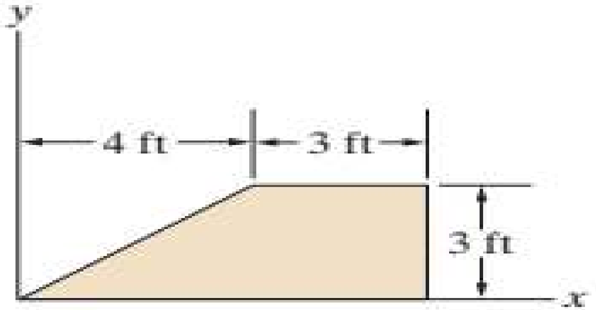
ENGINEERING MECHANICS
STATICS
Fifth Edition

Bedford | Fowler

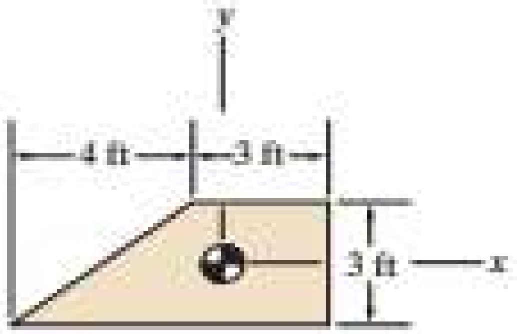
8.41 Determine I_x and k_x.

8.42 Determine J_O and k_O.

8.43 Determine I_{xy}.



Problems 8.41–8.43



Problems 8.44–8.46

8.47 Determine I_x and k_x.

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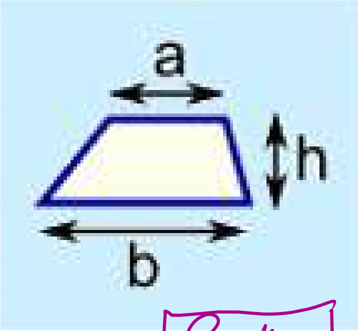
Prepared by Eng.Maged Kamel.



Trapezoid (US)

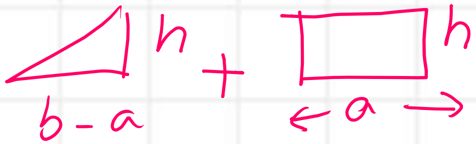
Trapezium (UK)

Area = ½(a+b) × h

h = vertical height

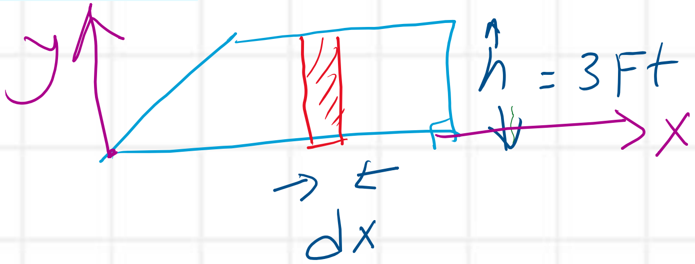
$\boxed{8.41}$

First part I_x and k_x.

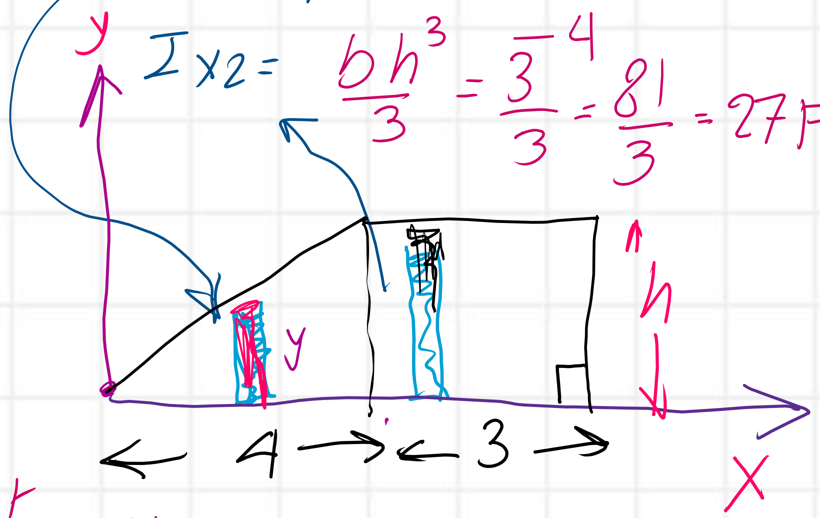


$a = 3$, $b - a = 4$, $b = 7$

Quick estimate I_{x_1}: $\frac{bh^3}{12} = \frac{4(3)^3}{12} = 9\ Ft^4$



$I_{x2} = \frac{bh^3}{3} = \frac{3^4}{3} = \frac{81}{3} = 27\ Ft^4$



A) Use a vertical strip (dx)(3).

B) Integerate about x-axis

$dI_x = dx\,(y)^3/3$

y is y ⇒ I_{x1} part

y = h = 3 → I_{x2} Part

$y = mx = \frac{3}{4}x$

$\int dI_x = I_{x_1} + I_{x2}$ $\quad I_{x_1} = \int^{4} \frac{y^3}{3}\,dx = \frac{1}{3}\int_{x=0}^{x=4} \left(\frac{3}{4}x\right)^3 dx$

$I_{x_1} = \frac{1}{3}\int_{x=0}^{x=4} \left(\frac{27}{64}\right) x^3\,dx = \frac{27}{3(64)} \left[\frac{x^4}{4}\right]_0^4 = \frac{27}{12(64)}\left[4\right]^4 = 9\ Ft^4$

matches quick Estimate

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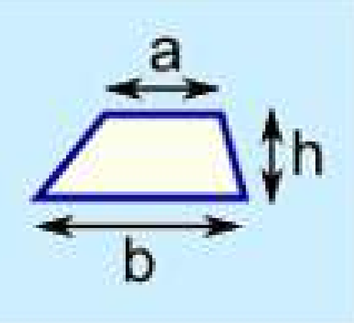

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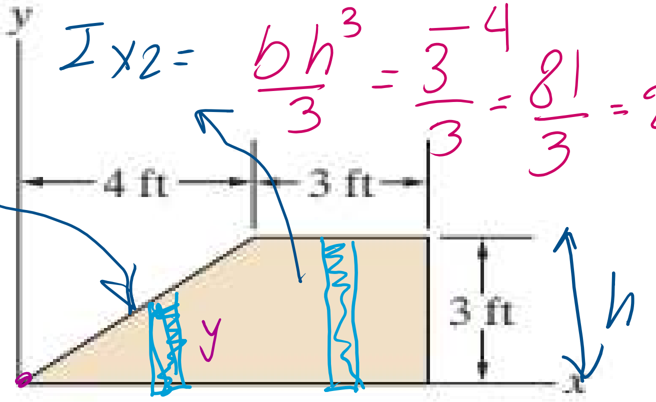
I_{x_1}

$\frac{bh^3}{12} = \frac{4(3)^3}{12} = 9\ Ft^4$

$I_{x_2} = \frac{bh^3}{3} = \frac{3^4}{3} = \frac{81}{3} = 27\ Ft^4$



Problems 8.41–8.43

I_{x_2} by integration $= \int_4^7 \frac{h^3}{3}\,dx = \frac{h^3}{3}\left[x\right]_4^7$

$I_{x_2} = \frac{(3)^3}{3}\left[7-4\right] = 27\ Ft^4 \rightarrow$ Same

$\sum I_x = 9 + 27 = 36\ Ft^4$

$\boxed{8.41}$ $k_x^2 = \frac{I_x}{A}$ → Area $= \frac{(3+7)}{2}(3) = 15\ Ft^2$

$k_x = \sqrt{\frac{36}{15}} = 1.55\ Ft$

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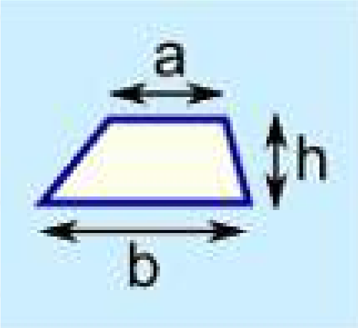

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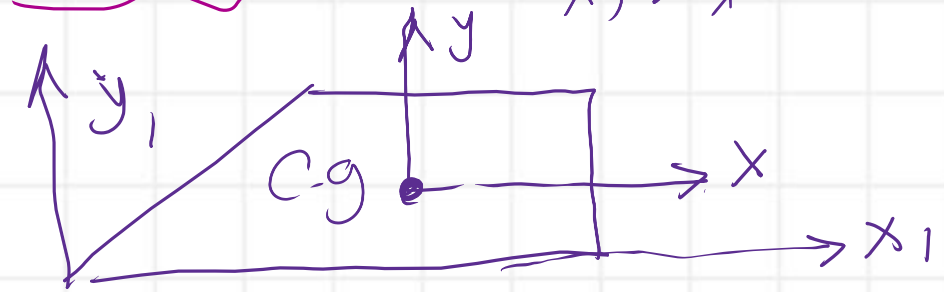
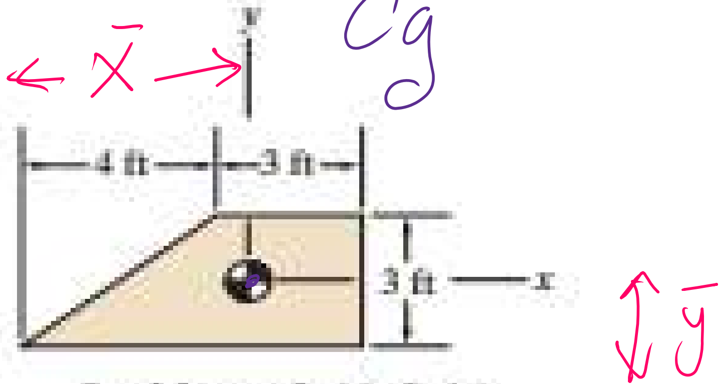
Example# 8-47

I_x, K_x about the Cg.





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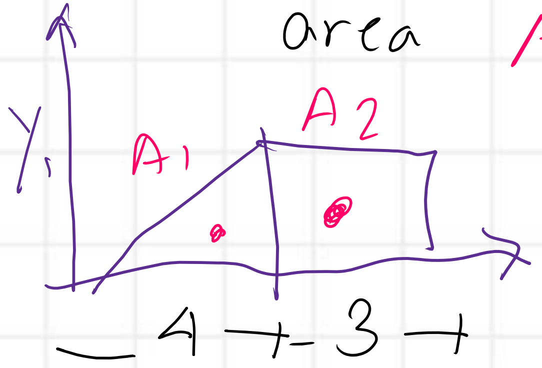


$h = 3$

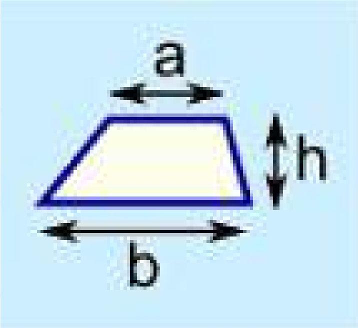
$A_1 = \frac{1}{2}(4)(3) = 6\ Ft^2$

$A_2 = 3(3) = 9\ Ft^2$

$$\bar{X}_1 = \frac{A_1 X_1 + A_2 X_2}{\Sigma A} = \frac{6\left(\frac{2}{3}\right)(4) + 9(7-1.50)}{15}$$

c.g

$$\bar{x} = \frac{1}{15}(16 + 49.5) = 4.3667$$



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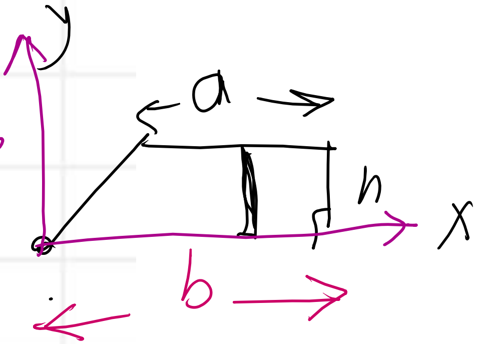
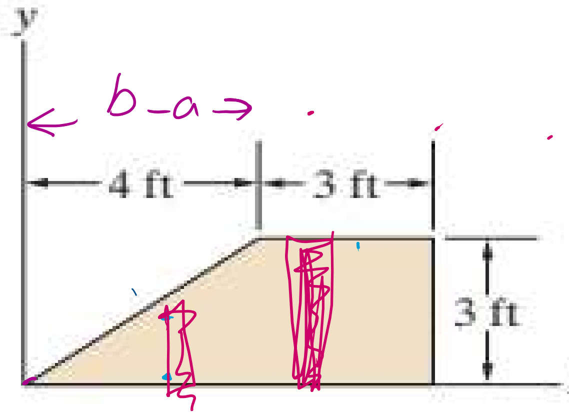
To derive a general Expression For I_x





Problems 8.41–8.43

$$I_{x_2} = \int_{b-a}^{b} dx\,\frac{h^3}{3} = \frac{h^3}{3}\left[x\right]_{b-a}^{b}$$

$$I_{x_2} = \frac{h^3}{3}(b-(b-a)) = \frac{a h^3}{3}$$

$$y = mx = \frac{h}{b-a}(x)$$

$$I_{x_1} = \int_{0}^{b-a} dx\left(\frac{h^3}{(b-a)^3}\right)\frac{x^3}{3} = \frac{h^3}{3(b-a)^3}\left[\frac{x^4}{4}\right]_{0}^{b-a} = \frac{h^3}{12(b-a)^3}(b-a)^4$$

$$I_x = \frac{a h^3}{3} + \frac{h^3}{12}(b-a) = \frac{h^3(4a+b-a)}{12} = \frac{h^3}{12}(3a+b)$$

For Example

$$I_x = \frac{3^3}{12}(9+7) = 36\ \text{Ft}^4$$

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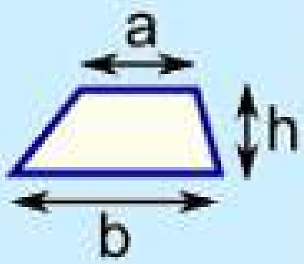

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Area = ½(a+b) × h

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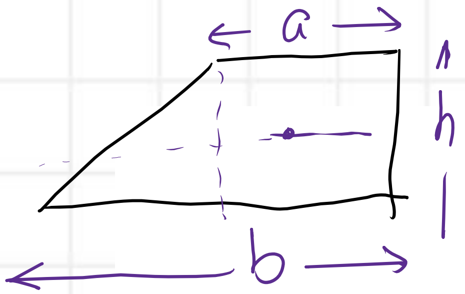

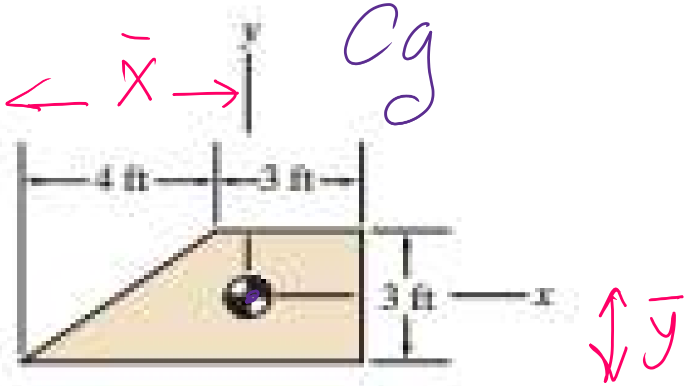

Problems 8.44–8.46

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I_x, k_x about the Cg.

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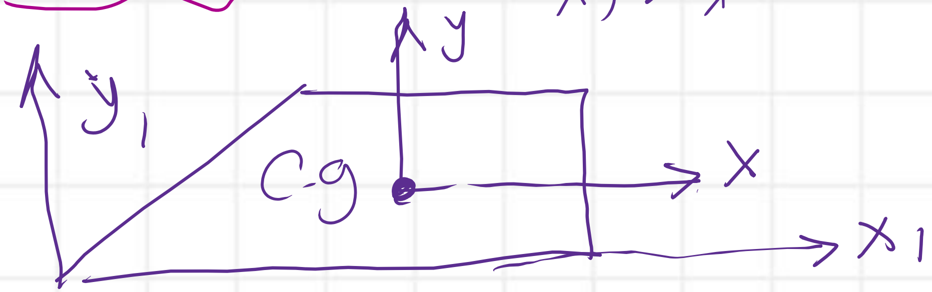
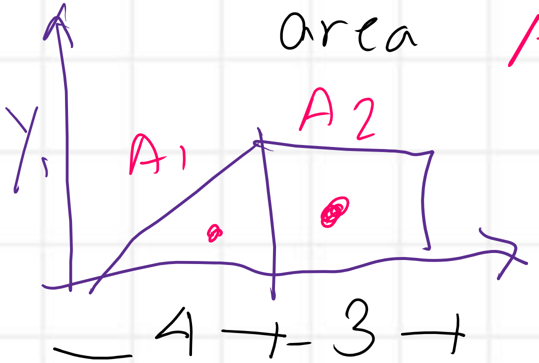




$h = 3$

area

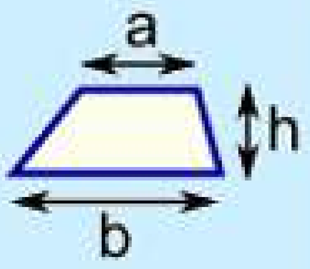
$A_1 = \frac{1}{2}(4)(3) = 6\ Ft^2$

$A_2 = 3(3) = 9\ Ft^2$

For $\bar{y} = A_1y_1 + A_2y_2 / \Sigma A$

$y_1 = 1'$, $y_2 = 1.5'$

$\bar{y} = \dfrac{6(1) + 9(1.50)}{15} = 1.3'$



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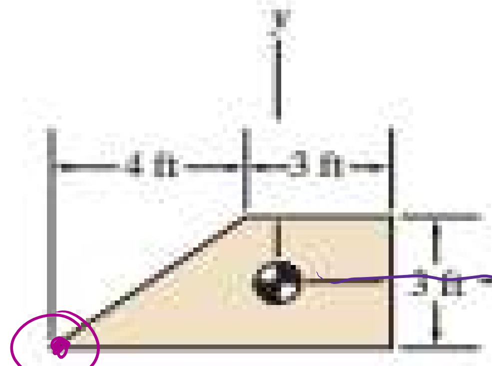

$I_{x\,cg}$

I_x

Problems 8.44–8.46

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From 8.41

I_x at Left Corner $= 36\ Ft^4$

$A = 15\ Ft^2$

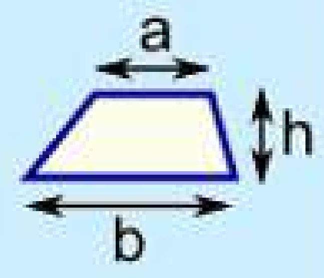
$\bar{y} = 1.3'$

$$I_{x\,c.g} = I_x - A \cdot \bar{y}_{c.g}^{\,2}$$

$$I_{x\,c.g} = 36 - (15)(1.3)^2$$

$$= 36 - 25.35 = 10.65\ Ft^4$$

$$K_{x\,cg} = \sqrt{\frac{I_{x\,c.g}}{A}} = \sqrt{\frac{10.65}{15}} = 0.843'$$



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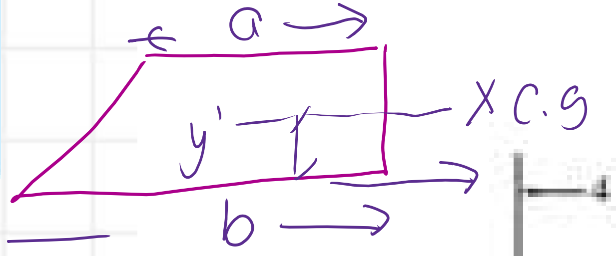
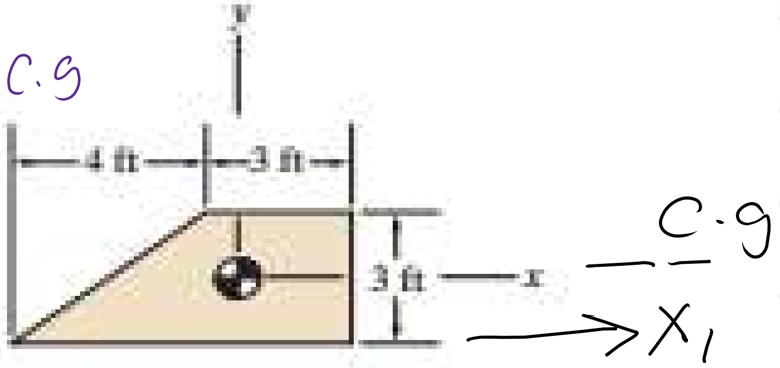
General expression





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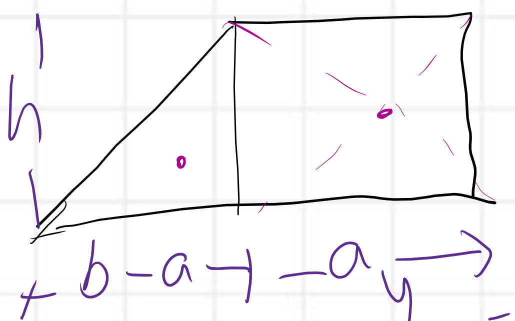
$$I_x = \frac{h^3}{12}(3a+b)$$

Deduct $A\bar{y}^2$



$$\sum Ay = \frac{1}{2}(b-a)h\cdot\frac{h}{3} + (ah)(h/2)$$

$$y_{c.g} = \frac{h^2}{6}\left[\frac{b-a+3a}{(a+b)h/2}\right] = \frac{h^2}{6}\frac{(b+2a)}{(a+b)}\frac{2}{h} = \frac{h}{3}\frac{(b+2a)}{a+b}$$

$$A\,y_{c.g}^2 = \frac{1}{2}(a+b)h\left[\frac{h^2}{9}\frac{(b+2a)^2}{(a+b)^2}\right] = \frac{h^3}{18}(b+2a)^2/(a+b)$$

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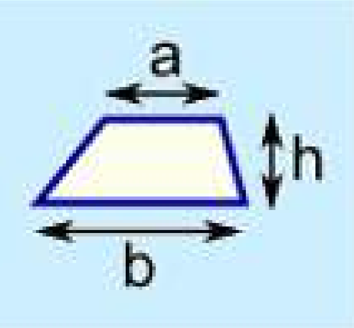

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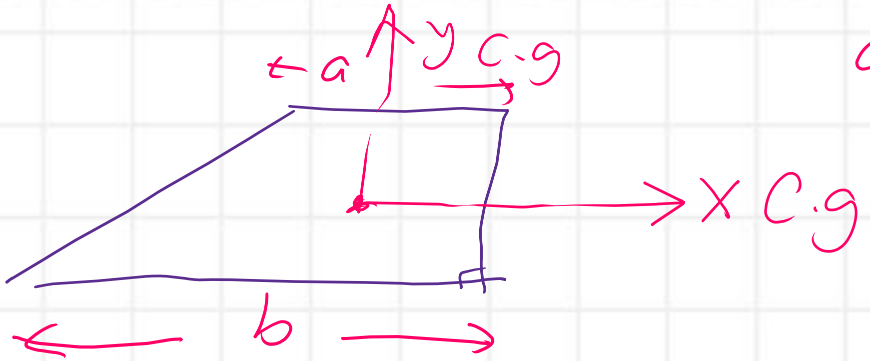

$a = 3'$

$h = 3'$

$b = 7'$

$$I_{xcg} = \frac{h^3}{12}(3a+b) - \frac{h^3}{18}\frac{(b+2a)^2}{(a+b)}$$

$$I_{xcg} = \frac{h^3\left[(1.5a+1.5b)(3a+b) - (b+2a)^2\right]}{18(a+b)}$$

$$I_{xc.g} = \frac{h^3(1)}{18(a+b)}\left[4.5a^2 + 6ab + 1.5b^2 - b^2 - 4ab + 4a^2\right]$$

$$I_{xc.g} = \frac{h^3}{18(a+b)}\left[0.5a^2 + 2ab + 0.50b^2\right) = \frac{h^3}{36}\frac{(a^2 + 4ab + b^2)}{(a+b)}$$

Check $\Rightarrow$ $$I_{xcg} = \frac{(3)^3}{36}\left(\frac{3^2 + 4(3)(7) + 7^2}{(10)}\right) = 10.65\ Ft^4$$