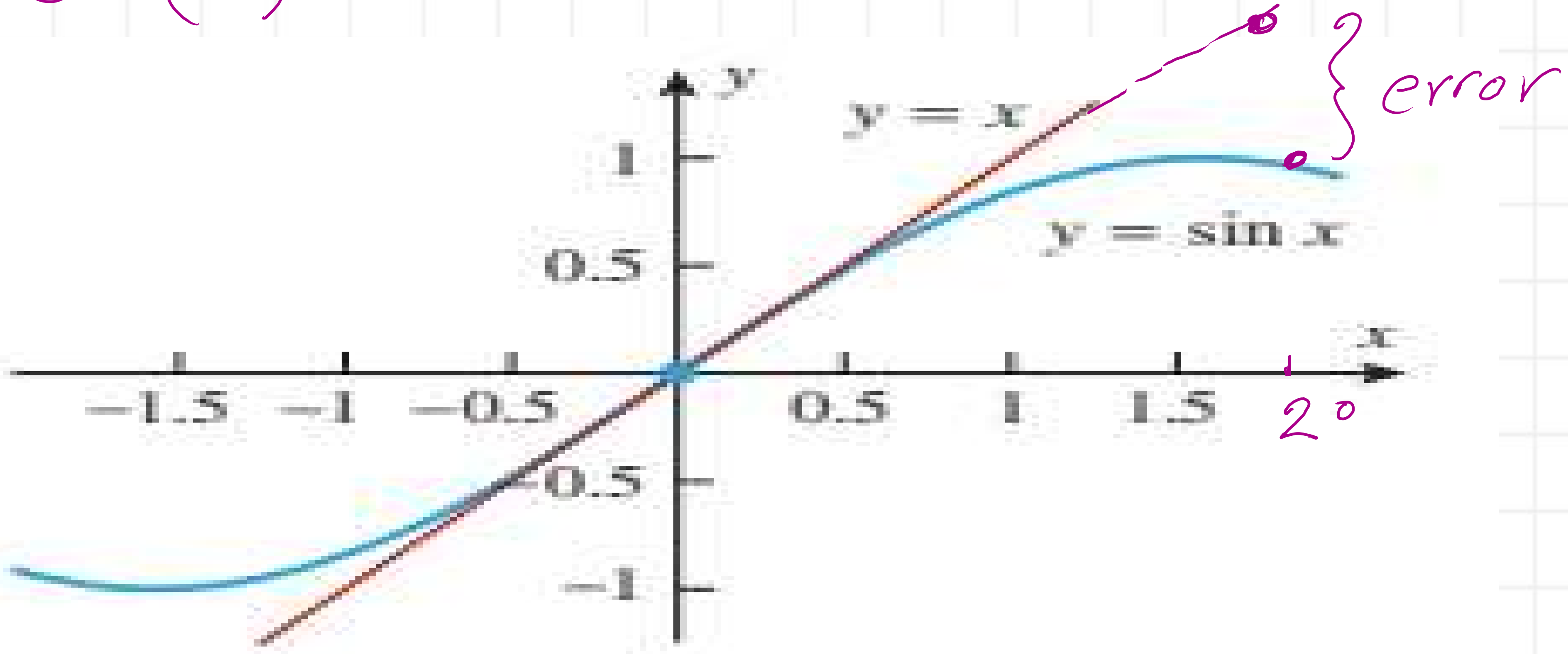
Solution: Part (b) $x_b = 2^\circ$

$L(x = 2^\circ) = f(2) = 2^\circ \rightarrow \text{rad} \Rightarrow \frac{2^\circ}{180} = \frac{\text{rad}(2)}{\pi}$

$\text{rad}(2) = \pi\frac{(2)}{180} = 0.0349066 \rightarrow L(2)$

$\pi = 3.14159$ $\quad f(2) = 0.0348995 \rightarrow \sin(2)$ actual

$f(x) = \sin(x)$



▲ Figure 2.9.3

Error absolute value

$$= |\sin x - (x)|$$

$$= |0.034899 - 0.0349066|$$

$$E = 7.6(10^{-6})$$

Example #2: Find L(x) of y=f(x)=X^2 at 2.50.

Solution:

$L(x) = f(a)+f'(a)(x_b - x_a)$

The nearest point which is known, is 2.

Steps: 1-Make a differentiation to the function $f'(x)=2.x$

2- substitute at x=2, and get f(2) *then* since $f(x)=X^2$ *then* $f(x=2)=(2)^2=4$

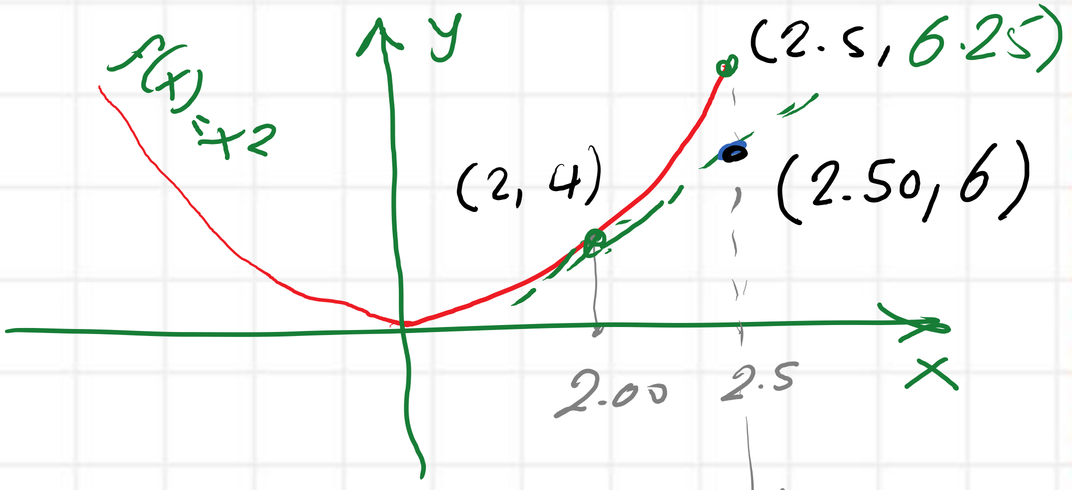
3- find the value of f'(2) as $f'(x=2)= 2*(2)=4$

4- back to linear approximation expression. $L(x) = f(a)+f'(a)(x_b - x_a)$, a=2.00, xb=2.50, f'(a)=f'(2) .

$L(x=2.50)=4+(4)*(2.50-2.00)=4+4*0.50=6$

Comparing with true value of 2.50^2=6.25

error =6.25-6.00 =0.25



Linear form

$L(x_b) = f(a)+f'(a)(x_b - x_a)$

$x_a = 2, \; x_b = 2.50$

$L(2.50) = 4 + 4(2.50-2)$
$= 6.00$

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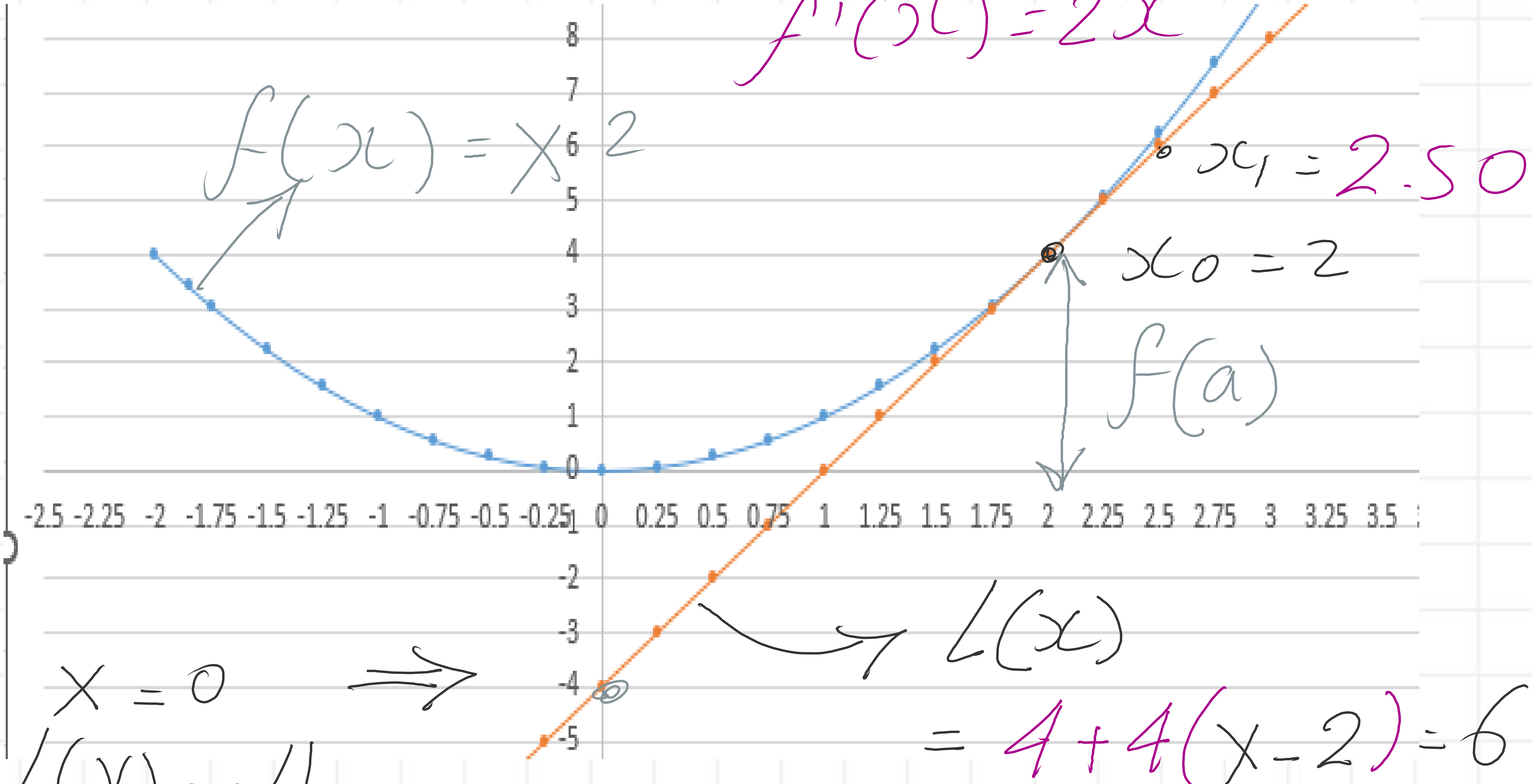
$L(x)$ Line

$f(x) = x^2$

$f'(x) = 2x$



$L(x) = -4$

Example # 3: Find $y=f(x)=\sqrt[3]{8.00}$ by approximation, hence use it for

$\sqrt[3]{8.20}$ & $\sqrt[3]{25.0}$.

Solution

$L(x) = f(a)+f'(a)(x_b - x_a)$

put x0=8

$f(x) = \sqrt[3]{x} = x^{+1/3}$

$x^n \Rightarrow n = 1/3$

$f'(x) = n x^{n-1}$

$n = 1/3$

$f'(x) = \frac{1}{3} x^{1/3 - 1}$

for $x_0 = 8$

$f(x_0=8) = \sqrt[3]{8} = 2$

$f'(x_0=8) = \frac{1}{3}(8)^{-2/3} = \frac{1}{3} \frac{1}{8^{2/3}} = \frac{1}{3}\left(\frac{1}{2^{3(\frac{2}{3})}}\right) = \frac{1}{12}$

$L(x) = 2 + \frac{1}{12}(x-8)$

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Example # 3: Find $y=f(x)=\sqrt[3]{8.00}$ by approximation, hence use it for

$\sqrt[3]{8.20}$ & $\sqrt[3]{25.0}$.

$f(8) = 2$

$f'(8) = 1/12$

Solution

$L(x) = f(a)+f'(a)(x_b - x_a)$

$L(x) = 2 + \frac{1}{12}(x - \underset{x_0}{8})$

for $x = 8.20 \rightarrow L(8.2) = 2 + \frac{1}{12}(8.2-8) =$

$L(8.2) = 2 + \frac{0.2}{12} = 2.0166 \Rightarrow 2.0165$ actual value

while for $L(25) = 2 + \frac{1}{12}(25-8)$

$L(25) = 3.4166 \rightarrow$ actual value $\rightarrow 2.4662$ actual

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Using the formula for x=8.20

$x_a = 8$
$x_b = 8.20$

$f(x=8)=2.00$, $f'(8.00)=\frac{1}{12}$

$L(x=8.20)=2+(\frac{1}{12})*(8.20-8.00)=2+(\frac{1}{12})*0.20= 2.0166$

$L(8.2) = f(8) + f'(8)(0.2)$

Comparing with true value of $\sqrt[3]{8.20} =2.0165$ Error =0

As for x b=25

$x_a = 8$
$x_b = 25$

$L(x)=f(x=8)+f'(8)*(25-8)$

$L(x=25) = 2 + \frac{1}{12}(17) = 3.41661$

Comparing with true value of $\sqrt[3]{25.0} = 2.92402$

Error =2.92402- 3.41661=-0.49259

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Example # 4 Use linear approximation method to find the value of f(x) $=\cos x$ at $\frac{\Pi}{3}$, *at starting point* at $x=\frac{\Pi}{4}$ *and estimate the relative error* %

$L(x_b) = f(a) + f'(a)(x_b - x_a)$

Solution

$f(x)= \cos(x)$

$f'(x)=d\cos(x)/dx = -\sin(x)$

$x_a = \pi/4 \quad x_b = \frac{\pi}{3}$

$f(x_0) = \cos\left(\frac{\pi}{4}\right) = +\frac{\sqrt{2}}{2}$

$f'(x_0) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$x_b - x_0 = \pi/12$

$L(x) = f(\underbrace{x=\tfrac{\Pi}{3}}_{}) = f(x=\tfrac{\Pi}{4}) + f'(x=\tfrac{\Pi}{4}) * (\tfrac{\Pi}{3} - \tfrac{\Pi}{4})$

$\overset{x_b}{}\quad \overset{x_a}{} \quad \overset{x_b - x_a}{}$

$f(x=\frac{\Pi}{3}) = \cos(x=\frac{\Pi}{4}) + (-\sin(x=\frac{\Pi}{4})) * (\frac{\pi}{12})$

=sqrt(2)/2+ (-sqrt(2)/2)*pi/12)=0.70711-0.18512=0.52199

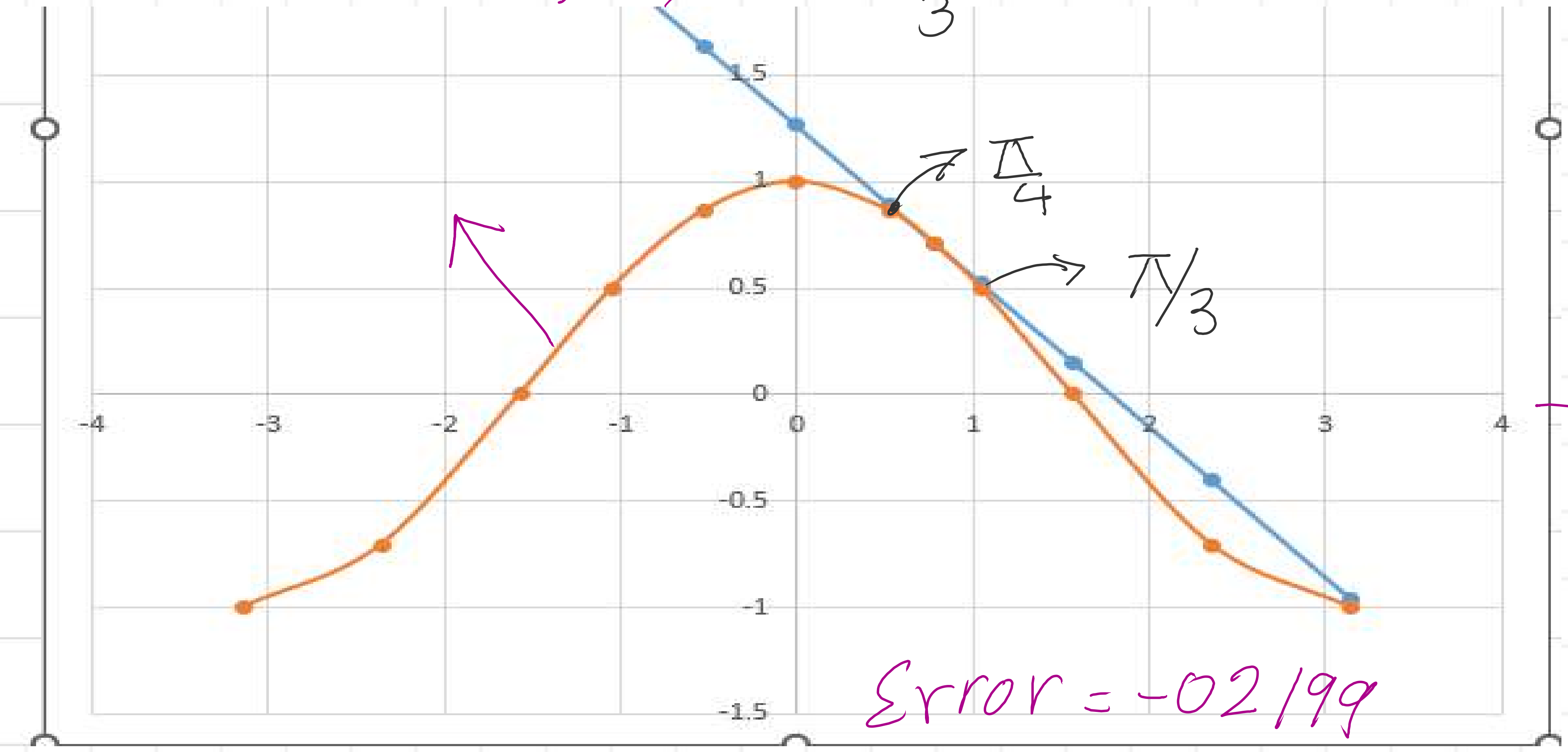
Actual value of $f(x=\frac{\Pi}{3})=0.50$

Relative error=abs((0.50-0.52199)/0.50)=+4.398 %=4.40%

$f(x) = \cos(x)$

$L(x) = +\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(\frac{\pi}{12}\right)$

$x = \frac{\pi}{3}$



Error = -02199