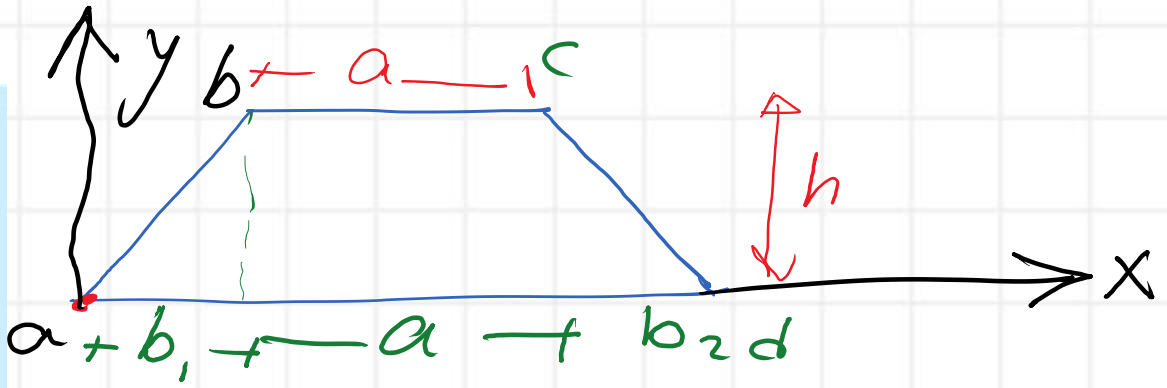
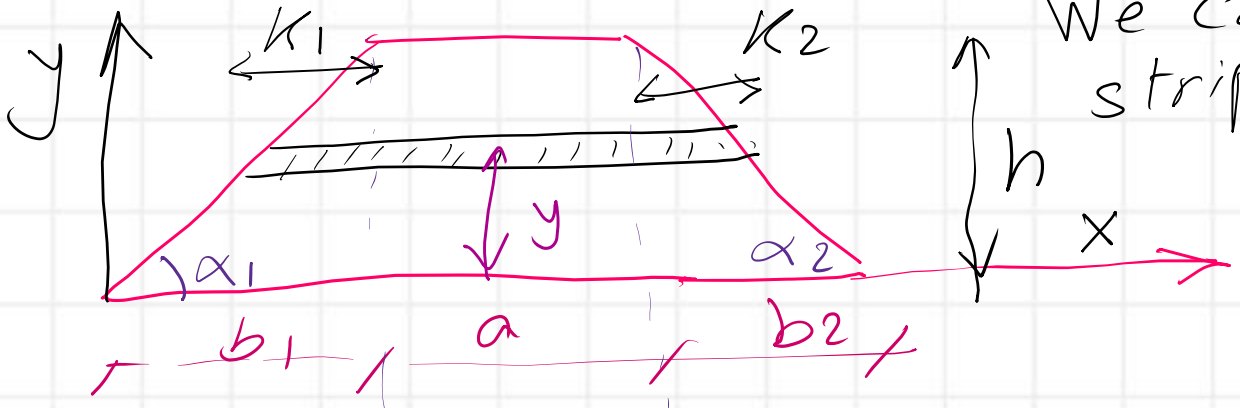
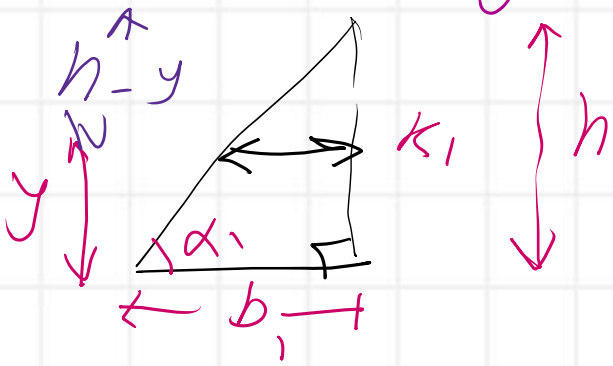
Moment of Inertia I_x for Trapezoid - option - 2

For Trapezoid a b c d





We can consider a horizontal strip of length $= k_1 + a + k_2$, with y distance from x-axis's

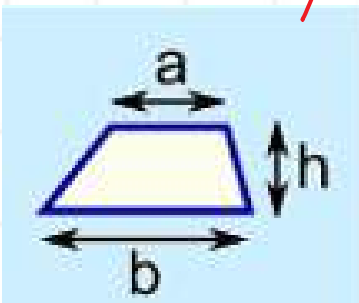


① Estimate k_1 as $\Rightarrow \frac{k_1}{b_1} = \frac{h-y}{h}$

$$k_1 = \frac{b_1}{h}(h-y)$$

Moment of Inertia I_x for Trapezoid – option-2

For Trapezoid a b c d

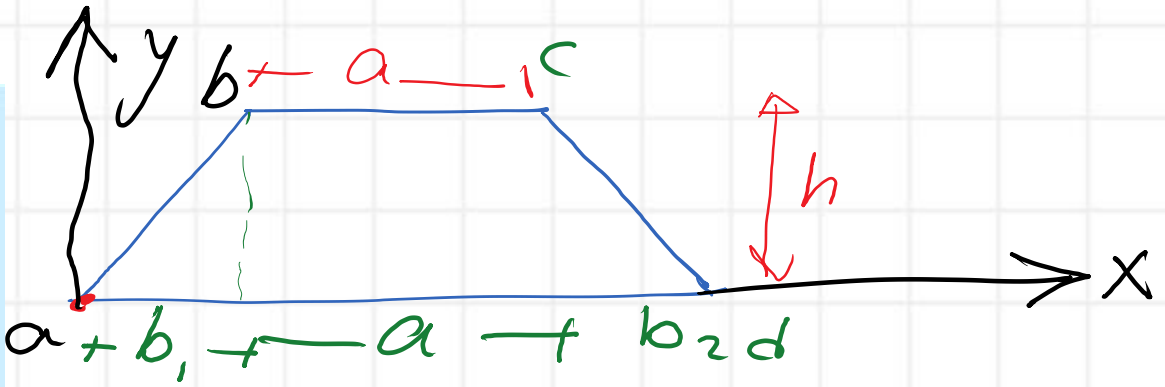


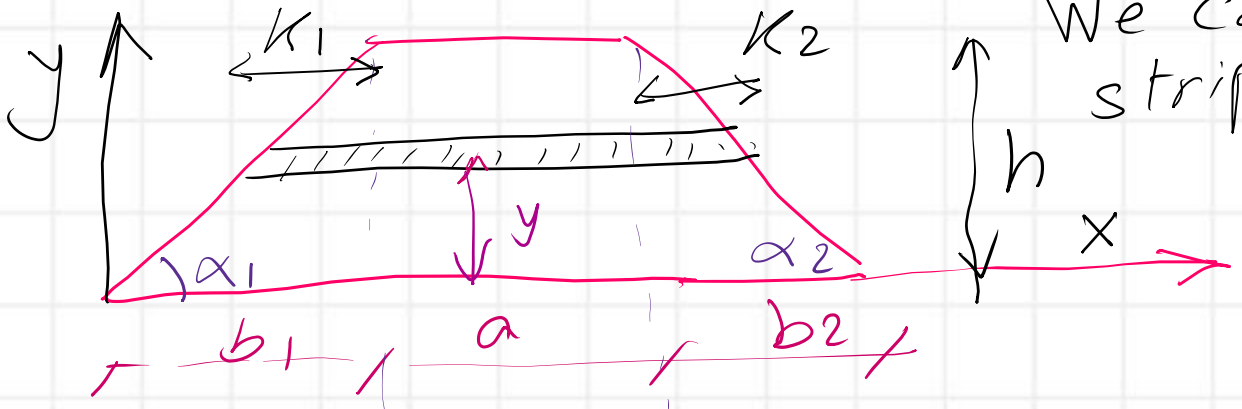
Trapezoid (US)

Trapezium (UK)

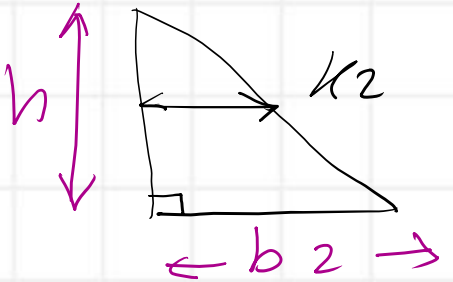
Area = ½(a+b) × h

h = vertical height





We can consider a horizontal strip of length $= k_1 + a + k_2$ apart by a distance y from x-axis

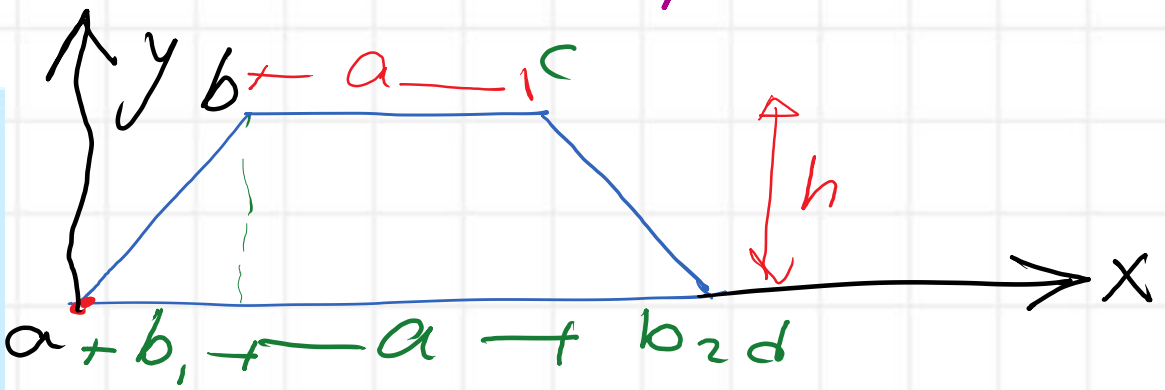


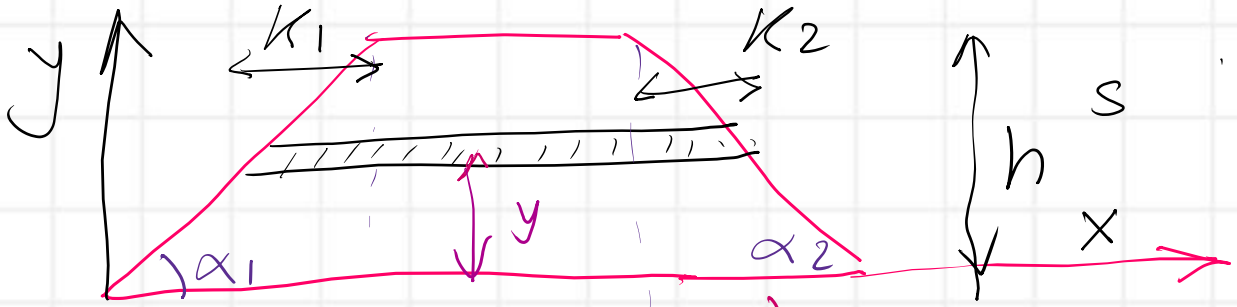
② Estimate k_2 as $\Rightarrow \frac{k_2}{b_2} = \frac{h-y}{h}$

$$k_2 = \frac{(h-y)}{h} \cdot b_2$$

Moment of Inertia I_x for Trapezoid - option - 2

For Trapezoid a b c d





Integrate from $y = 0$ to $y = h$

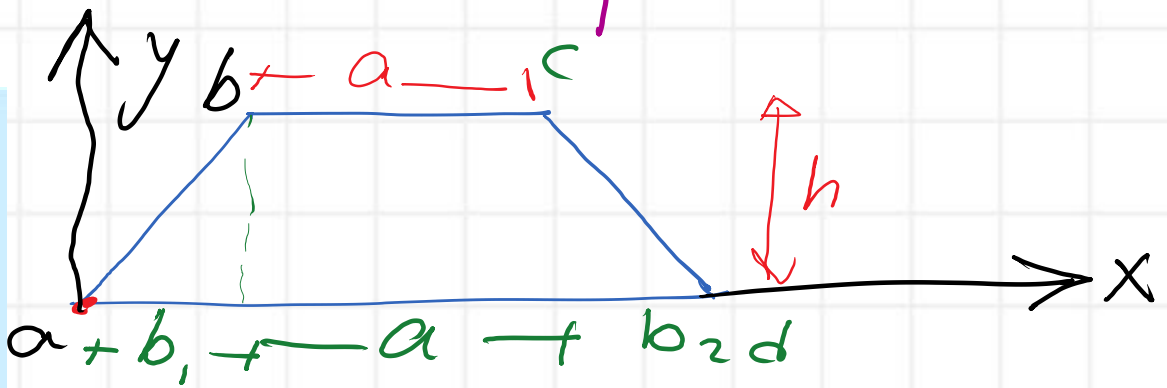
$$dI_x = (a + K_1 + K_2)\, dy\, (y^2)$$

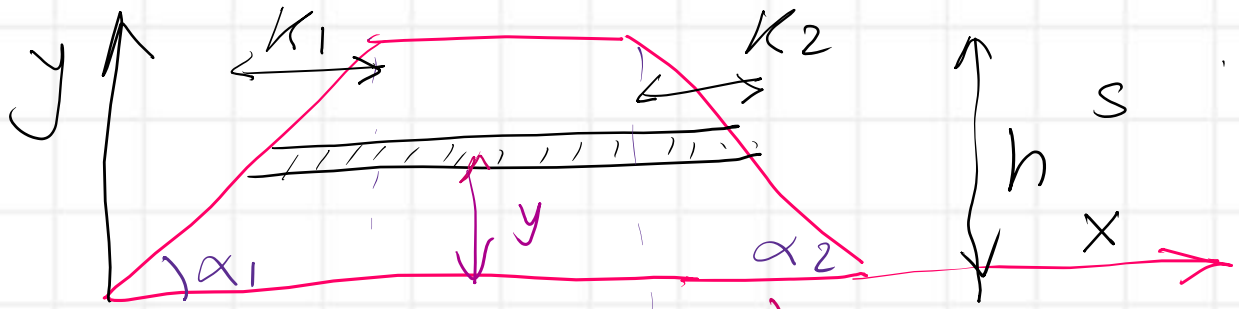
$$I_x = \int_0^h y^2 dy \left(a + \frac{b_1}{h}(h-y) + \frac{b_2}{h}(h-y)\right)$$

$$I_x = \int_0^h \left(\frac{b_1}{h} + \frac{b_2}{h}\right)(h-y)\, y^2 dy + \int_0^h a\, y^2 dy$$

Moment of Inertia I_x for Trapezoid option-2

For Trapezoid abcd





Integrate from $y = 0$ to $y = h$

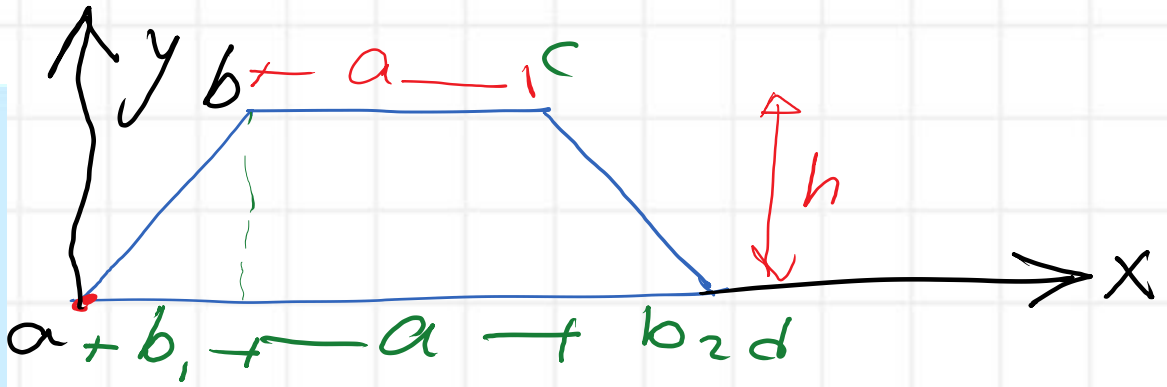
$$I_x = \int_0^h \left(\frac{b_1}{h} + \frac{b_2}{h}\right)(h - y)\,y^2\,dy + \int_0^h a\,y^2\,dy$$

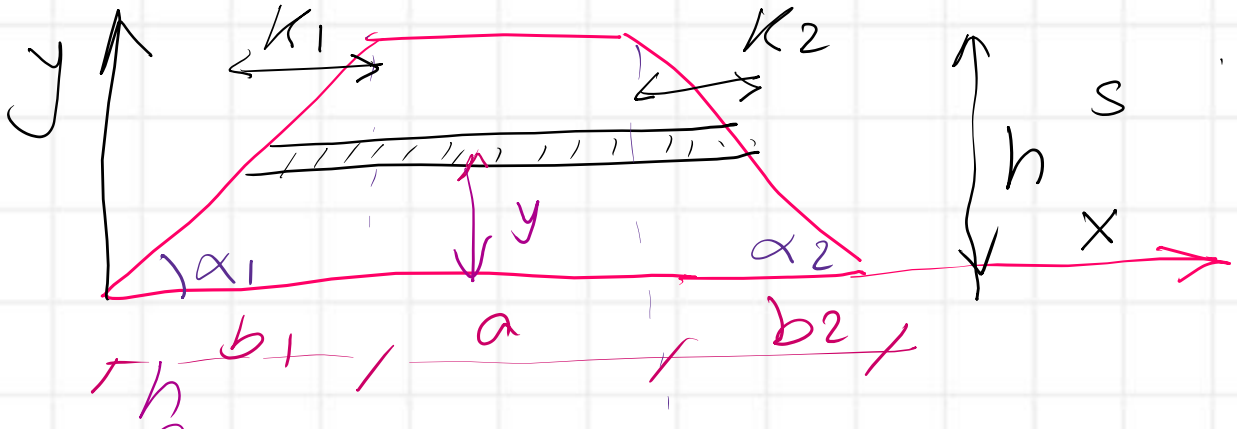
$$I_{x_1} = \left(\frac{b_1 + b_2}{h}\right)\int\left(hy^2\,dy - y^3\,dy\right)$$

$$I_{x_1} = \left(\frac{b_1 + b_2}{h}\right)\left(h\,\frac{y^3}{3}\Big|_0^h - \frac{y^4}{4}\Big|_0^h\right) = \frac{b_1 + b_2}{h}\left(\frac{h^4}{3} - \frac{h^4}{4}\right) = \frac{(b_1 + b_2)\,h^3}{12}$$

Moment of Inertia I_x for Trapezoid - option-2

For Trapezoid a b c d





Integrate from $y = 0$ to $y = h$

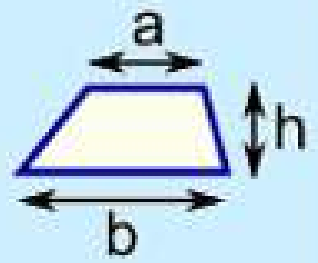
$$I_x = \int_0^h \left(\frac{b_1}{h} + \frac{b_2}{h}\right)(h-y)\,y^2\,dy + \int_0^h a\,y^2\,dy$$

$$I_x = \int a\,y^2\,dy = a\,\frac{y^3}{3}\Big|_0^h = \frac{a\,h^3}{3} = \frac{4\,a\,h^3}{12}$$

Prepared by Eng.Maged Kamel.

Moment of Inertia I_x for Trapezoid option-2

For Trapezoid a b c d

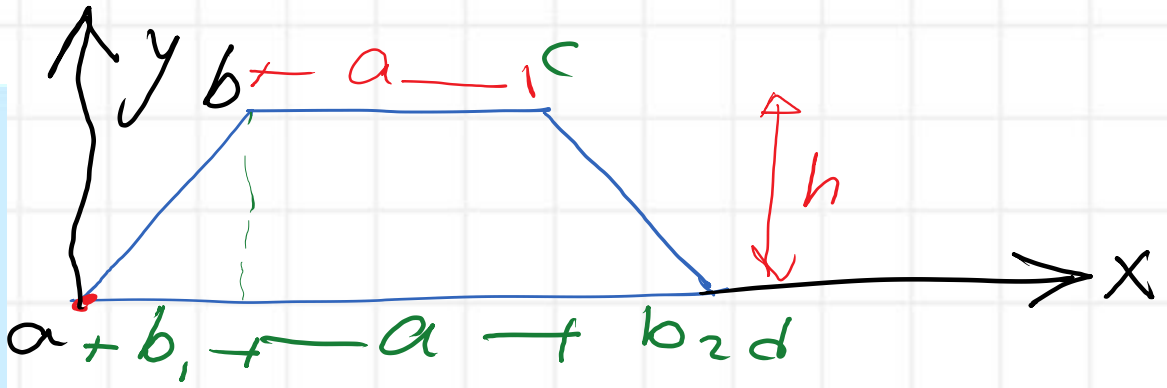


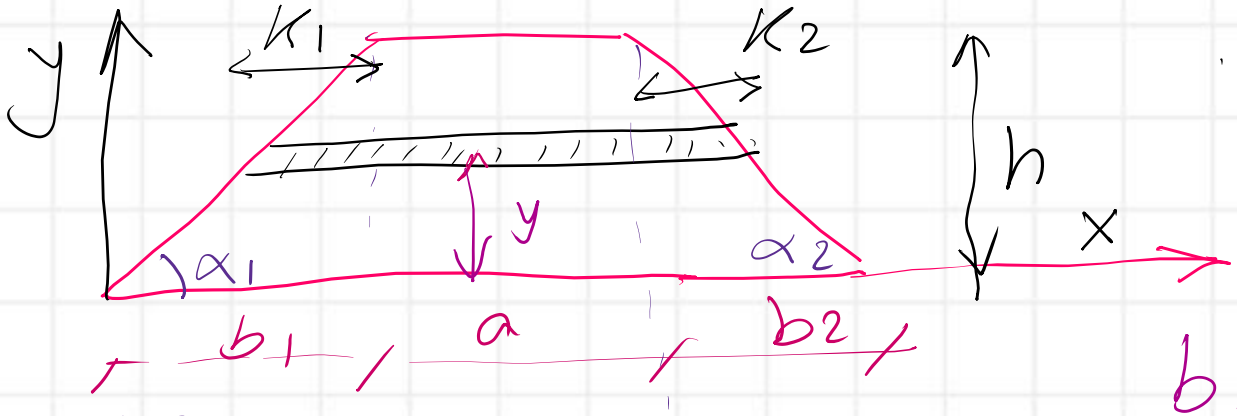
Trapezoid (US)

Trapezium (UK)

Area = ½(a+b) × h

h = vertical height





$$I_x = I_{x1} + I_{x2}$$

$$= \frac{b_1 + b_2}{12}(h^3) + 4\frac{a h^3}{12}$$

$$b = a + b_1 + b_2$$

$$I_x = \frac{h^3}{12}\left[b_1 + b_2 + 4a\right] = \frac{h^3}{12}\left((a + b_1 + b_2) + 3a\right] =$$

$$I_x = \frac{h^3}{12}(b + 3a)$$

Final Expression

Prepared by Eng.Maged Kamel.