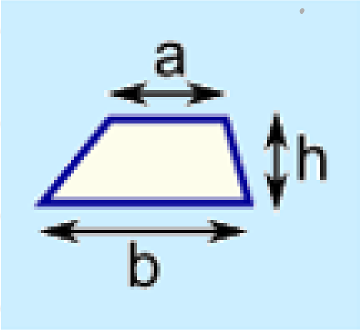
Moment of inertia For Trapezium



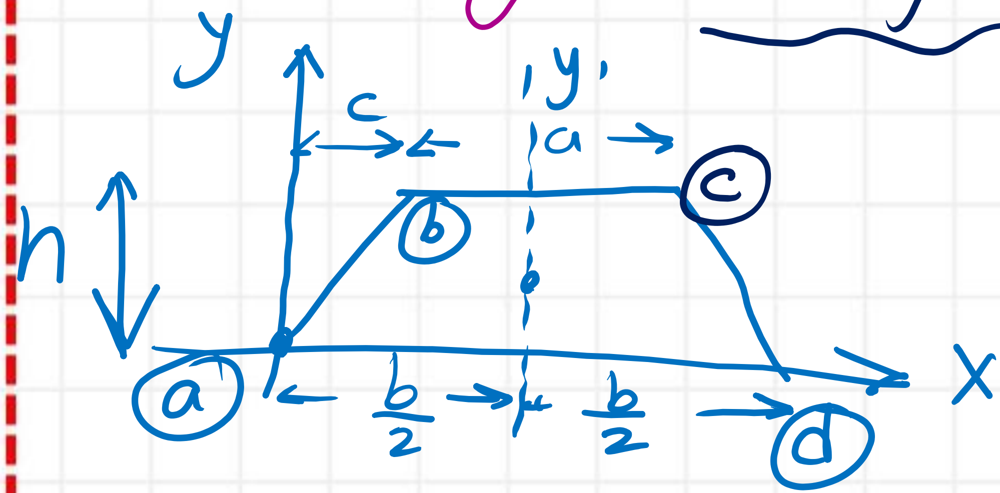
Trapezoid (US)
Trapezium (UK)
Area = ½(a+b) × h
h = vertical height

about y' mid axis
while y is the external axis at a

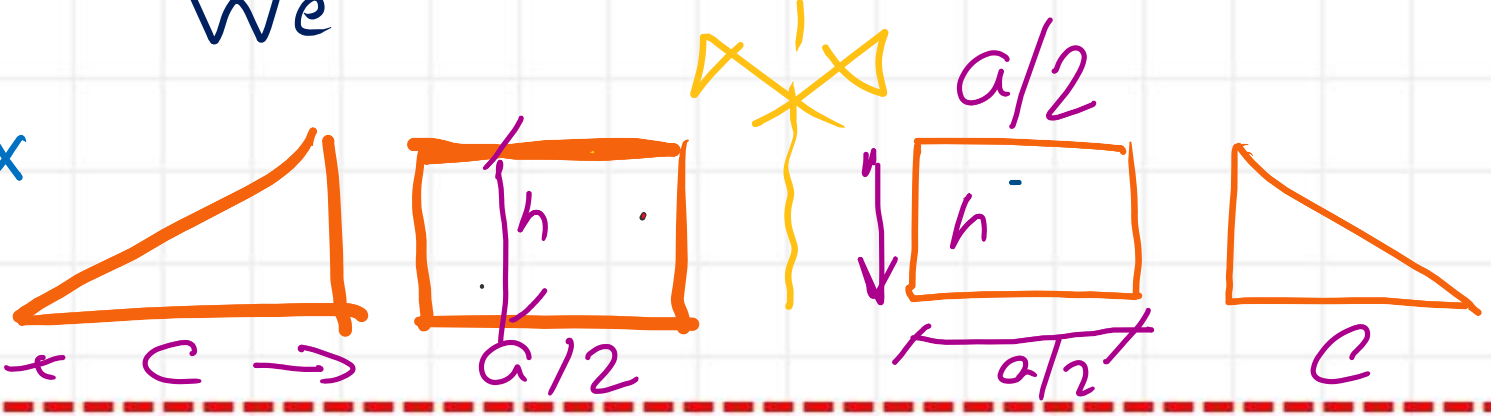
y' Case of Symmetry $I_{y'}$ $\quad \boxed{b/2 = c + \frac{a}{2}}$

Due to symmetry

We



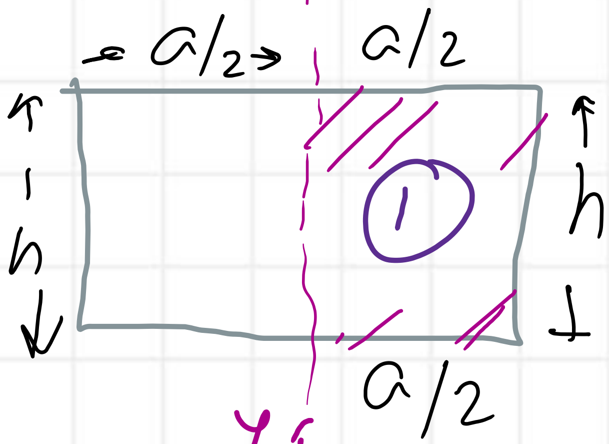
Corners a, b, c, d



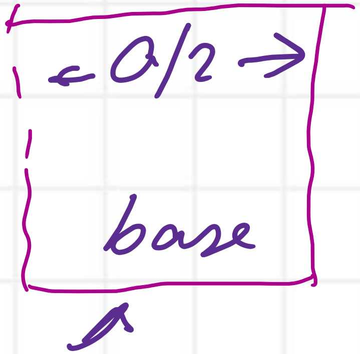
Two Cases

one hatched rectangle

Dim: $\frac{a}{2}(h)$



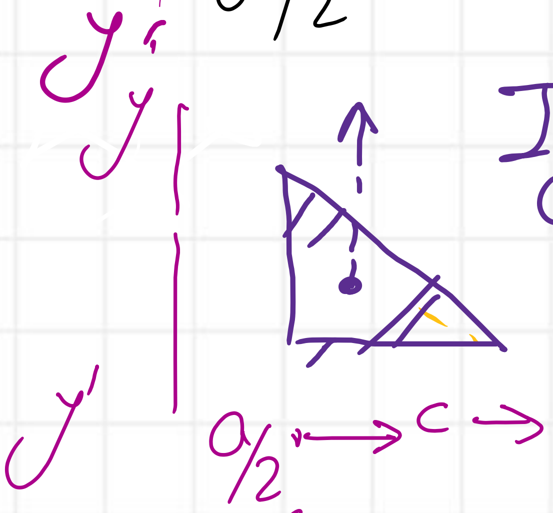
$I_{y'}$ at cg = Height $\frac{(base)^3}{3}$



$= \frac{h}{3}\left(\frac{a}{2}\right)^3$

$= \frac{ha^3}{24}$

(b) one triangle



$I_{y'_2} = \frac{hc^3}{36} + \text{area}\,(x_{c.g})^2$

$x_{cg} = \frac{a}{2} + \frac{c}{3} =$

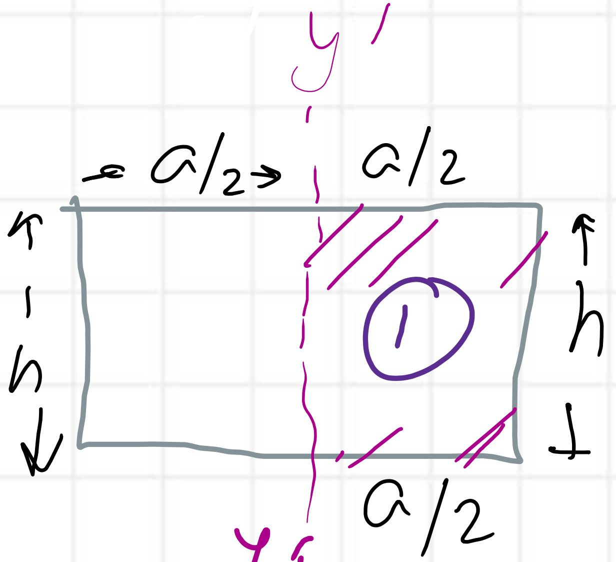
$x_{cg} = \frac{3a+2c}{6}$

$I_{y'_2} = \frac{hc^3}{36} + \frac{1}{2}\,\frac{a \cdot c}{36}(3a+2c)^2$

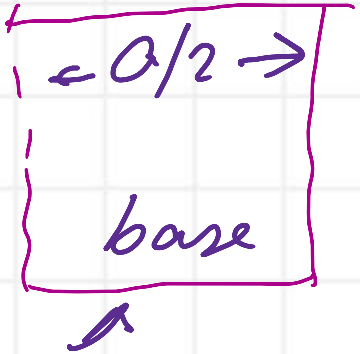
Two Cases

one hatched rectangle

Dim: $\frac{a}{2}(h)$



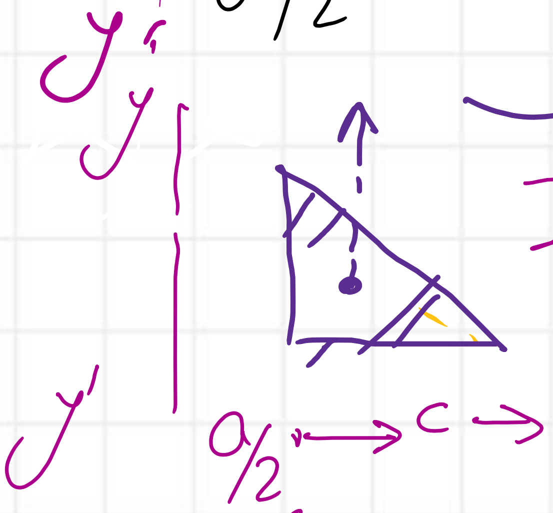
$$I_{y'} \text{ at cg} = \text{Height}\frac{(\text{base})^3}{3}$$



$$= \frac{h}{3}\left(\frac{a}{2}\right)^3$$

$$= \frac{h a^3}{24}$$

(b) one triangle



$$I_{y'_2} = \frac{hc^3}{36} + \frac{1}{72}ac\,(9a^2 + 12ac + 4c^2)$$

$$I_{y'_2} = \frac{hc^3}{36} + \frac{1}{2}\frac{a.c}{36}(3a+2c)^2$$

$$\frac{hc}{72}(2c^2 + 9a^2 + 12ac + 4c^2)$$

$$I_{y'_2} = \frac{hc}{72}\left(6c^2 + 12ac + 9a^2\right)$$ Triangle

For right Triangle, while $\boxed{I_{y'_{rect}} = \frac{ha^3}{24}}$

For Two Triangles and Two rectangles

add and multiply by 2

$$I_{y'} = 2I_{y'_1} + 2I_{y'_2} = \frac{2ha^3}{24} + \frac{2hc}{72}\left(6c^2 + 12ac + 9a^2\right)$$

where

$$c = \frac{b}{2} - \frac{a}{2} = \frac{1}{2}(b-a)$$

$$I_{y'} = 2\frac{ha^3}{24} + \frac{12hc^3}{72} + 24\frac{ac^2h}{72} + \frac{18a^2hc}{72}$$

$$C = \frac{b-a}{2}$$

Check Similar items

$$I_{y'} = \frac{2}{72}\left(6ha^3 + 6hc^3 + 12hac^2 + 9ha^2c\right)$$

$$6hc^3 = 6h\left(\frac{b-a}{2}\right)^3 = \frac{6h}{2}(b-a)\frac{(b^2 - 2ba + a^2)}{4}$$

$$\frac{1}{8}(6hb - 6ha)(b^2 - 2ba + a^2)$$

$$\frac{1}{8}(6hb^3 - 12hb^2a + 6hba^2 - 6hab^2 + 12ha^2b - 6ha^3)$$

$6hc^3$ Term) others ($6ha^3$ & $12hac^2$ & $9ha^2c$

$$\frac{6}{8}hb^3 - \frac{18}{8}hb^2a + \frac{18ha^2b}{8} - \frac{6ha^3}{8}$$

$$12hac^2 = 12ha\left(\frac{b-a}{2}\right)^2 = \frac{12ha}{4}(b^2-2ab+a^2)$$

$$12hac^2 = \frac{12hab^2}{4} - \frac{24}{4}ha^2b + \frac{12ha^3}{4}$$

$$= 3hab^2 - 6ha^2b + 3ha^3$$

$$9ha^2c = 9ha^2\left(\frac{b-a}{2}\right) = 4.5ha^2b - 4.50ha^3$$

$$6ha^3 = +6ha^3$$

$$I_{y'} = \left(\frac{2}{72}\right)\left(\frac{6hb^3 - 18hb^2a + 18ha^2b - 6ha^3}{8}\right)$$

$$+ \frac{2}{72}\left(6ha^3 + 3hab^2 - 6ha^2b + 3ha^3\right)$$

$$+ \frac{2}{72}\left(4.5\,ha^2b - 4.5\,ha^3\right)$$

$$ha^3 \Rightarrow \frac{2}{72}\left(-\frac{6}{8} + 6 - 4.5\right) = \frac{3}{4}$$

$$ha^2b \Rightarrow \frac{2}{72}\left(\frac{18}{8} - 6 + 4.5\right) = \frac{3}{4}\left(\frac{1}{36}\right)$$

$$2/72\left(\frac{6}{8}\right) \qquad = \frac{3}{4}\left(\frac{1}{36}\right)$$

$$hb^2a \rightarrow \frac{2}{72}\left(\frac{-18}{8}+3\right) = \frac{2}{72}\left(\frac{6}{8}\right) = \frac{1}{48}hb^2a$$

$$I_{y'} = \frac{3}{4}\left(\frac{1}{\cancel{36}_{12}}\right)ha^3 + \frac{3}{4}\left(\frac{1}{36}\right)ha^2b + \frac{3}{4}\left(\frac{1}{36}\right)hb^3 + \frac{1}{48}hb^2a$$

$$I_{y'} = \frac{1}{48}\left(ha^3 + ha^2b + hb^3 + hb^2a\right)$$

$$I_{y'} = \frac{h}{48}(a+b)(a^2+b^2)$$

$\longrightarrow$ y' Symmetric axis

This is the Expression which we have

Moment of Inertia about the y_c axis

I_{yc}

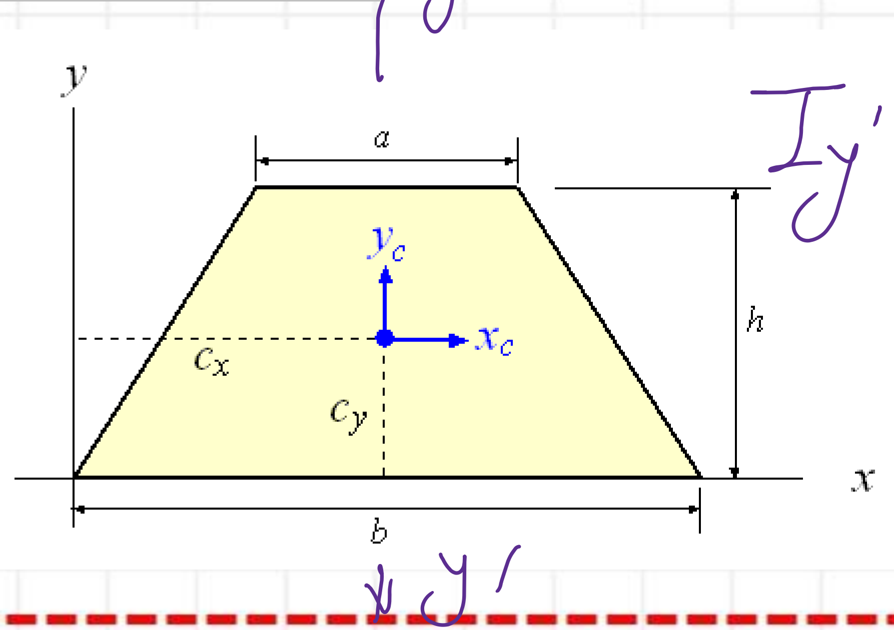
$$\frac{h(a+b)(a^2+b^2)}{48}$$

https://www.efunda.com/math/areas/IsosTrapezoid.cfm

$$k_y^2 = \frac{I_{ycg}}{A}$$

$$= \frac{\frac{h(a+b)(a^2+b^2)}{48}}{\frac{(a+b)h}{2}}$$

$$k_y^2 = \frac{a^2+b^2}{24}$$



$$I_{y'} = \frac{h(a+b)(a^2+b^2)}{48}$$

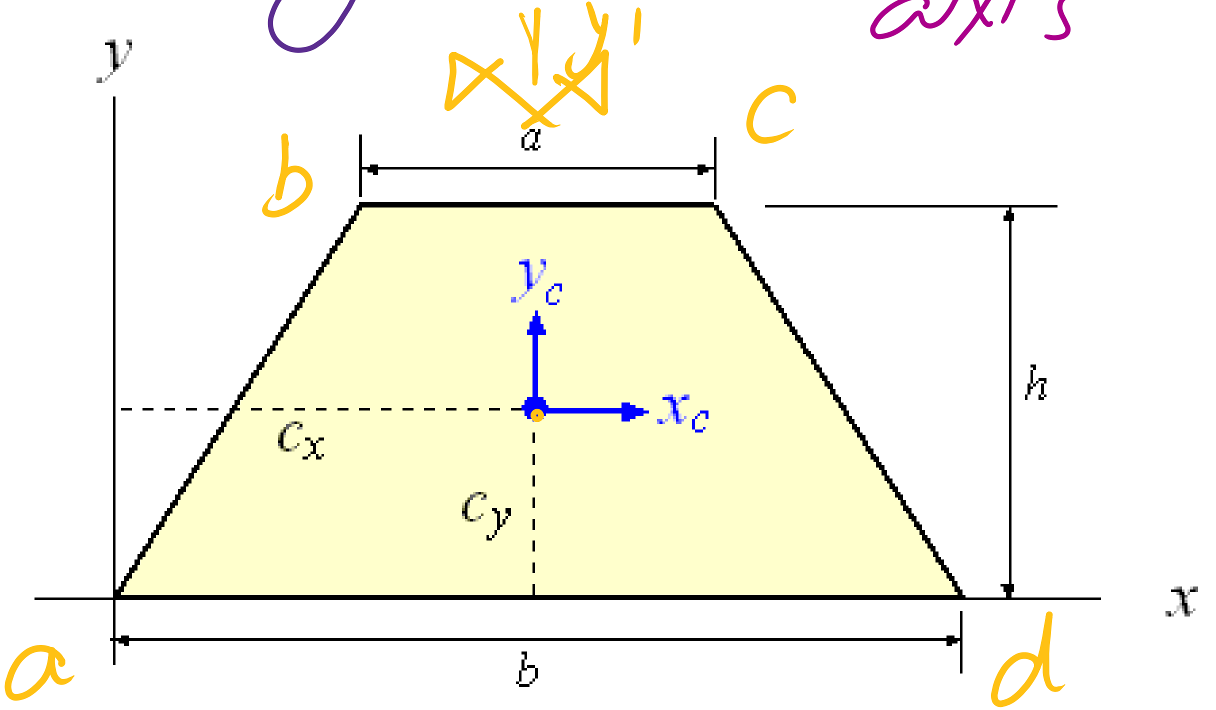
$$\text{area} = \frac{(a+b)h}{2}$$

For I_y about an External axis

$$I_y = I_{y_c} + A\bar{x}_{cg}^2$$

$$x_{cg} = \frac{b}{2}$$

$$\text{Area} = \frac{1}{2}(a+b)h$$



$$I_y = \frac{h}{48}(a+b)(a^2+b^2) + \frac{1}{2}(a+b)(h)\left(\frac{b^2}{4}\right)$$

$$I_y = \frac{h}{48}(a+b)(a^2+b^2) + \frac{6h}{48}(b^3+b^2a)$$

I_y about an External axis

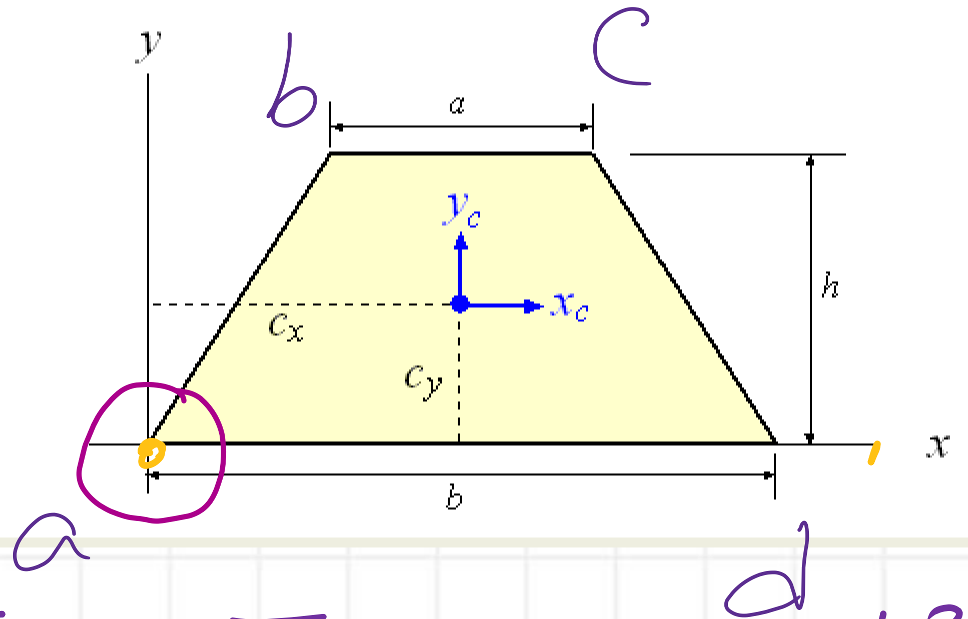
$$I_y = \frac{h}{48}(a+b)(a^2+b^2) + \frac{6h}{48}(b^3+b^2a)$$

$$I_y = \frac{h(a^3 + ba^2 + ab^2 + b^3)}{48} + \frac{h}{48}(6b^3 + 6b^2a)$$

$$I_y = \frac{h}{48}(a^3 + ba^2 + ab^2 + 6ab^2 + b^3 + 6b^3)$$

$$I_y = \frac{h}{48}(a+b)(a^2+7b^2) \rightarrow A = \frac{a+b}{2}(h)$$

$$K_y^2 = I_y/A = \frac{h(a+b)(a^2+7b^2)}{48(a+b)\frac{1}{2}(h)} = \frac{1}{2}(a^2+7b^2)$$



Polar moment of Inertia

at left Corner a

$$I_p = I_x + I_y = \frac{h^3}{12}(3a+b) + \frac{h}{48}(a+b)(a^2+7b^2)$$

$$I_p = \frac{h}{48}(12ah^2 + 4bh^2 + a^3 + ba^2 + 7ab^2 + 7b^3)$$

$$\frac{h\left(4h^2b + 12h^2a + 7b^3 + 7ab^2 + ba^2 + a^3\right)}{48}$$

Same Expression