Numerical analysis

Bracketing method

Bisecting *method*.

1-This method depends on two values for F(x) of opposite signs, with a -ve sign at point a & at point b with a +ve sign.

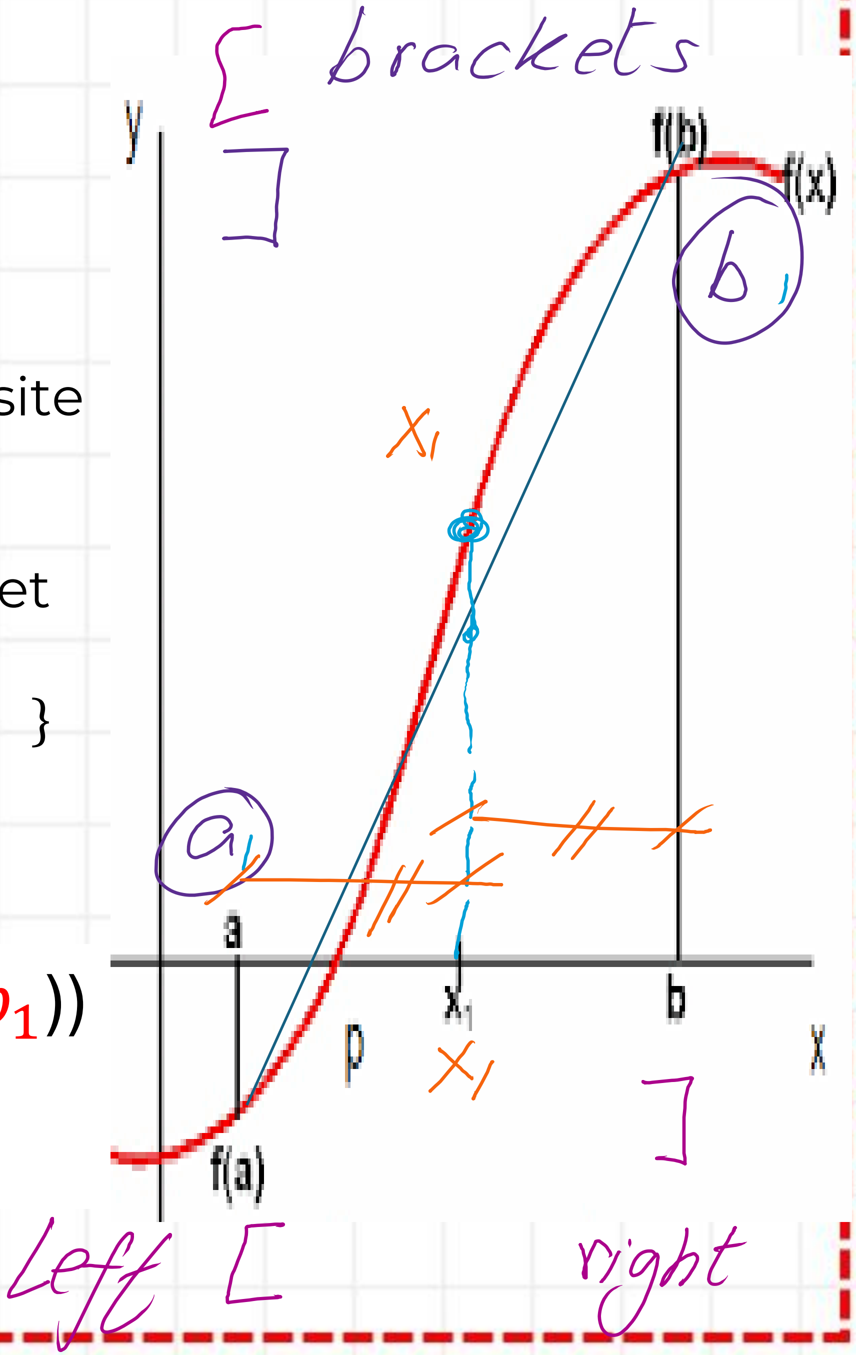
when approaching from –ve to +ve; assign left bracket {

And ending with +ve value, assign right hand bracket }

2- Our zero value must be in between, so we take an intermediate point between (a_1 & $b1$).

3- Our (x_1) , which is the first trial=0.50*((a_1+(b_1))

4- check f((x_1) .



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Bracketing method

Bisecting *method*.-a1&b1

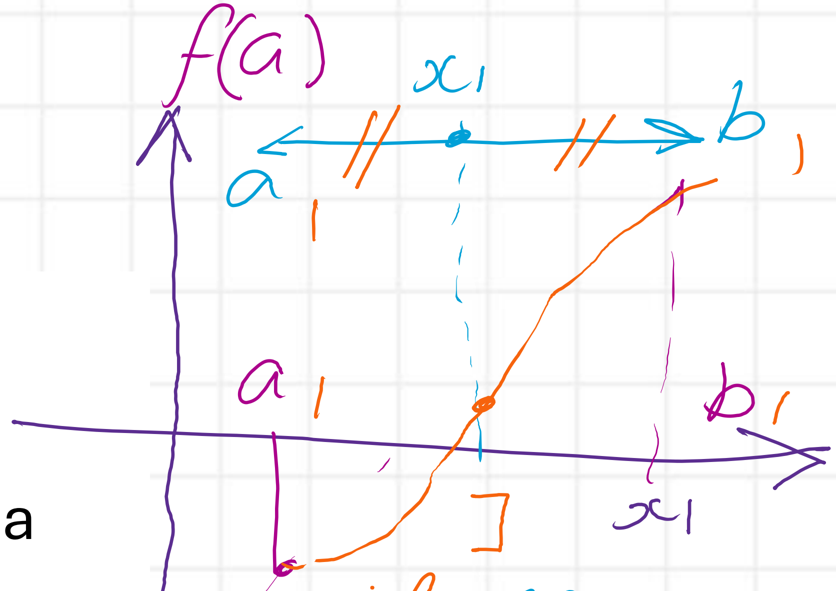
5- if f((a_1)*f(x_1) < 0 , then they are opposite to each other , (x_1 right-have right hand bracket }

We will bisect the distance between a_1 & x_1 and get a new point x_2. If f((a_1) f(x_2) >0 . X2 will have a left bracket, get new point. Take intermediate point ,between x_2 and b_1 with

as 2nd trial= 0.50*((x_1+(b_1) ,then estimate $f(x_2)$.

6- Check if f((x_2)*f(b_1)) < 0,then these points will be opposite to each other .

7- Continue the process by bisecting the distance between points having opposite f(x) values until you approach the zero point.



[if f(a1).f(x1) is -ve]

else [

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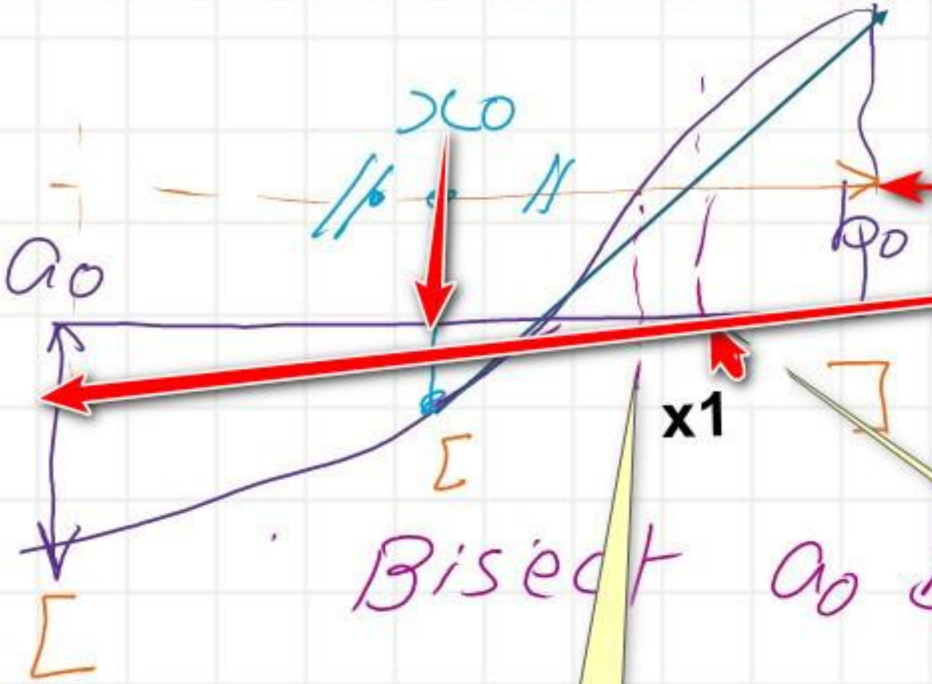


a_1 or b_1

$f(a_0) = -ve$

$f(b_0) = +ve$

a0 b0

a_0 : left point

b_0 : right point

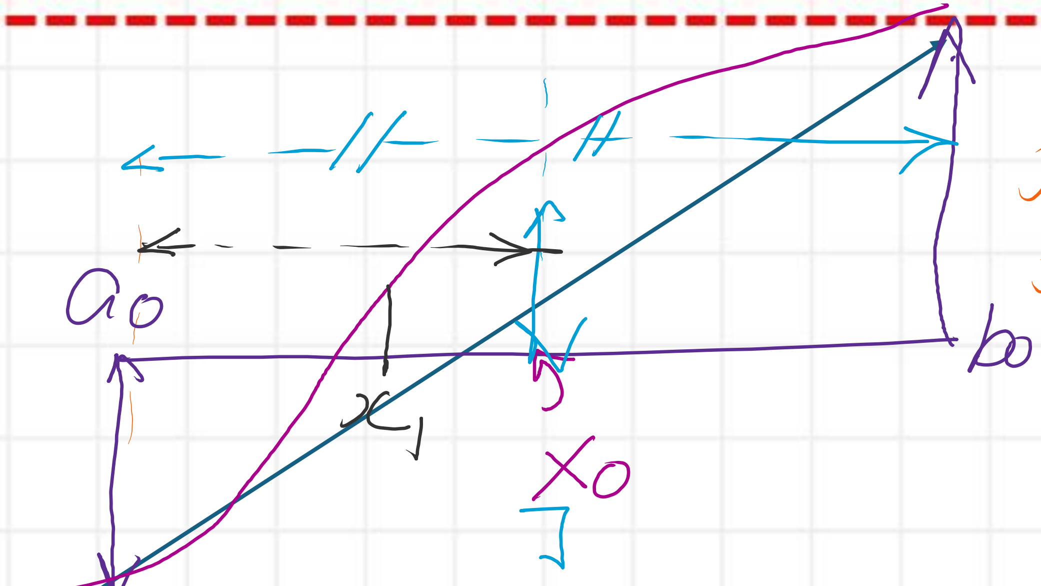
Bisect $a_0 b_0 \rightarrow x_0$

Get $f(x_0) \Rightarrow$ if $\underbrace{f(a_0) f(x_0)}_{-ve} < 0 \rightarrow x_0$ Left bracket [

Bisect the distance between x0 and b0, select a new point call it x1.

Find f(x1). if f(x1)*f(b0)<0 then bisect; b0x1 to another point call x2 and continue the procees.

Find f(x1). if f(x1)*f(b0)>0 This point is our new right bracket



$f(a_0) = -ve$

$f(b_0) = +ve$

a_0 : left point

b_0 : right point

a_1 or a_0 $\quad$ b_1 or b_0

Bisect $a_0 b_0 \rightarrow x_0$

Get $f(x_0) \Rightarrow$ if $f(a_0)f(x_0) > 0 \rightarrow x_0$ is a new right bracket

Bisect $a_0 x_0 \longrightarrow$ new Point x_1

check $f(a_0)f(x_1) > 0$ +ve x_1 is a new right bracket

Bisect again till $f(a_0)f(x) = 0$

Our example is :f(x) =$x^3 - 6x^2 + 11 * \mathrm{x} - 6$, find the zeros for the function by Analytical method and further by Numerical method.

Solution:

Leading coefficient ± 1

Constant coefficient $\pm 1,6,2,3$

Our choices: $\pm \left(\frac{1,6,2,3}{1}\right)$ as zeros

2	1	-6	11	-6
		2	-8	+6
	1	-4	3	0

If x=2 our first choice i,e X-2=0

(x-2)*(x^2-4x+3)=0

This formula can be again simplified as (x-2)*(x-3)*(x-1)=0

We have three solutions X1=2 &X2=3 & X3=1

Example f(x)=(X^3-6X^2+11X-6).

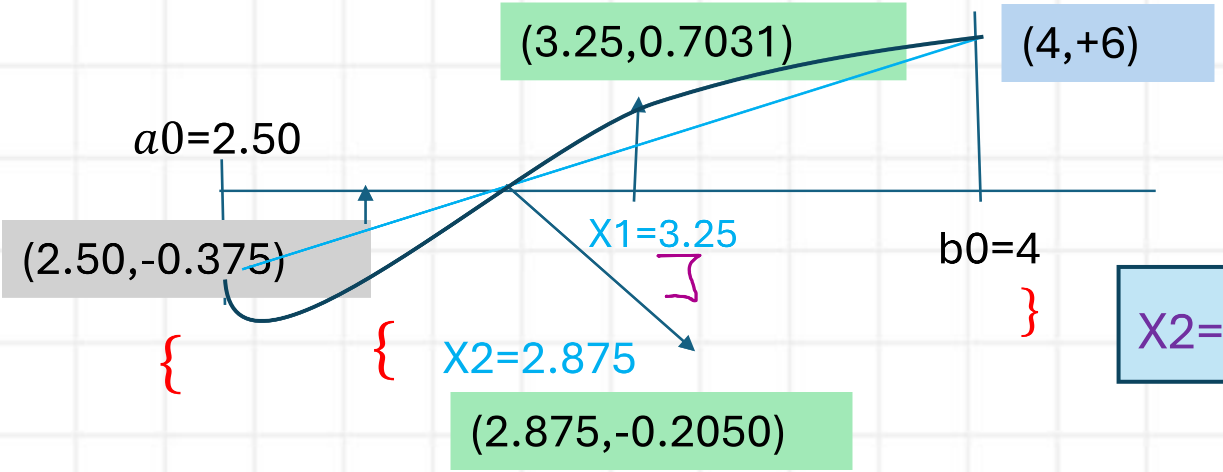
Find the zeros for the function by the bisecting method.

Solution:

1- Try several values of x that give different values of f(x)

for X =4   f(4) = 4^3-6*4^2+11*(4)-6=64-96+44-6=+6

for x=2.50 then   f(2.50)= 2.5^3 - 6*2.5^2+11*(2.5)-6=15.625-37.50+27.50-6=-0.375



X3=0.50*(2.875+3.25)=3.0625

f(3.0625)=+0.1369

X2=0.50*(2.5+3.25)=2.875 ,f(2.875)=-0.2050

X4=0.50*(2.875+3.0625)=2.96875

f(2.96875=-0.0596

X2=2.875 { } X1=3.25

X2=2.875 { } X3=3.0625

X4=2.968 { } X3=3.0625

} X5=3.015

Zero point at x=3

X5=0.50*(2.968+3.0625)=3.015

f(3.015)=0.032

Bisect aobo distance

$a_0 = 2.5$ [

$b_0 = 4$]

Take X1=0.50*(4+2.50)=3.25 ⟵ find $f(x_1)$

1st iteration.

f(3.25)=+0.7031

f(2.50)*f(3.25)=- 0.375*(+0.7031)=- 0.26367

$f(a_0) \cdot f(x_1) = -ve$

They are opposite to each other

x_1]

2nd iteration.

Bisect a_0, x_1 } x_2

Take X2=0.50*(2.50+3.25)=2.875

f(2.875)=-**0.20508** ⟶ $f(x_2)$

+ve

f(2.50)*f(2.875)=- 0.375*(-0.20508)=0.0769

$f(a_0) f(x_2) = +ve$

X2 [

f(2.875)*f(4)=-0.20508*6 =-1.23047 ⟶ Confirm with point bo

new Left

3rd iteration.

Take X3=0.50*(4+2.875)=3.0625 x_3

$f(x_3)$ f(3.0625)=+0.1369

]

f(2.875)*f(3.0625)=-0.20508*(+0.1369)=-0.03

$f(x_2) \cdot f(x_3) \Rightarrow -ve$

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x_3 new right

$x_3 = 3.0625$]
$x_2 = 2.875$ [

4rth iteration.

Take X4=0.50*(2.875+3.0625)=2.968 ← Bisect $x_2\ x_3$

f(2.968) =-0.06 find $f(x_4)$

f(3.0625)*f(2.968)=+0.1369*(-0.06)=-0.01

] [

Check $f(x_3) \cdot f(x_4) = -ve$

5th iteration

Take X5=0.50*(2.968+3.0625)=3.015

Bisect x_4 [Left bracket

x_4 x_3

f(3.015) =+0.032 →]

f(3.015)*f(2.968)=+0.032*(-0.06)=-0.002

x_5] new right bracket

Check $f(x_5) \cdot f(x_4) = -ve$

Solution is converging to x=3.00

x_4 [Left bracket

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Error estimation

Number of iteration $n \geq \log_{10}(b_0 - a_0) - \log_{10}(\varepsilon) / (\log_{10} 2)$

In our example $a_0 = 2.50$ & $b_0 = 4$ *for error tolerance of* $\epsilon = 0.001$

$$n \geq \log_{10}(4 - 2.5) * (-3) / (\log_{10} 2)$$

$$(-0.166091 - (-3) * {}^{1}/_{0.301029} = 10.55$$

Relative error =(x new-x old)/x new for x is value estimated for zero f(x) value

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