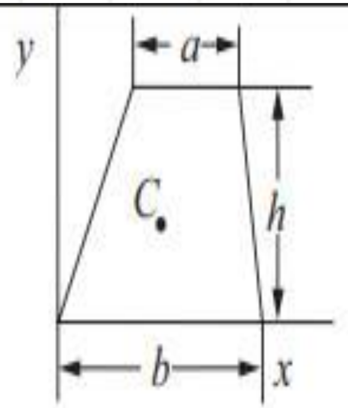
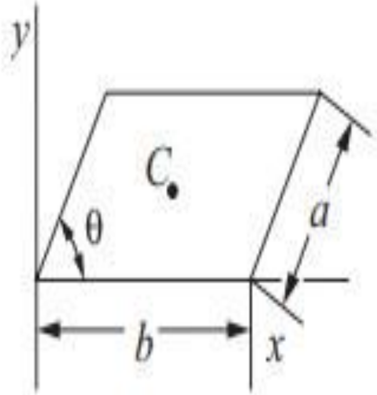
Ix for Trapezoid

Ix at Cg

Ix at base

	$A = h(a+b)/2$ $y_c = \dfrac{h(2a+b)}{3(a+b)}$	$I_{x_c} = \dfrac{h^3(a^2+4ab+b^2)}{36(a+b)}$ $I_x = \dfrac{h^3(3a+b)}{12}$	$r_{x_c}^2 = \dfrac{h^2(a^2+4ab+b^2)}{18(a+b)}$ $r_x^2 = \dfrac{h^2(3a+b)}{6(a+b)}$
	$A = ab\sin\theta$ $x_c = (b + a\cos\theta)/2$ $y_c = (a\sin\theta)/2$	$I_{x_c} = (a^3 b \sin^3\theta)/12$ $I_{y_c} = [ab\sin\theta(b^2 + a^2\cos^2\theta)]/12$ $I_x = (a^3 b \sin^3\theta)/3$ $I_y = [ab\sin\theta(b + a\cos\theta)^2]/3 - (a^2 b^2 \sin\theta\cos\theta)/6$	$r_{x_c}^2 = (a\sin\theta)^2/12$ $r_{y_c}^2 = (b^2 + a^2\cos^2\theta)/12$ $r_x^2 = (a\sin\theta)^2/3$ $r_y^2 = (b + a\cos\theta)^2/3 - (ab\cos\theta)/6$

Housner, George W., and Donald E. Hudson, *Applied Mechanics Dynamics*, D. Van Nostrand Company, Inc., Princeton, NJ, 1959. Table reprinted by permission of G.W. Housner & D.E. Hudson.

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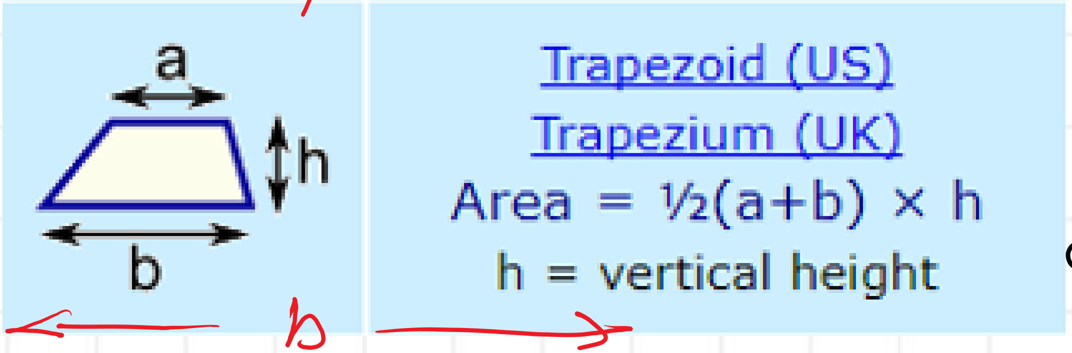
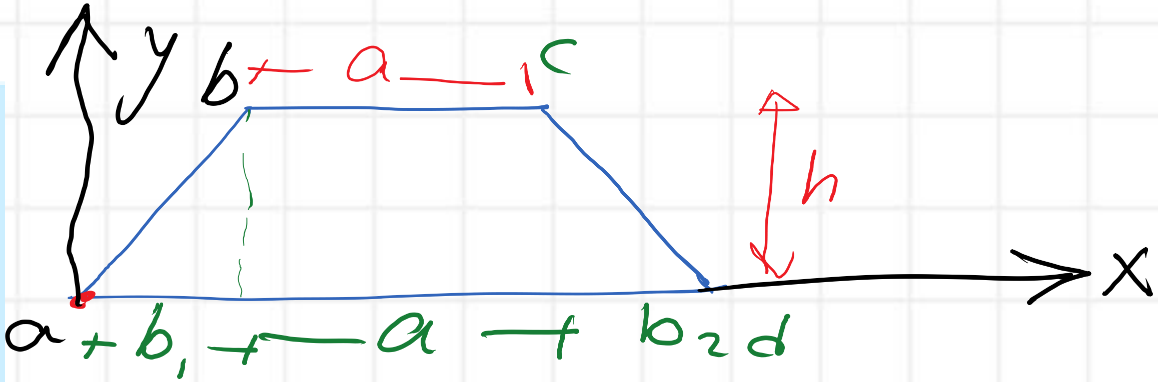
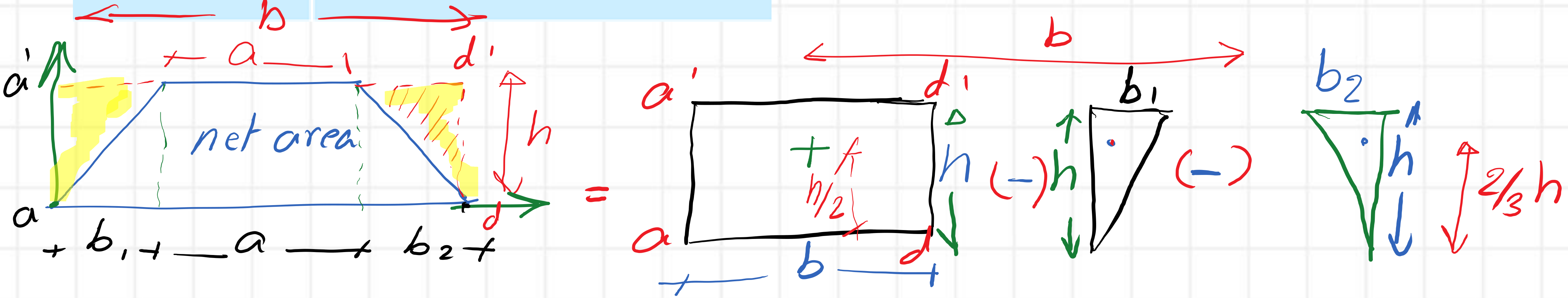
Moment of Inertia I_x For Trapizoid

For Trapezoid abcd







We WiLL Consider the Trapeziod as Composed of

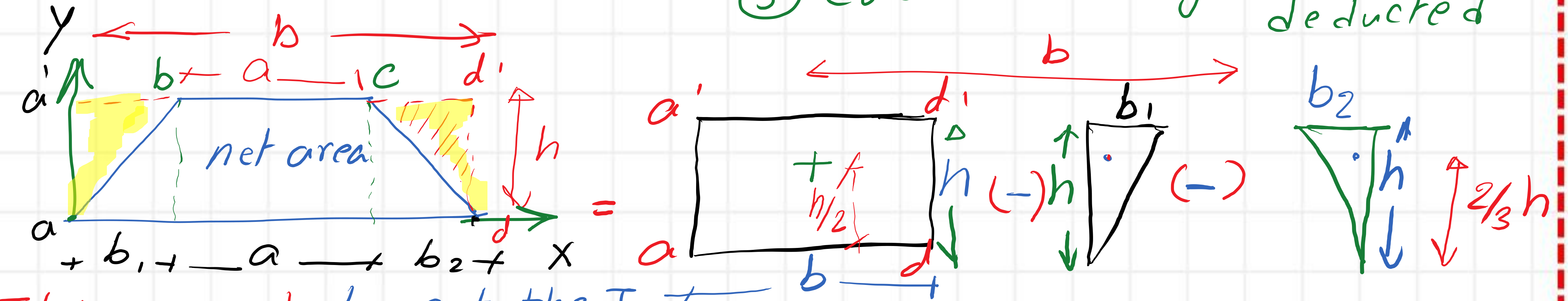
① big rectangle a a'd d' : base = b , height = h

Moment of Inertia I_x For Trapizoid

For Trapezoid abcd

② aa'b : Triangle to be deducted

③ cdd': a Triangle to be deducted



It is required to get the I_x about axis x at the base

For the rectangle Our $I_x = \frac{bh^3}{3}$ as proved earlier

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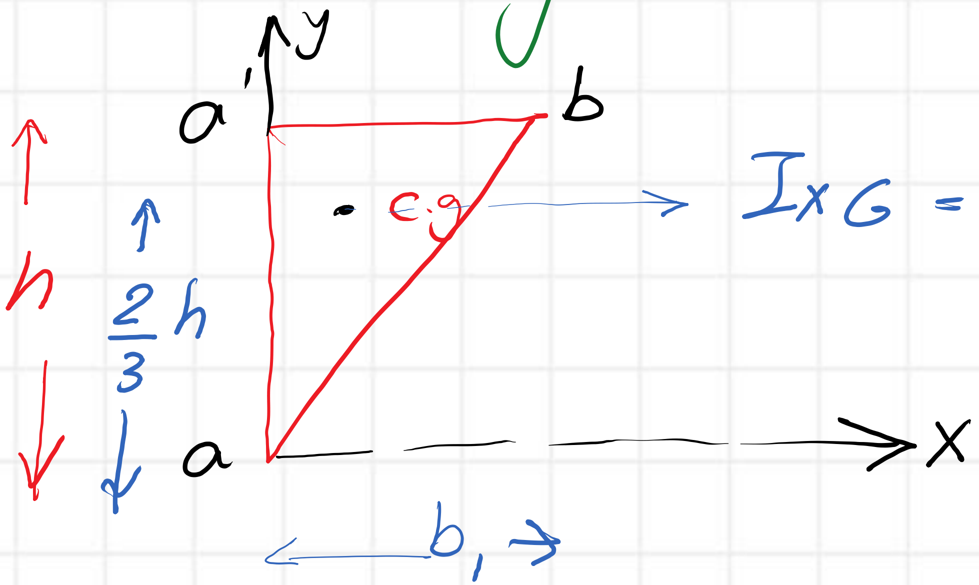
② Now For triangle $aa'b$, base $= b_1$ & height $= h$



$$I_{xG} = b_1 \frac{h^3}{36}, \text{ then } I_x = I_{xG} + A \cdot \bar{y}^2$$

$$\bar{y} = \frac{2}{3}h \Rightarrow \bar{y}^2 = \left(\frac{2}{3}h\right)^2 = \frac{4h^2}{9}$$

$$A = \frac{1}{2} b_1 h$$

$$I_{x_{Tr}} = \frac{b_1 h^3}{36} + \frac{1}{2} b_1 h \left(\frac{4h^2}{9}\right) = b_1 h^3 \left[\frac{1}{36} + \frac{8}{36}\right] = b_1 h^3 / 4$$

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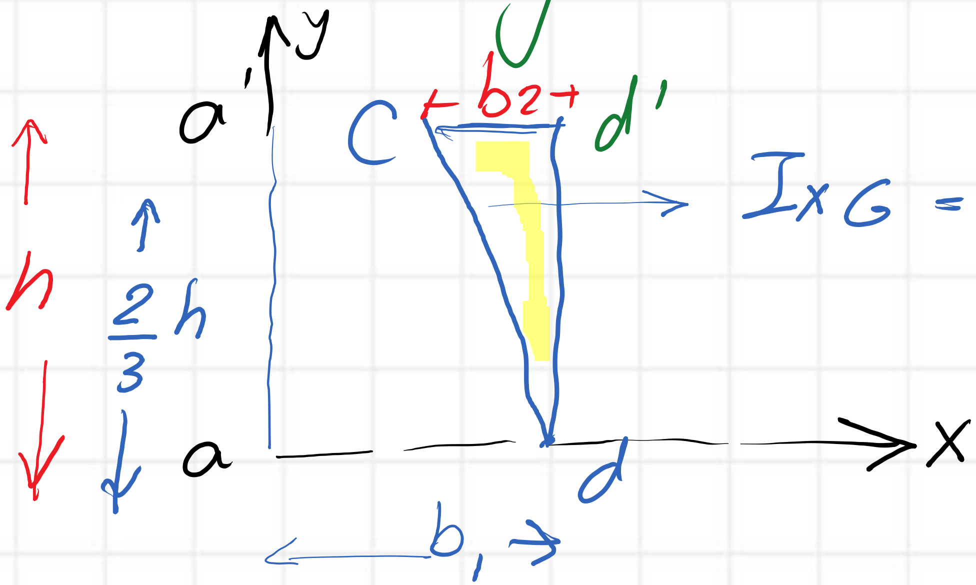
③ Now For triangle cdd', base $= b_2$ & height $= h$



$I_{xG} = \frac{b_2 h^3}{36}$, then $I_x = I_{xG} + A \cdot \bar{y}^2$

$\bar{y} = \frac{2}{3}h \Rightarrow \bar{y}^2 = (\frac{2}{3}h)^2 = \frac{4h^2}{9}$

$A = \frac{1}{2} b h$

$$I_{x(Tr)} = \frac{b_2 h^3}{36} + \frac{1}{2} b_2 h \left(\frac{4h^2}{9}\right) = b_2 h^3 \left[\frac{1}{36} + \frac{8}{36}\right] = b_2 h^3/4$$

The Final $I_x = I_{x\,rect} - I_{x\,Triangle} - I_{x\,triangle\ 2}$

$$I_x = \frac{bh^3}{3} - \frac{b_1 h^3}{4} - \frac{b_2 h^3}{4} = \frac{1}{12} h^3 (4b - 3b_1 - 3b_2)$$

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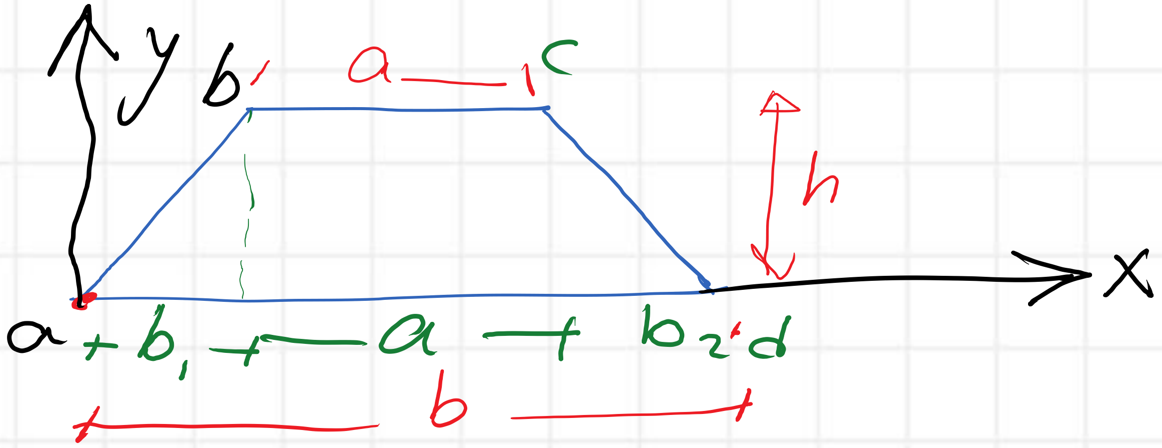

recall $b = (b_1 + a + b_2)$

$$I_x = \frac{bh^3}{3} - \frac{b_1 h^3}{4} - \frac{b_2 h^3}{4} = \frac{1}{12} h^3 (4b - 3b_1 - 3b_2)$$

$$I_x = \frac{h^3}{12}\left(4(b_1 + a + b_2) - 3b_1 - 3b_2\right)$$

$$= \frac{h^3}{12}\left(b_1(4-3) + 4a + b_2(4-3)\right)$$

$$= \frac{h^3}{12}(b_1 + b_2 + 4a) = \frac{h^3}{12}(b_1 + b_2 + a + 3a)$$

$$I_x = \frac{h^3}{12}(3a + b)$$

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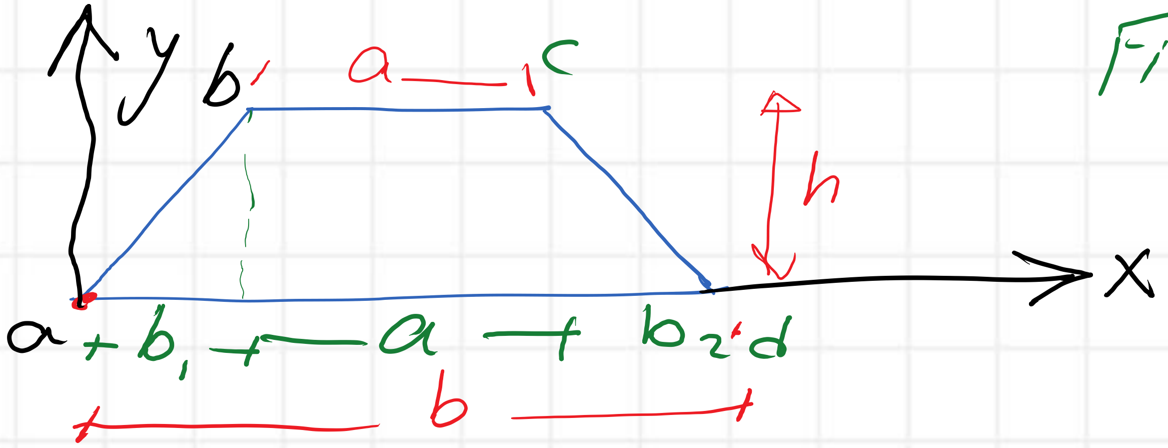
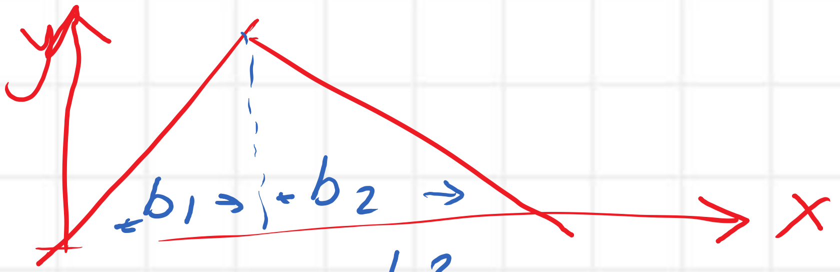


Finally $I_x = \frac{h^3}{12}(3a+b)$

if $a = 0 \Rightarrow b = b_1 + b_2$



$I_x = \frac{h^3}{12}((3(0)+(b_1+b_2))$

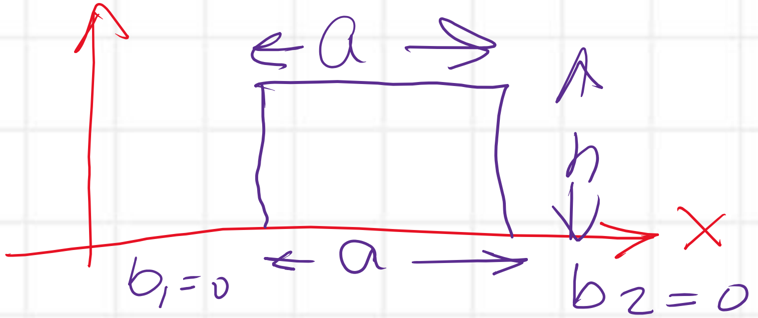
$I_x = \frac{h^3}{12}(b_1+b_2)$ which is the case of Triangle

Check the expression

Case of a triangle

For a rectangle

$b = a$



$\Rightarrow I_{x_{rec}} = \frac{h^3}{12}(4a) = \frac{h^3 a}{3}$

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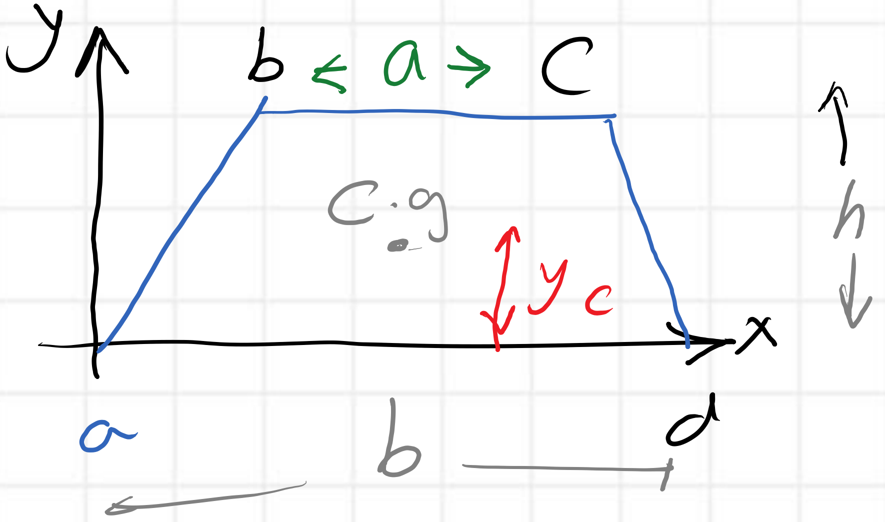
To get I_{xG} For Trapeziod : Use Parallel axes theorem



$$y_c = \frac{h(2a+b)}{3(a+b)}$$

$$I_x = I_{x_{cg}} + A y_c^2$$

$$I_x = \frac{h^3}{12}(3a+b)$$

$$A_{Tr} = \left(\frac{a+b}{2}\right)h$$

$$I_{x_g} = I_x - A \cdot \bar{y}_c^2 = \frac{h^3}{12}(3a+b) - \frac{1}{2}(a+b).h\left[\frac{h^2(2a+b)^2}{9(a+b)^2}\right]$$

$$I_{xg} = \frac{h^3}{18 \times 12}\left[\frac{(3a+b)(18)(a+b)^2 - 12(a+b)(2a+b)^2}{(a+b)^2}\right]$$

take $(a+b)$ as a Common Factor

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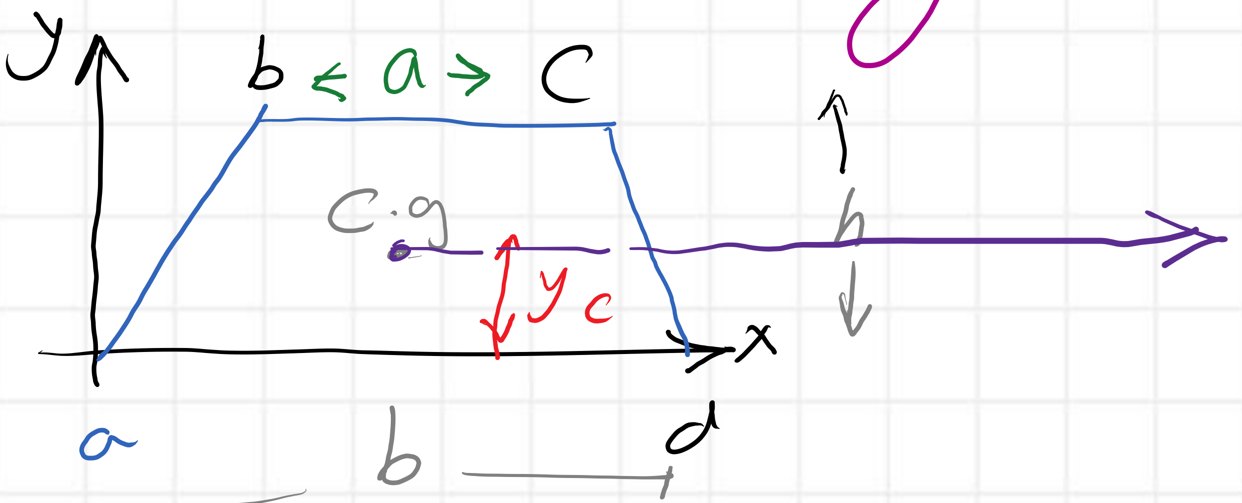
I_x at the Cg for a Trapezoid



$$I_{xg} = \frac{h^3}{18 \times 12}\left[\frac{(3a+b)(18)(a+b)^2 - 12(a+b)(2a+b)^2}{(a+b)^2}\right]$$

take $a+b)$ as a Common Factor

$$I_{xg} = \frac{h^3}{18(12)}\frac{(a+b)}{(a+b)^2}\left[(3a+b)(18a+18b) - 12(2a+b)^2\right]$$

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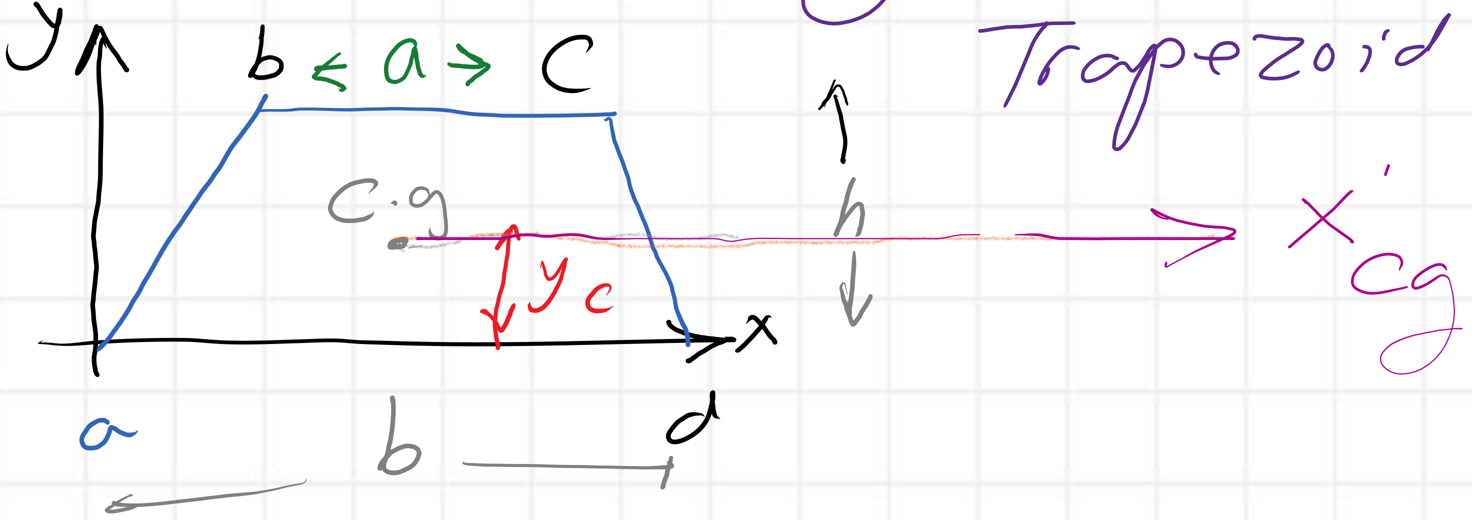
I_x at the Cg of Trapezoid

Continue Estimating



$$I_{xg} = \frac{h^3}{18(12)} \frac{(a+b)}{(a+b)^2} \left[(3a+b)(18a+18b) - 12(2a+b)^2 \right]$$

$$I_{xg} = \frac{h^3 \quad (1)}{18(12)(a+b)} \left[54a^2 + 72ab + 18b^2 - 48a^2 - 48ab - 12b^2 \right]$$

$$\left[a^2(54-48) + ab(72-48) + b^2(18-12) \right]$$

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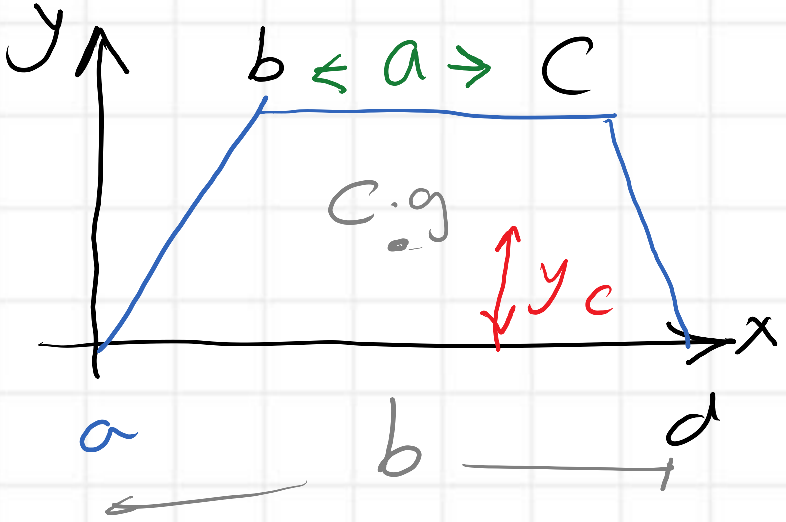
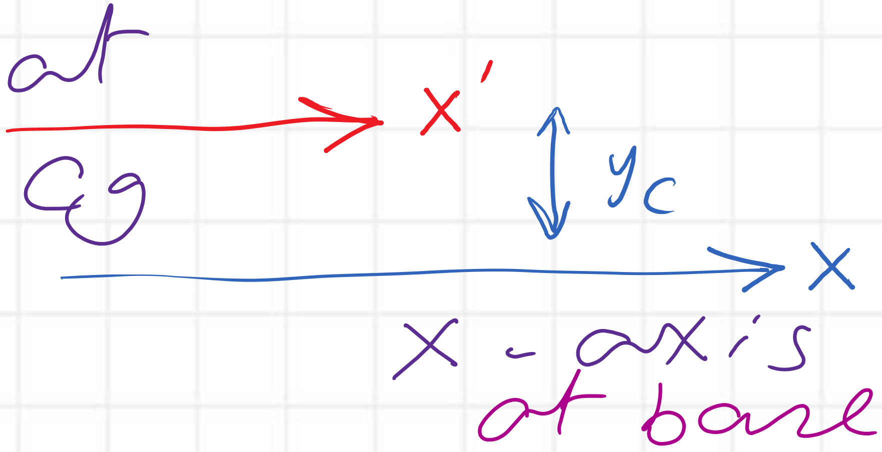
To get I_{xG} For Trapeziod





$$I_{xg} = \frac{h^3}{18}\left(\frac{1}{12(a+b)}\right)[6a^2+24ab+6b^2)$$

$$I_{xg} = \frac{6h^3}{18(12)}\,\frac{1}{a+b}[a^2+4ab+b^2)$$

$$I_{xg} = \frac{h^3}{36}\left(\frac{1}{a+b}\right)[a^2+4ab+b^2)$$

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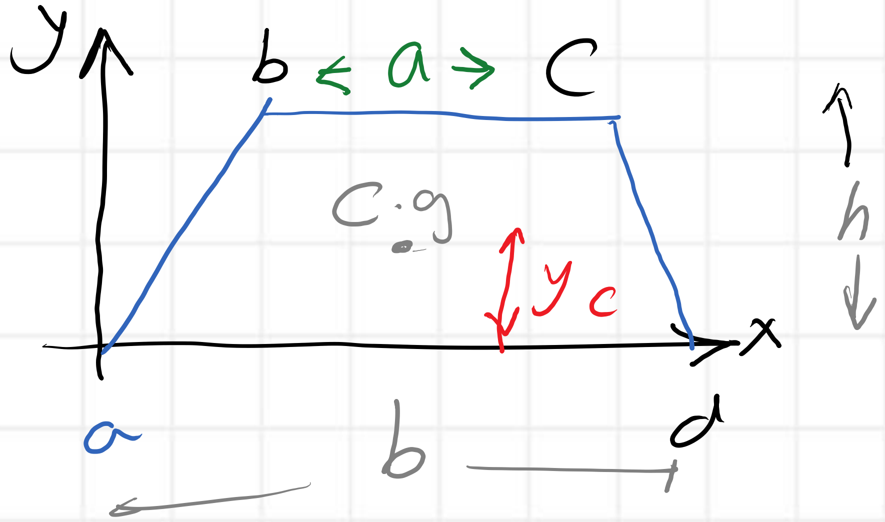
Radius of Gyration

First about X-axis



$$I_x = \frac{h^3}{12}(3a+b)$$

Since

$$A_{Tr} = \frac{1}{2}(a+b)h$$

$$I_x = A_{Tr}(r_x^2) \Rightarrow r_x^2 = \frac{I_x}{A} = \frac{h^3}{12}(3a+b)\left(\frac{1}{\frac{(a+b)h}{2}}\right)$$

$$r_x^2 = \frac{h^2}{6}(3a+b)\left(\frac{1}{a+b}\right)$$

$$r_{x_c}^2 = \frac{h^2(a^2+4ab+b^2)}{18(a+b)}$$

$$r_x^2 = \frac{h^2(3a+b)}{6(a+b)}$$

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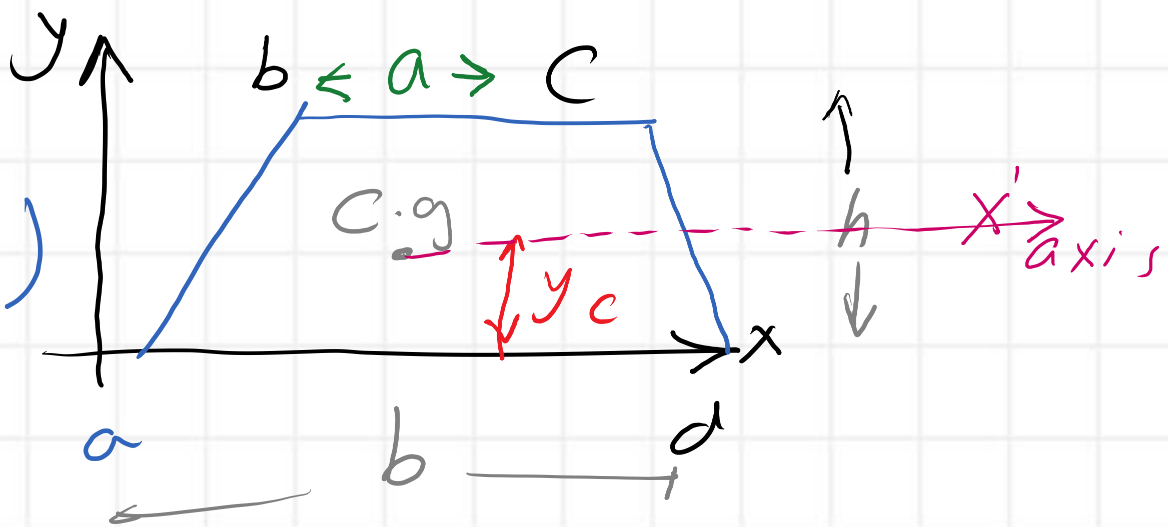
Radius of Gyration about Cg

$$I_{xg} = \frac{h^3}{36}\left(\frac{1}{a+b}\right)\left[a^2 + 4ab + b^2\right)$$



Since

$$A_{Tr} = \frac{1}{2}(a+b)h$$

$$I_x = A_{Tr}(r_x^2) \Rightarrow r_x^2 = \frac{I_x}{A} =$$

$$r^2_{x\,c.g} = \frac{h^3}{36}\left(\frac{1}{a+b}\right)\frac{(a^2+4ab+b^2)}{1} \Big/ \frac{1}{2}(a+b)h$$

$$r^2_{x\,cg} = \frac{h^2}{18}(a^2 + 4ab + b^2)$$

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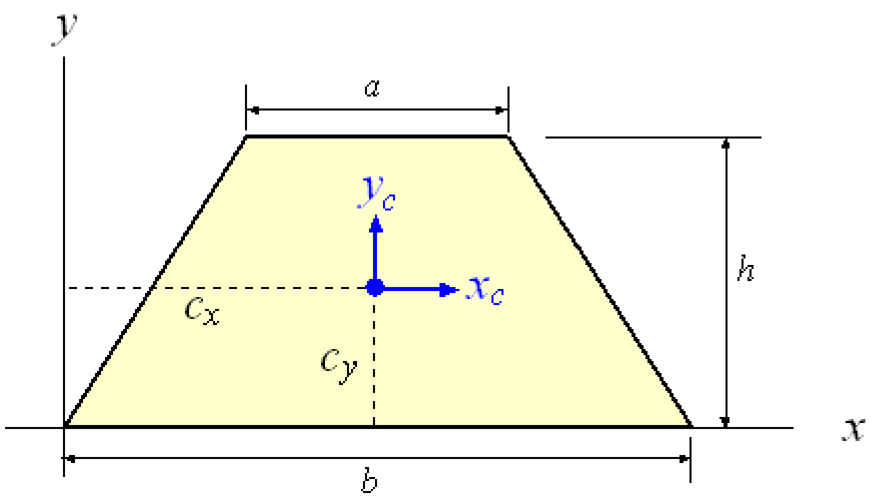

Moment of Inertia
about the *x* axis
I_x

$$\frac{h^3(3a+b)}{12}$$

Moment of Inertia
about the x_c axis
I_{xc}

$$\frac{h^3(a^2+4ab+b^2)}{36(a+b)}$$

Final Expression
for Ixc

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