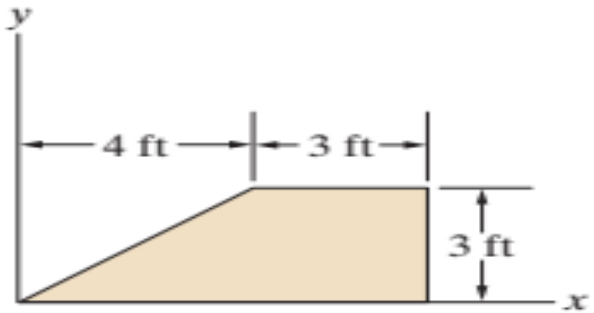
ENGINEERING MECHANICS
STATICS
Fifth Edition

Bedford | Fowler

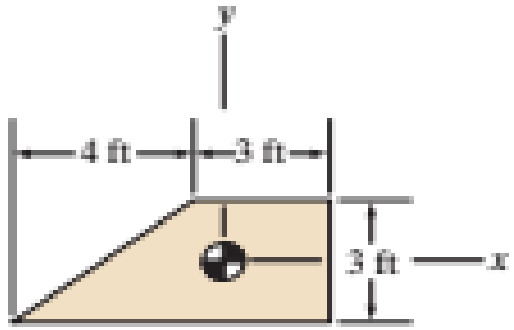
8.41 Determine I_x and k_x.

8.42 Determine J_O and k_O.

8.43 Determine I_{xy}.



Problems 8.41–8.43



Problems 8.44–8.46

8.47 Determine I_x and k_x.

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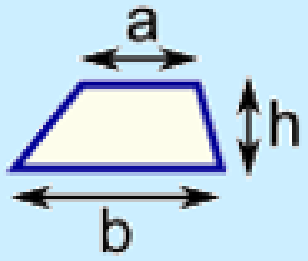
8.49 Determine I_{xy}.



Trapezoid (US)
Trapezium (UK)
Area = ½(a+b) × h
h = vertical height

8.41 Determine I_x and k_x

8.42 Determine J_O and k_O.

8.43 Determine I_{xy}

$I_x = 36\ Ft^4$ From previous post

For 8.42 $J_o = I_x + I_y$, we need to Find I_y

By integration $I_y = I_{y_1} + I_{y_2}$.

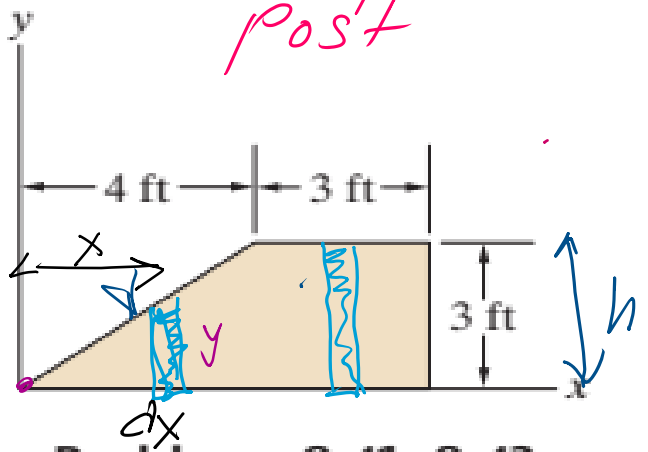
$dI_{y_1} = dA(x^2) = (dx)\,y\,(x^2)$

again $\Rightarrow y = \frac{3}{4}x$

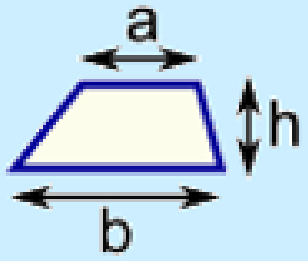
$dI_{y_1} = (\frac{3}{4}x)(x^2)\,dx = \frac{3}{4}x^3\,dx$

$$\int_{x=0}^{x=4} \frac{3}{4}x^3\,dx = \frac{3}{4}\left(x^4\right)_0^4\left(\frac{1}{4}\right) = \frac{3}{16}(4)^4 = \frac{3}{16}(16)(16) = 48\ Ft^4$$

Quick Estimate: $\frac{b^3h}{4} = \frac{4^3}{4}(3) = 48\ Ft^4$



Problems 8.41–8.43



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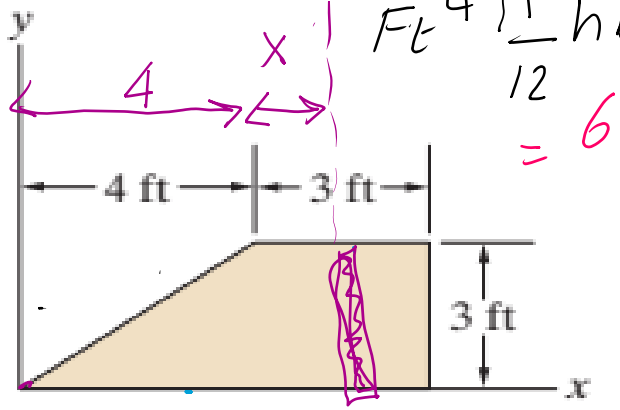
I_{y_2} Quick

$bh(5-5)^2$ × c.g

$9(5-5)^2 = 272.25$ Ft^4 | $\frac{1}{12}hb^3$



Problems 8.41–8.43

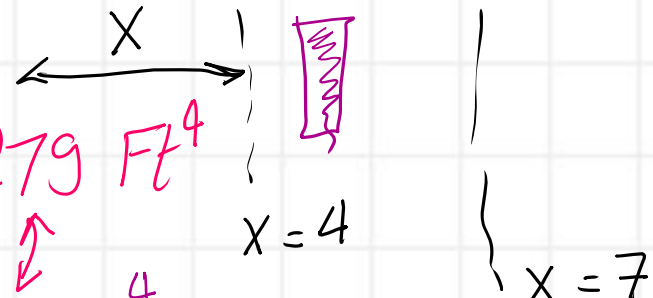
$= 6.75$ FT^4

For 8.42 $I_{y_2} = \int_0^3 dx\,(h)(4+x)^2$

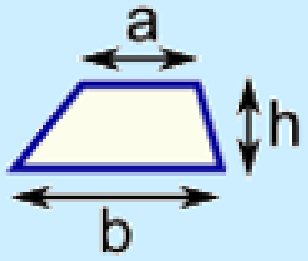
$I_{y_2} = h\int(16 + 8x + x^2)\,dx$

$I_{y_2} = h\left[16x\Big|_0^3 + \frac{8x^2}{2}\Big|_0^3 + \frac{x^3}{3}\Big|_0^3\right]$

$3\left[48 + (4)(9) + \frac{(27)}{3}\right] = 3\left[48 + 36 + 9\right] = 279\ Ft^4$



or $I_{y_2} = \int_4^7 dx\,h(x)^2 = h\left[\frac{x^3}{3}\right]_4^7 = \frac{3}{3}\left[7^3 - 4^3\right] = 279\ Ft^4$



Trapezoid (US)
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Area = ½(a+b) × h
h = vertical height

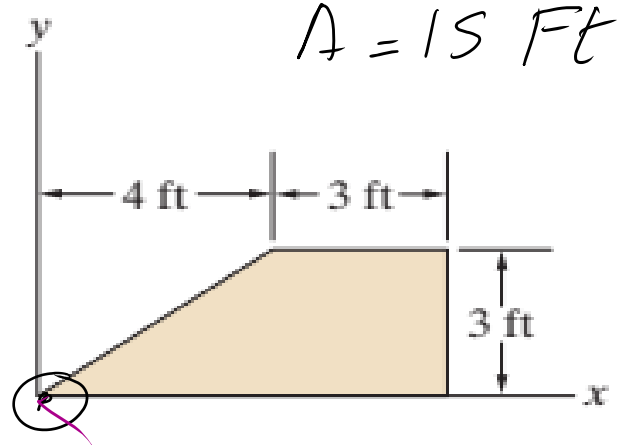
8.41 Determine I_x and k_x.
8.42 Determine J_O and k_O. → Left Corner
8.43 Determine I_{xy}.

$I_{x\,Final} = 36\ Ft^4$

$A = 15\ Ft^2$

$I_{y\,Final} = I_{y1} + I_{y2}$

$= 48 + 279 = 327\ Ft^4$

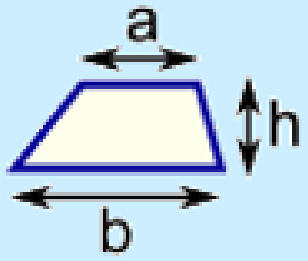


Problems 8.41–8.43

$J_O = I_x + I_y = 36 + [327] = 363\ Ft^4$

$K_O = \sqrt{\frac{J_O}{A}} = \sqrt{\frac{363}{15}} = 2.218\ Ft$

J_O



Trapezoid (US)

Trapezium (UK)

Area = ½(a+b) × h

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8.41 Determine I_x and k_x.

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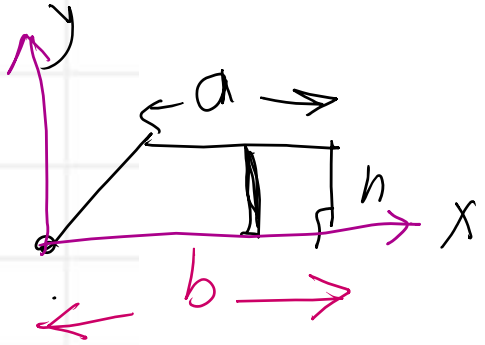
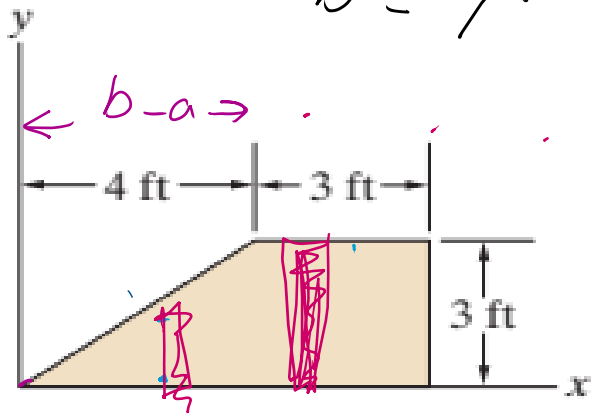
$a = 3'$

$b - a = 7 - 3 = 4$

$b = 7'$





Problems 8.41–8.43

Iy general Expression

$$I_{y1} = \int_0^{b-a} (dx)(y)(x^2)$$

$$= \int_0^{b-a} \left(\frac{h}{b-a}\right) x^3\,dx = \frac{h}{(b-a)}\,\frac{x^4}{4} = \frac{h}{4}(b-a)^3$$

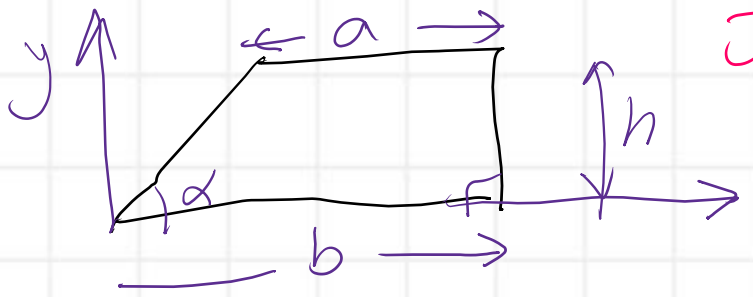
$$y = mx = \left(\frac{h}{(b-a)}\right)x$$

$$I_{y2} = \int_{b-a}^{b} (dx)\,h\,(x^2) = \frac{h}{3}\left[x^3\right]_{b-a}^{b} = \frac{h}{3}\left[(b)^3 - (b-a)^3\right]$$

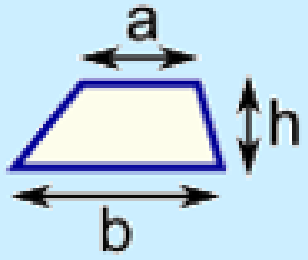
$$I_y = \frac{h}{4}(b-a)^3 + \frac{h}{3}\left[b^3 - (b-a)^3\right] \rightarrow \frac{3}{4}(4)^3 + \frac{3}{3}\left(7^3 - (4)^3\right) = 48 + 279$$

$327\ Ft^4$

check



J_o by general Expression



Trapezoid (US)
Trapezium (UK)
Area = ½(a+b) × h
h = vertical height

$$J_o = I_x + I_y = \underbrace{\frac{h^3}{12}(3a+b)}_{I_x} + \underbrace{\frac{h}{4}(b-a)^3 + \frac{h}{3}\left[b^3-(b-a)^3\right]}_{I_y}$$

$$J_o = \frac{h^3}{12}(3a+b) + \frac{h}{12}\left(3(b-a)^3 + 4b^3 - 4(b-a)^3\right]$$

$$J_o = \underbrace{\frac{h^3}{12}(3a+b)}_{I_x} + \underbrace{\frac{h}{12}\left[4b^3-(b-a)^3\right]}_{I_y}$$

8.41 Determine I_x and k_x.
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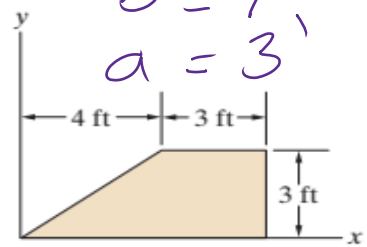
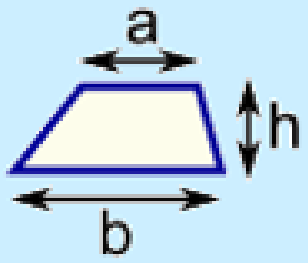
$h = 3'$
$b = 7'$
$a = 3'$



Problems 8.41–8.43

$$J_o = \frac{3^3}{12}(9+7) + \frac{3}{12}\left[4(7^3-(7-3)^3\right] = 36+327$$

$$J_o = 363\ Ft^4$$



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Area = ½(a+b) × h

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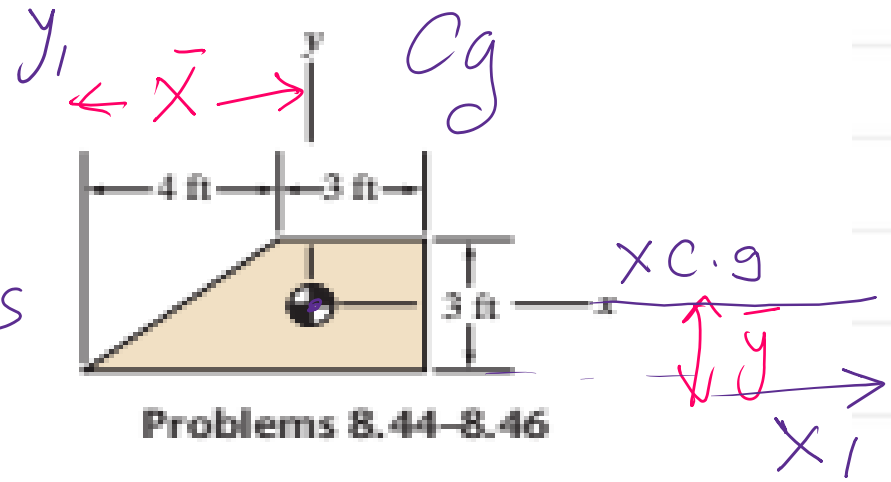
J_o and k_o at the c.g



Problems 8.44–8.46

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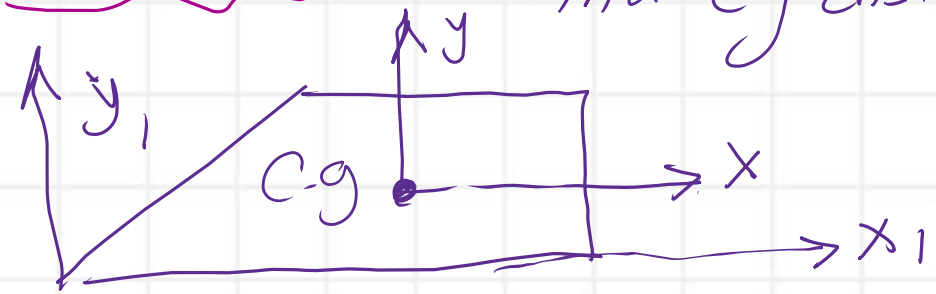
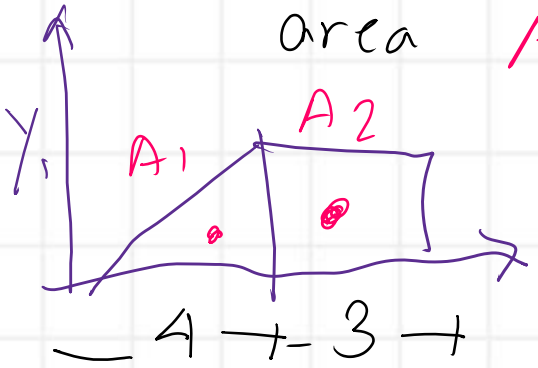
Example 8-48

Find cg distance to y_1-axis





area $A_1 = \frac{1}{2}(4)(3) = 6\ Ft^2$

$A_2 = 3(3) = 9\ Ft^2$

$\bar{X}_1 = \frac{A_1 X_1 + A_2 X_2}{\Sigma A} = \frac{6\left(\frac{2}{3}\right)(4) + 9(7 - 1.50)}{15}$

c.g

$\bar{x} = \frac{1}{15}(16 + 49.5) = 4.366'$

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From 8.42

I_y ∂ external axis

$= 327\ Ft^4$

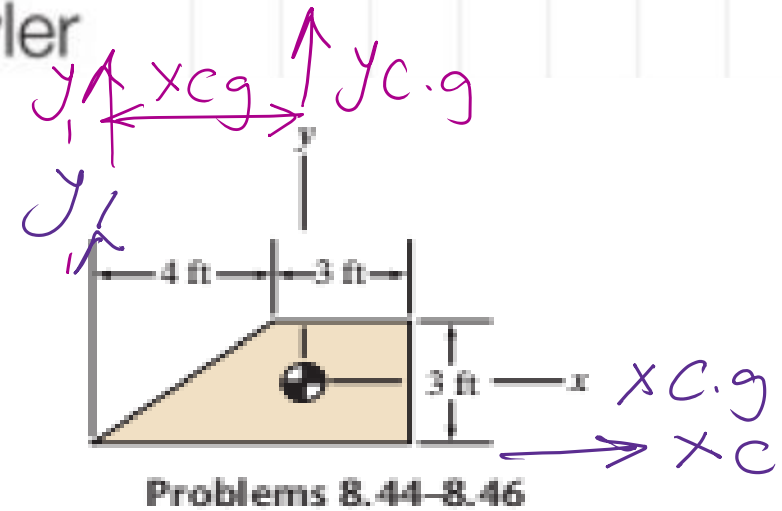
To get $I_{y\,\partial C.g}$ deduct $A(\bar{X}^2_{C.g})$

$A = \frac{(3+7)(3)}{2} = 15\ Ft^2$

$\bar{X}^2_{C.g} = (4.3667)^2 = 19.068\ Ft^2$

$I_{yC.g} = 327 - (15)(19.068)$

$= 40.98\ Ft^2$



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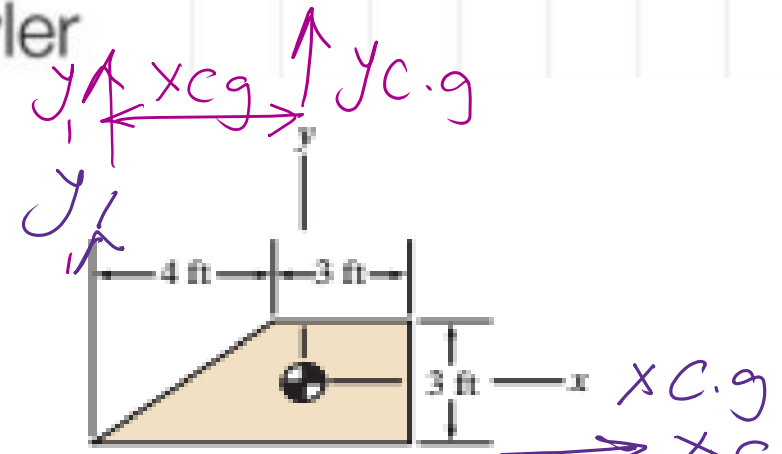
$A = 15\ Ft^2$

$J_{O_{C.g}} = I_{x_{cg}} + I_{y_{C.g}}$

$= 10.65 + 40.98 = 51.63\ Ft^4$

$k_O = \sqrt{\frac{I_{O_{cg}}}{A}} = \sqrt{\frac{51.63}{15}} = 1.855$



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