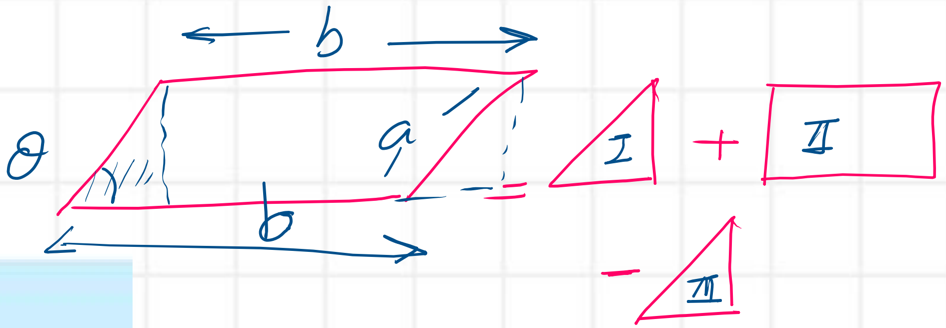
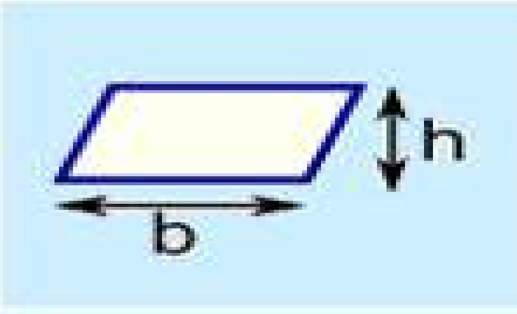
I_{xy} For Parallelogram



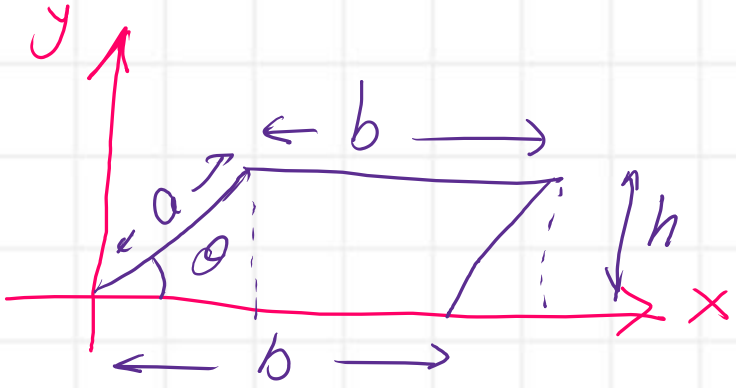


Parallelogram

Area = b × h

b = base

h = vertical height



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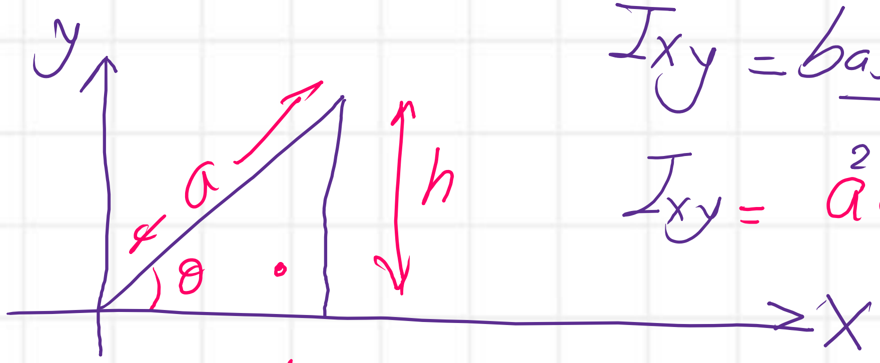


Triangle-1

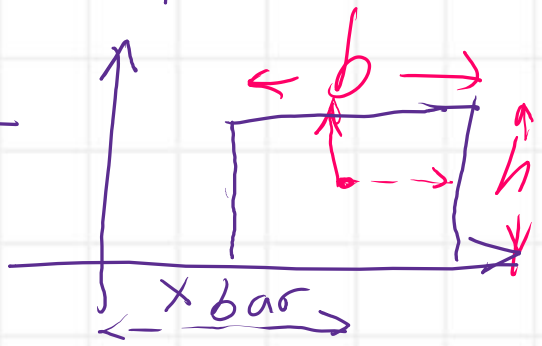
$I_{xy} = \frac{base^2 (heigh)^2}{8}$

$I_{xy} = \frac{a^2\cos^2\theta\,(h)^2}{8}$

$X_{b_r} = \left(\frac{b}{2} + a\cos\theta\right)$

$I_{xy_{rect}} = bh\left(\frac{b}{2} + a\cos\theta\right)\frac{h}{2}$

+



$I_{xy_g} = 0$

$y_{c.g} = h/2$

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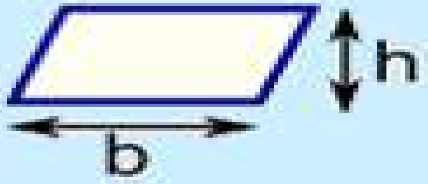
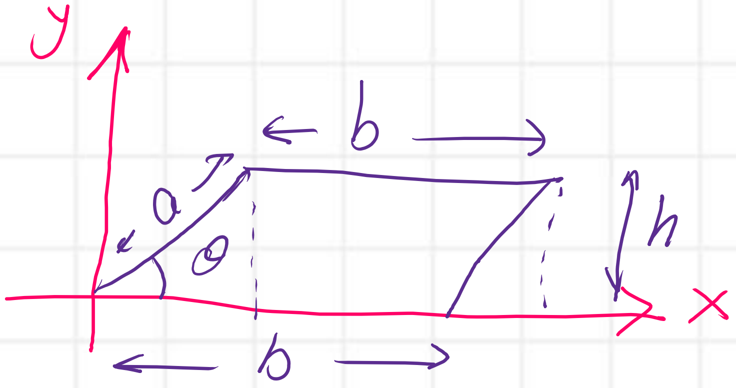
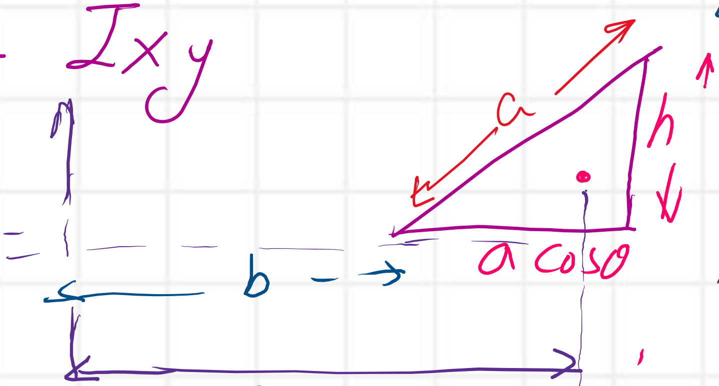
I_{xy} For Parallelogram



Parallelogram
Area = b × h
b = base
h = vertical height



$- I_{xy}$



III shape

$I_{xy_g} = (a\cos\theta)^2 \frac{h^2}{72}$,

$x_{c.g} =$

$b + \frac{2}{3} a\cos\theta$

$y_{c.g} = \frac{h}{3}$

$b + a\cos\theta$

$$I_{xy} = a^2\cos^2\theta \, \frac{h^2}{72} + \frac{1}{2}(a\cos\theta)h \, * \left(b + \frac{2}{3} a\cos\theta\right) \frac{h}{3}$$

3rd shape

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I_{xy} For Parallelogram

I_{xy} From Triangle | I_{xy} From rectangle | $(-)$ I_{xy} 2nd triangle

$$I_{xy} = \frac{a^2 \cos^2\theta\, (h)^2}{8} \qquad I_{xy} = bh\left(\frac{b}{2} + a\cos\theta\right)\frac{h}{2} \qquad I_{xy} = \frac{a^2\cos^2\theta\, h^2}{72} + \frac{1}{2}a(\cos\theta)h\left(\frac{h}{3}\right)\left(b + \frac{2}{3}a\cos\theta\right)$$

$$I_{xy} = \sum_{i=1}^{i=3} I_{xy_i} \Rightarrow \text{Similar terms}$$

$$\rightarrow a^2\cos^2\theta\,\frac{h^2}{8} - (a^2\cos^2\theta)\frac{h^2}{72} = \frac{8a^2\cos^2\theta}{72}(h^2)$$

$$+ \frac{b^2h^2}{4} + ba\cos\theta\left(\frac{h^2}{2}\right) - \frac{1}{2}a\cos\theta\,\frac{h^2}{3}\left(\frac{3b + 2a\cos\theta}{3}\right)$$

$$8a^2\cos^2\theta\,\frac{h^2}{72} - \frac{1}{18}\left(ab\cos\theta\, h^2(3)\right) - \frac{1}{18}\left(2a^2\cos^2\theta\, h^2\right)$$

$$+ \frac{b^2h^2}{4} + \frac{ba}{2}\cos\theta\, h^2$$

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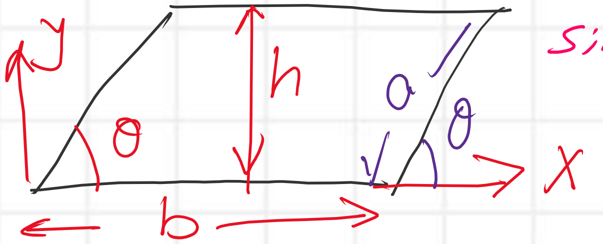
$$I_{xy} = 8a^2\cos^2\theta\,\frac{h^2}{72} - \frac{1}{18}\left(ab\cos\theta\, h^2(3)\right) - \frac{1}{18}\left(2a^2\cos^2\theta \cdot h^2\right) + \frac{b^2h^2}{4} + \frac{ba}{2}\cos\theta\, h^2$$

$$I_{xy} = \frac{a^2\cos^2\theta}{72}\, h^2(8-8) + \frac{b^2h^2}{4} + ba\cos\theta\, h^2\left(\frac{1}{2} - \frac{1}{6}\right)$$

$$I_{xy} = \frac{b^2h^2}{4} + \frac{1}{3}\, ba\cos\theta\,(h^2) = \frac{h^2 b(3b + 4a\cos\theta)}{12}$$



since $h = a\sin\theta$, $A = (ab\sin\theta)$

$$I_{xy} = \frac{a^2\sin^2\theta}{12}\, b(3b + 4a\cos\theta)$$

$$\overline{r_{xy}}^2 = \frac{I_{xy}}{A} = \frac{a\sin\theta}{12}(3b + 4a\cos\theta)$$

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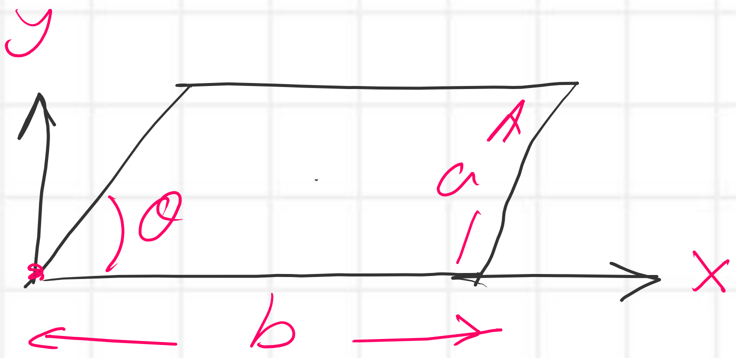
$$I_{xy} = \frac{a^2 \sin^2\theta\, b}{12}\left(3b + 4a\cos\theta\right)$$

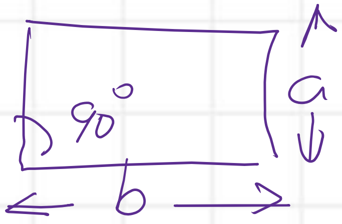


Since a rectangle is a special case when $\theta = 90^\circ$

Check I_{xy} for $\theta = 90^\circ \longrightarrow$ rectangle

$$I_{xy} = \frac{a^2 \sin^2(90)}{12} b\left(3b + 4a\cos(90)\right)$$

$$= a^2(1)\frac{b}{12}(3b+0) = \frac{3a^2b^2}{12} = \frac{1}{4}a^2b^2 \Rightarrow ok$$



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$$I_{xy_{C.g}} = I_{xy} - A\bar{x}\bar{y}$$

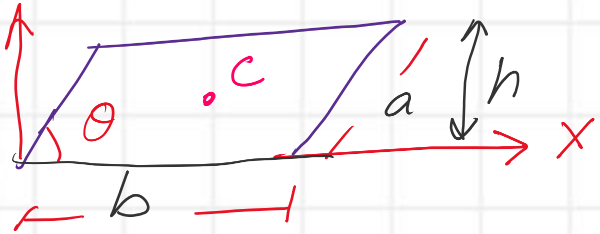
$$I_{xy} = \frac{a^2\sin^2\theta\, b}{12}(3b + 4a\cos\theta)$$

$$A = ab\sin\theta$$

$$\bar{x} = \left(\frac{b + a\cos\theta}{2}\right)$$

$$\bar{y} = \frac{a\sin\theta}{2}$$

$$I_{xy_{C.g}} =$$

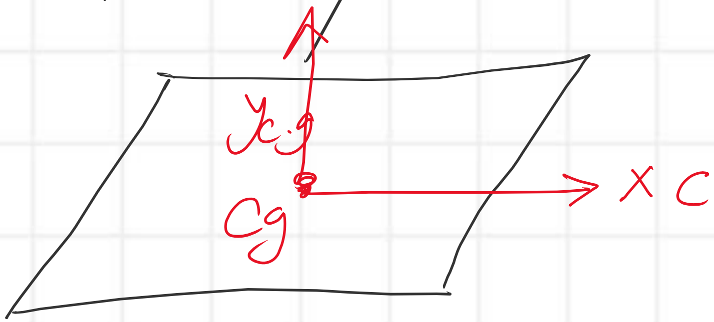


$$(A\bar{x}\bar{y}) = \frac{a^2 b\sin^2\theta}{2\,(2)}(b + a\cos\theta) = \frac{3a^2b^2\sin^2\theta + 3a^3\sin^2\theta\cos\theta(b)}{12}$$

$$I_{xy} - A\bar{x}\bar{y} = \frac{3b^2a^2\sin^2\theta}{12} + \frac{4a^3\sin^2\theta\cos\theta(b)}{12} - \frac{3a^2b^2\sin^2\theta - 3a^3b\sin^2\theta\cos\theta}{12}$$

$$I_{xy_G} = \frac{0 + a^3\sin^2\theta\cos\theta(b)}{12} = \frac{a^3b}{12}[\sin^2\theta)(\cos\theta)$$

The product of Inertia at C.g



$$I_{xy_G} = \frac{a^3 b}{12}\left(\sin^2\theta \cos\theta\right)$$

Since rectangle is a case of Parallelgram but with $\theta = 90^\circ$

$$I_{xy_G} = \frac{a^3 b}{12}\left[\sin^2(90)\cos(90) = 0\right.$$ $\Rightarrow$ Same as Proved

Prepared by Eng.Maged Kamel.