

Trapezoid (US)

Trapezium (UK)

$$\text{Area} = \frac{1}{2}(a+b) \times h$$

h = vertical height

8.41 Determine I_x and k_x

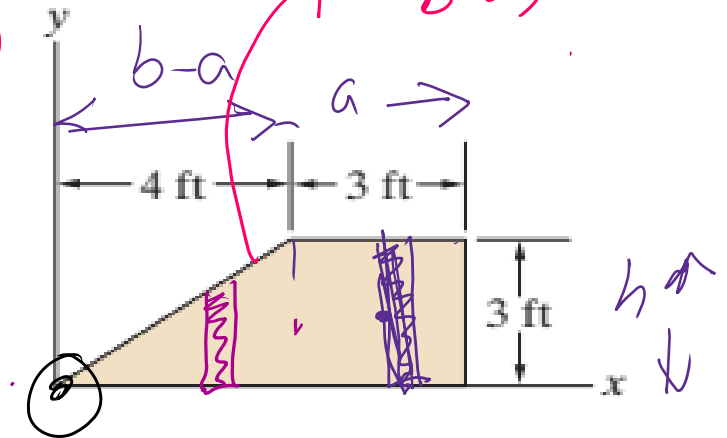
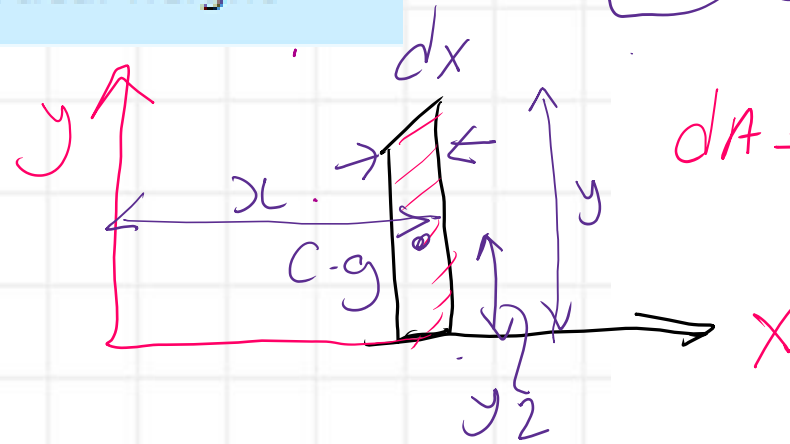
8.42 Determine J_O and k_O .

8.43 Determine I_{xy}

Left
Corner

$$y = \left(\frac{h}{b-a}\right)x$$

$$dA = (dx \cdot y)$$



Problems 8.41–8.43

8.43 I_{xy}

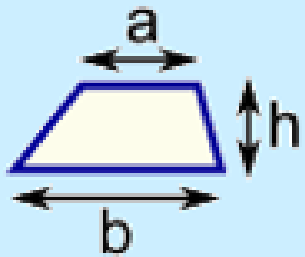
$$I_{xy} = \int_0^{b-a} dA (x) \left(\frac{y}{2}\right)$$

$$I_{xy_1} = \frac{1}{2} \int_0^{b-a} (dx) (y) (x) (y) = \frac{1}{2} \int_0^{b-a} \left(\frac{h}{b-a}\right)^2 x^2 (x) dx$$

$$I_{xy_1} = \frac{h^2}{2(b-a)^2} \int_0^{b-a} x^3 dx = \frac{h^2}{2(b-a)^2} \left. \frac{x^4}{4} \right|_0^{b-a} = \frac{h^2}{8(b-a)^2} (b-a)^4 = \frac{h^2}{8} (b-a)^2$$

$$b-a = 4', h = 3' \Rightarrow I_{xy_1} = \frac{3^2}{8} (4)^2 = 18 \text{ Ft}^4$$

Prepared by Eng. Maged Kamel.



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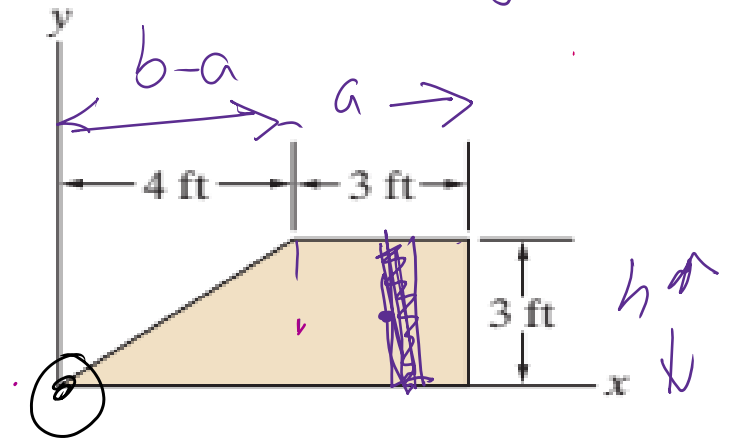
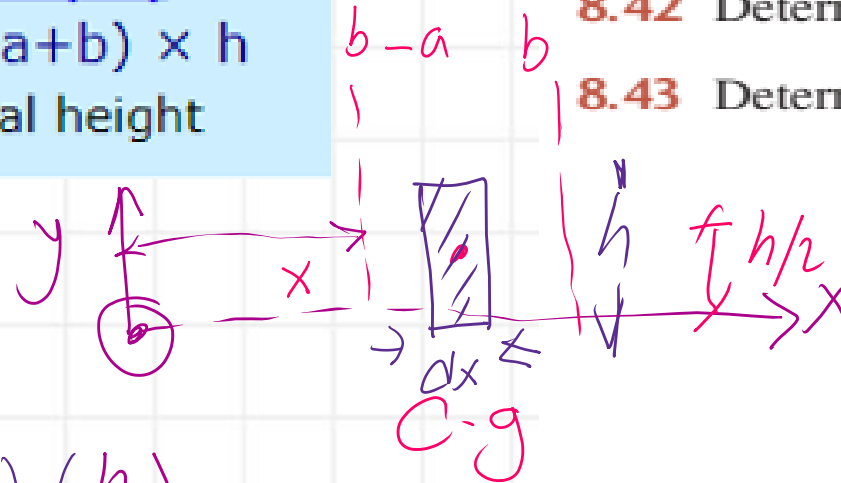
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$$\left. \begin{array}{l} a = 3' \\ b = 7' \end{array} \right\} h = 3'$$



Problems 8.41–8.43

8.43 (I_{xy})₂

$$(I_{xy})_2 = \int (dx \cdot h)(x) \left(\frac{h}{2}\right)$$

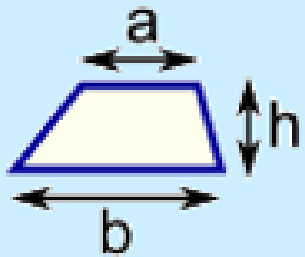
$$I_{xy}_2 = \frac{h^2}{2} \left[x^2 \right]_{b-a}^b \left(\frac{1}{2}\right) = \frac{h^2}{4} [b^2 - (b-a)^2]$$

$$(I_{xy})_2 = \frac{h^2}{4} [b^2 - b^2 + 2ba - a^2] = \frac{h^2}{4} (2ba - a^2)$$

$$(I_{xy})_2 = \frac{(3)^2}{4} (2(3)(7) - (3)^2) = \frac{9}{4} (42 - 9) = 74.25 \text{ Ft}^4$$

$$(I_{xy})_F = \frac{4}{4} 18 + 74.25 = 92.25 \text{ Ft}^4$$

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8.43

To derive a general Expression
 I_{xy} general expression

$$I_{xy} = \left(\frac{h^2}{8}\right)(b-a)^2 + \frac{h^2}{4}(2ba - a^2)$$

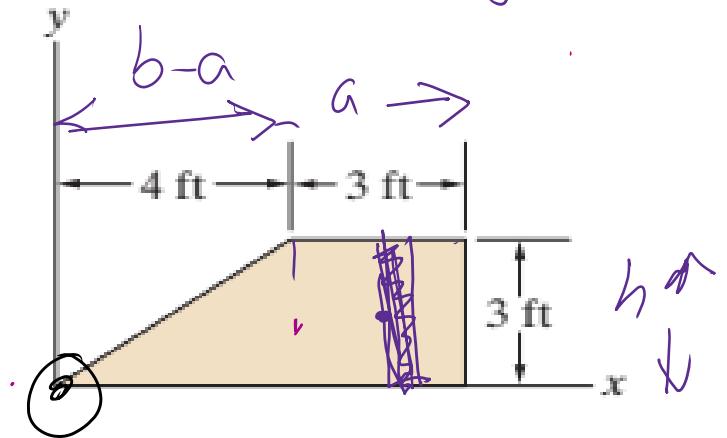
Simplify

$$I_{xy} = \frac{h^2}{8} [(b-a)^2 + 2(2ba - a^2)]$$

$$I_{xy} = \frac{h^2}{8} [(b-a)^2 - 2a^2 + 4ba]$$

$$I_{xy} = \frac{h^2}{8} [b^2 - 2ba + a^2 - 2a^2 + 4ba]$$

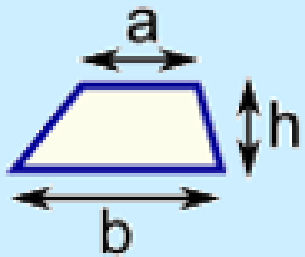
$$I_{xy} = \frac{h^2}{8} [(b+a)^2 - 2a^2]$$



Problems 8.41–8.43

$\leftarrow b \rightarrow$

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8.43 I_{xy} general expression

$$I_{xy} = \frac{h^2}{8} [(b+a)^2 - 2a^2]$$

$$I_{xy} = \frac{3^2}{8} [10^2 - 2(3^2)]$$

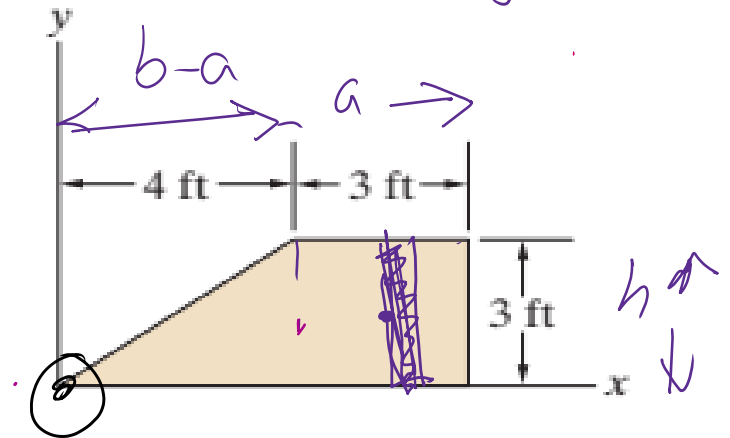
$$I_{xy} = \frac{9}{8} [100 - 18] = 92.25 \text{ Ft}^4$$

$$h = 3'$$

$$b + a = 10'$$

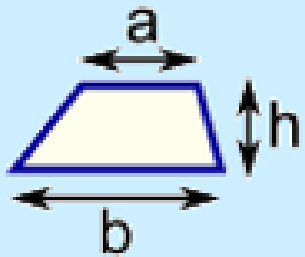
$$a = 3'$$

$$\left. \begin{array}{l} a = 3' \\ b = 7' \end{array} \right\} h = 3'$$



Problems 8.41–8.43

$\leftarrow b \rightarrow$



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8.49 I_{xy} at C.g

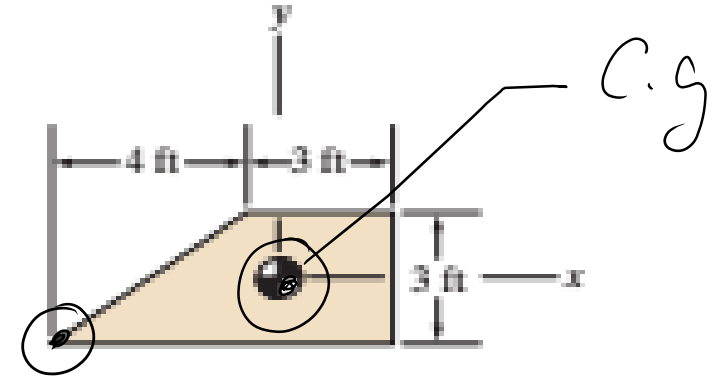
I_{xy} at left corner: 92.25 Ft^4

$$\bar{X} = 4.3667'$$

$$\bar{Y} = 1.3'$$

$$A = 15 \text{ Ft}^2$$

$$\begin{aligned} I_{xy \text{ C.g}} &= I_{xy} - A(\bar{X})(\bar{Y}) = 92.25 - (15)(4.3667)(1.3) \\ &= 92.25 - 85.1507 = 7.099 \text{ Ft}^4 \\ I_{xy \text{ C.g}} &\approx 7.10 \text{ Ft}^4 \end{aligned}$$



Problems 8.44–8.46

8.47 Determine I_x and k_x

8.48 Determine J_O and k_O .

8.49 Determine I_{xy} ←

