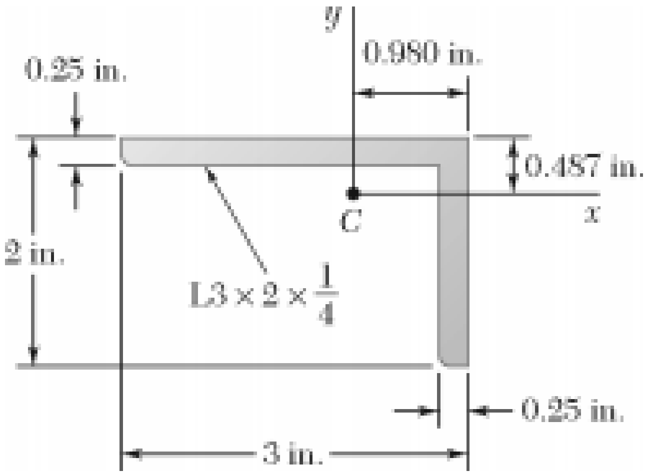
Vector mechanics For Engineers → BEER



PROBLEM 9.83

Determine the moments of inertia and the product of inertia of the L3×2×$\frac{1}{4}$-in. angle cross section of Problem 9.74 with respect to new centroidal axes obtained by rotating the x and y axes 30° clockwise.

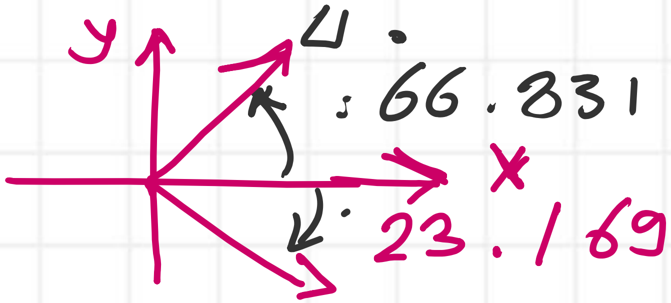
Solution From part-1 we have

$I_x = 0.39$ inch^4

$I_y : 1.09$ inch^4

$I_{xy} : -0.3798$ inch^4

$R : 0.5166$ inch^4



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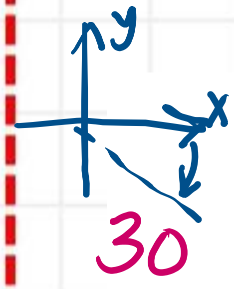

α :

Given direction

$2\theta = 2\alpha = -60$ $\qquad \cos(-60) = \frac{1}{2}$

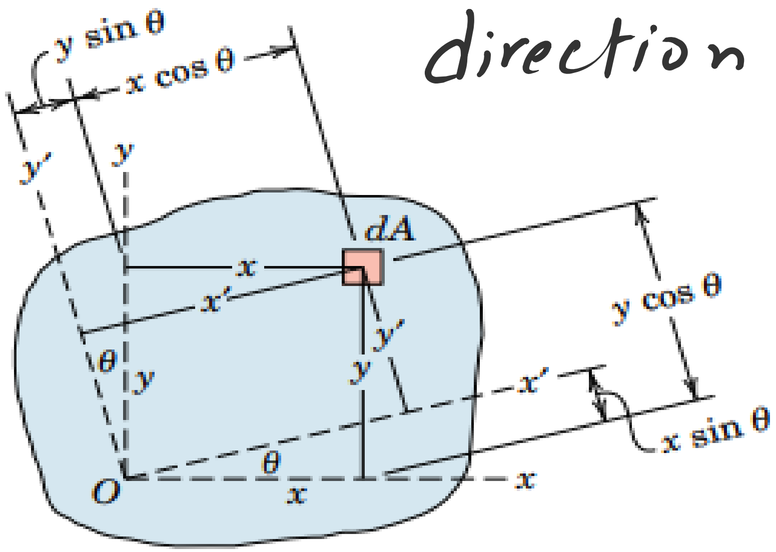

Figure A/6

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x}$$

$I_x : 0.39$ inch^4

$I_y : 1.09$ inch^4

$I_{xy} = -0.3798$ inch^4

$I_p = 1.48$ inch^4

$I_x + I_y$

$\sin(-60) = -0.8660$

$$I_{x'} = \left(\frac{0.39+1.09}{2}\right) + \left(\frac{0.39-1.09}{2}\right)\left(\frac{1}{2}\right) - (-0.3798)(-0.8660)$$

$$= 0.74 - 0.175 - 0.3289 = 0.2361 \text{ inch}^4$$

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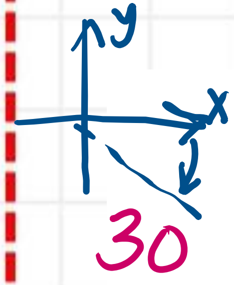

For a given direction

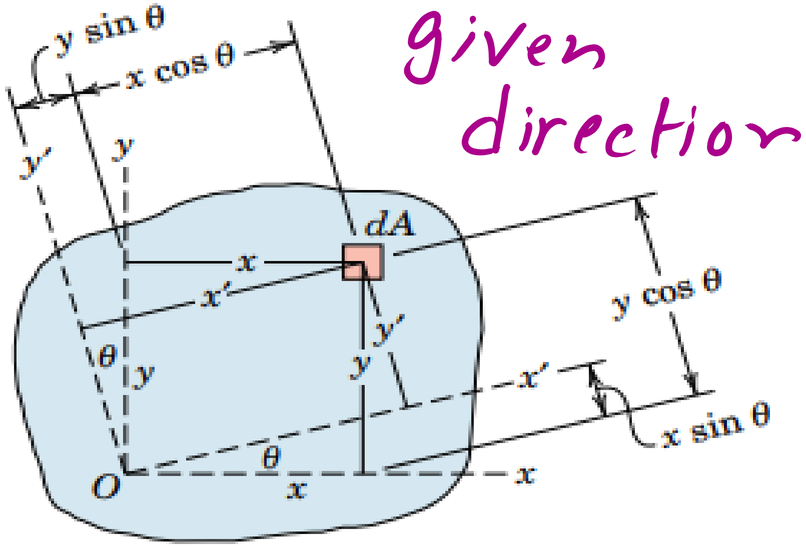

Figure A/6

$2\theta = 2\alpha = -60$

$\cos(-60) = \frac{1}{2}$

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$I_{y'}$

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x}$$

$I_x : 0.39$ inch^4

$I_y : 1.09$ inch^4

$I_{xy} = -0.3798$ inch^4

$I_p = 1.48$ inch^4

$+ I_y$

$\sin(-60) = -0.8660$

$I_{y'} = \left(\frac{0.39 + 1.09}{2}\right) - \left(\frac{0.39 - 1.09}{2}\right)(1/2) + (-0.3798)(-0.8660)$

$I_{y'} = 0.74 + 0.175 + 0.3289 = 1.2439$ inch^4

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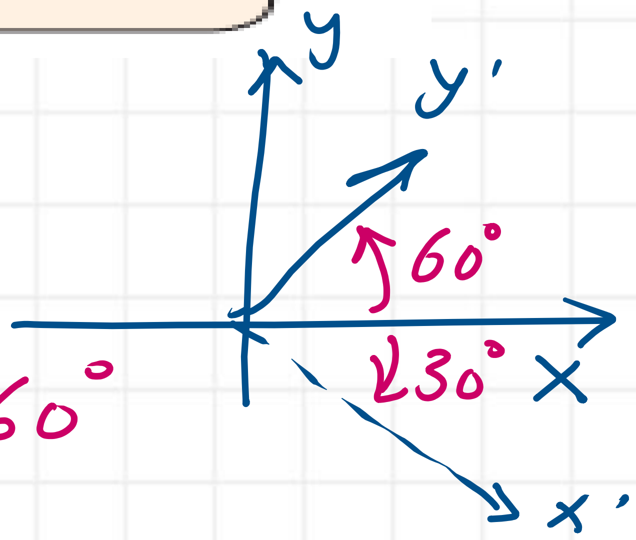
$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$I_x : 0.39 \text{ inch}^4$

$I_y : 1.09 \text{ inch}^4$

$I_{xy} : -0.3798 \text{ inch}^4$



$2\alpha = 2(30) = 60^\circ$ clockwise $\Rightarrow -60^\circ$

$$I_{x'y'} = \left(\frac{0.39 - 1.09}{2}\right)\sin(-60) + (-0.3798)\cos(-60)$$

$$= -0.35(-0.8660) - 0.3798\left(\tfrac{1}{2}\right)$$

$$I_{x'y'} = +0.3031 - 0.1899 = 0.1132 \text{ inch}^4$$

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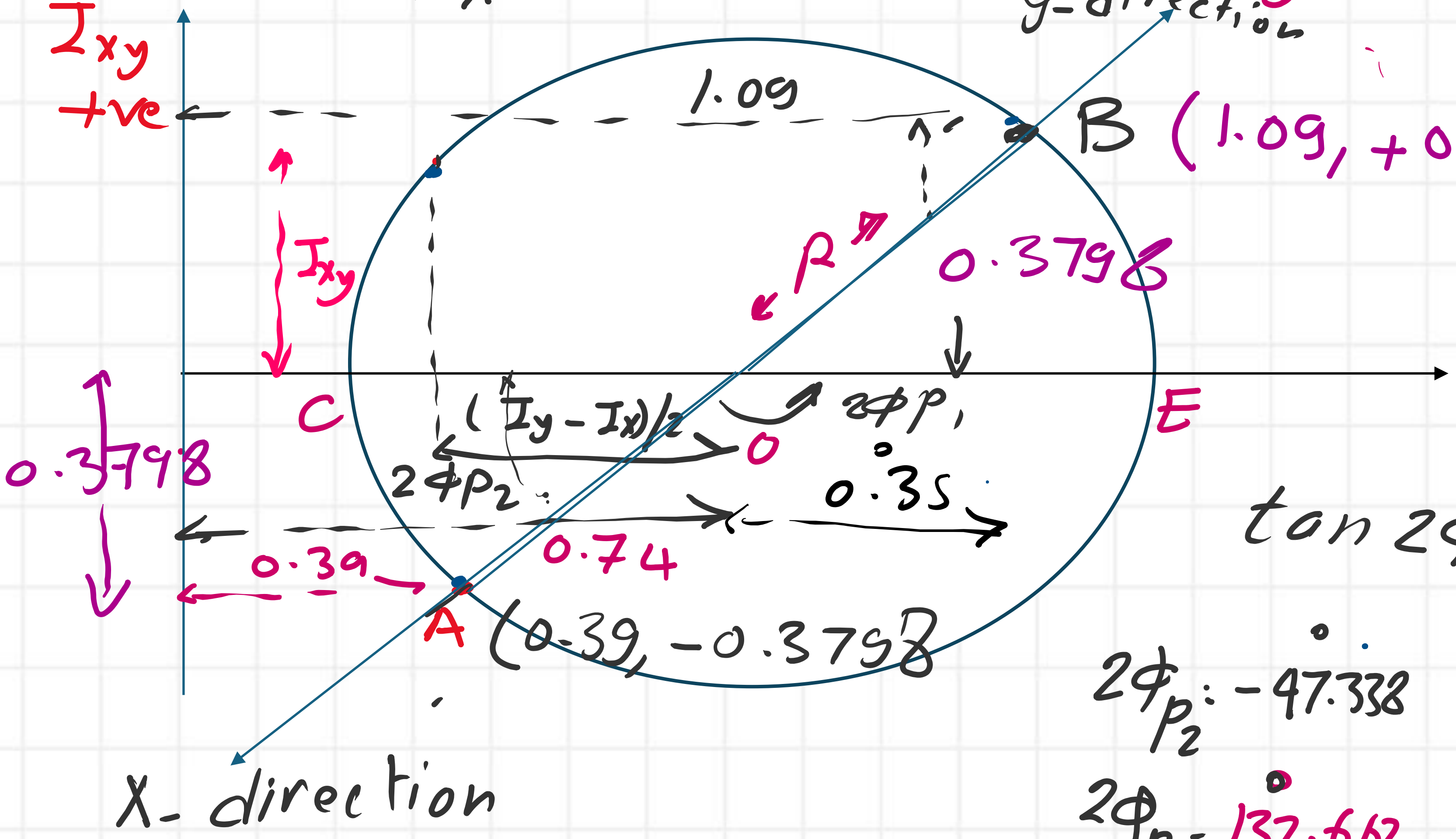
Case # 4 $I_x < I_y$, I_x is -ve

$I_p = 1.48\ inch^4$

$I_x : 0.39\ inch^4$

$I_y : 1.09\ inch^4$, $I_{xy} = -0.3798\ inch^4$



$$\tan 2\phi p = \frac{I_{xy}}{\left(\frac{I_y - I_x}{2}\right)} = \frac{-0.3798}{\left(\frac{1.09 - 0.3}{2}\right)} = -1.0851$$

$$\tan 2\phi = \frac{I_{xy}}{\left(\frac{I_y - I_x}{2}\right)} = \frac{-0.3798}{\frac{1}{2}(1.09 - 0.39)} = -1.0851$$

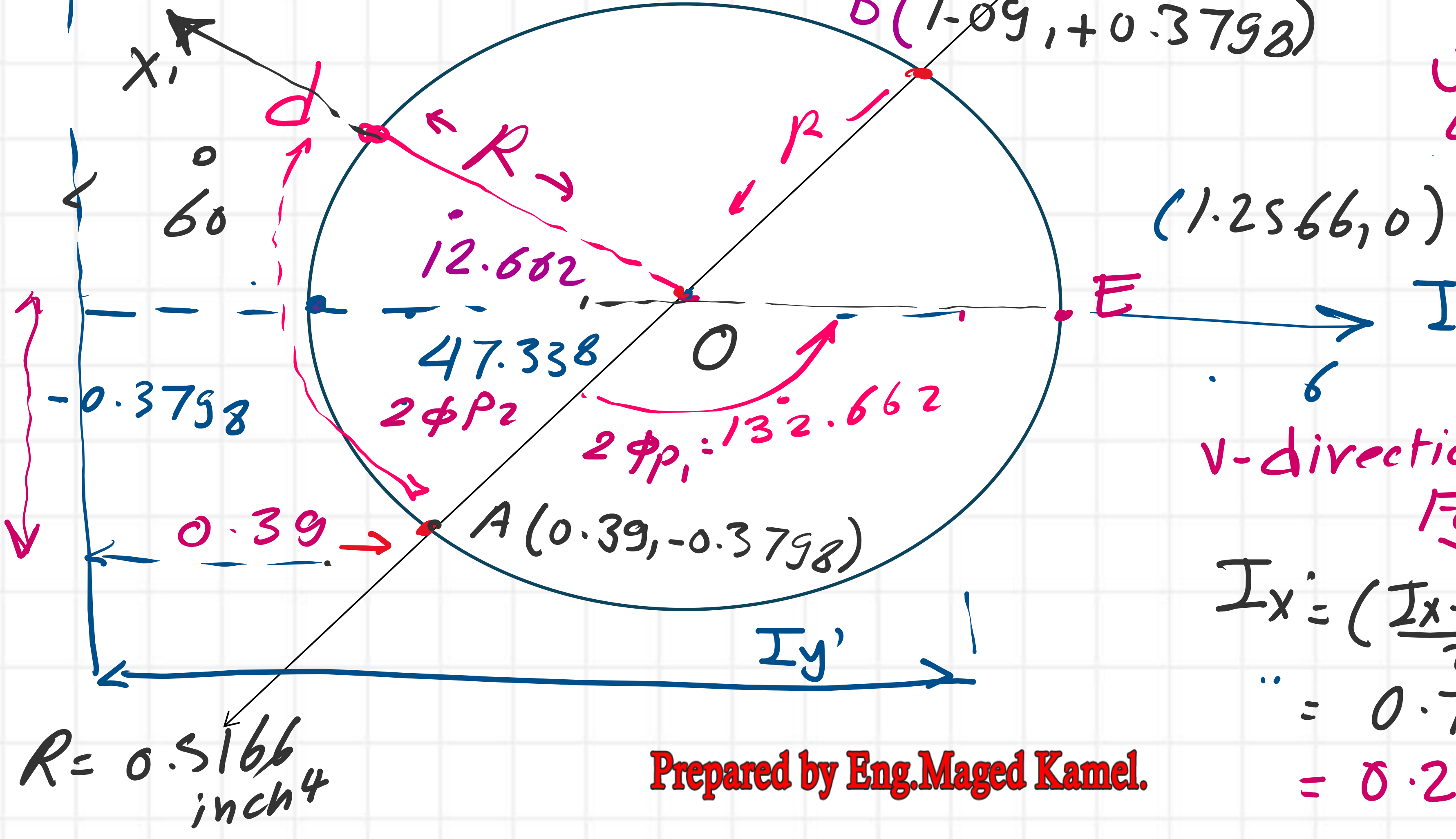
$2\phi_{p_2} : -47.338^\circ$

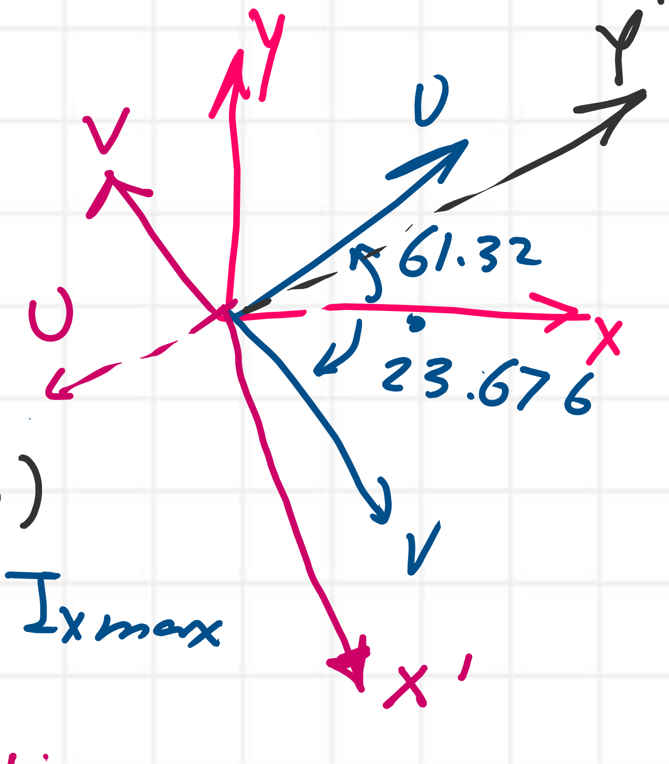
$2\phi_{p_1} = 132.662^\circ$

$I_x : 0.39 \text{ inch}^4$, $I_{xy} : -0.3798 \text{ inch}^4$

$I_y : 1.09 \text{ inch}^4$

$R : 0.5166 \text{ inch}^4$





v-direction

For point d

$$I_{x'} = \left(\frac{I_x + I_y}{2}\right) - R\cos(12.662^\circ)$$

$$= 0.74 - (0.5166)(0.9756)$$

$$= 0.2359 \text{ inch}^4$$

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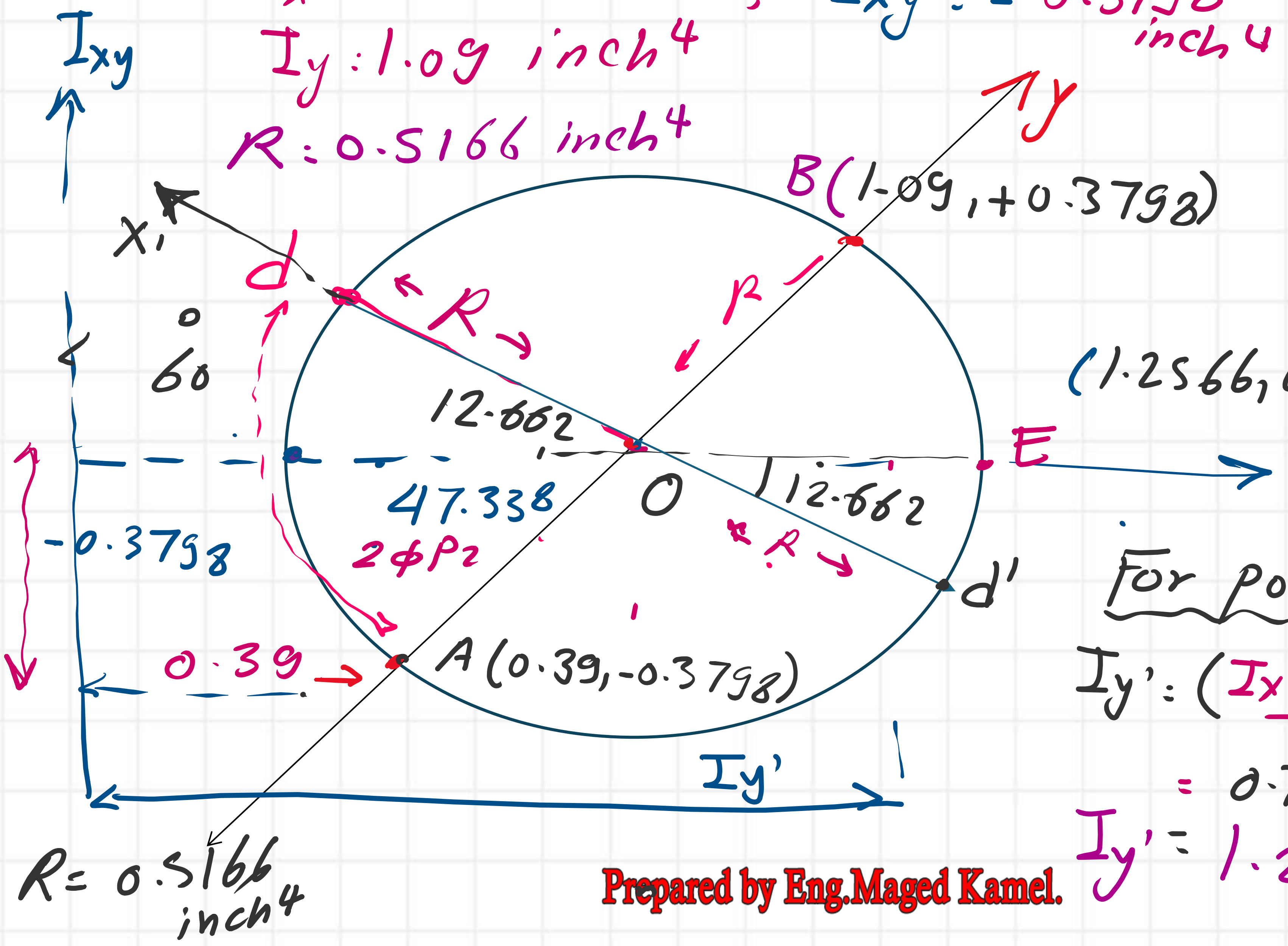
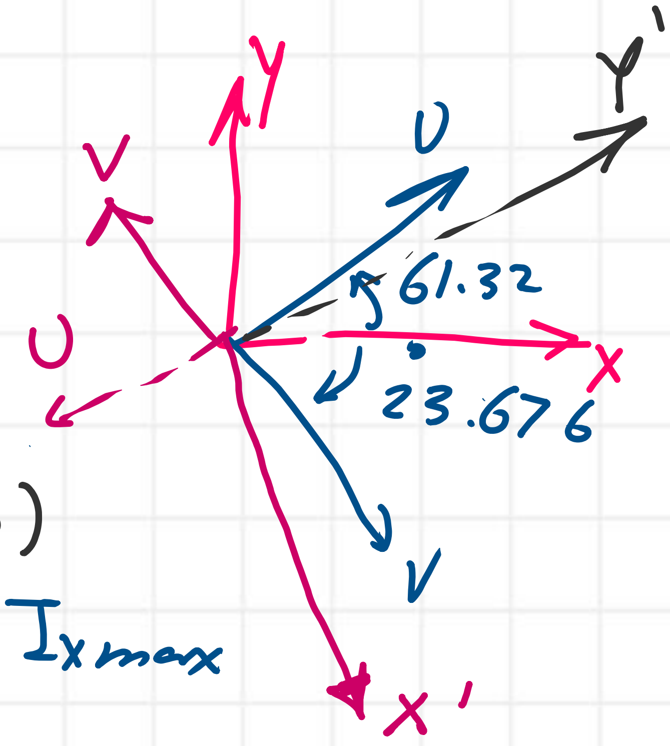
$I_x : 0.39\ inch^4$, $I_{xy} : -0.3798\ inch^4$

$I_y : 1.09\ inch^4$

$R : 0.5166\ inch^4$





For point d'

$$I_{y'} : \left(\frac{I_x+I_y}{2}\right) + R\cos(12.662)$$

$$= 0.74 + (0.5166)(0.97568)$$

$$I_{y'} = 1.244\ inch^4$$

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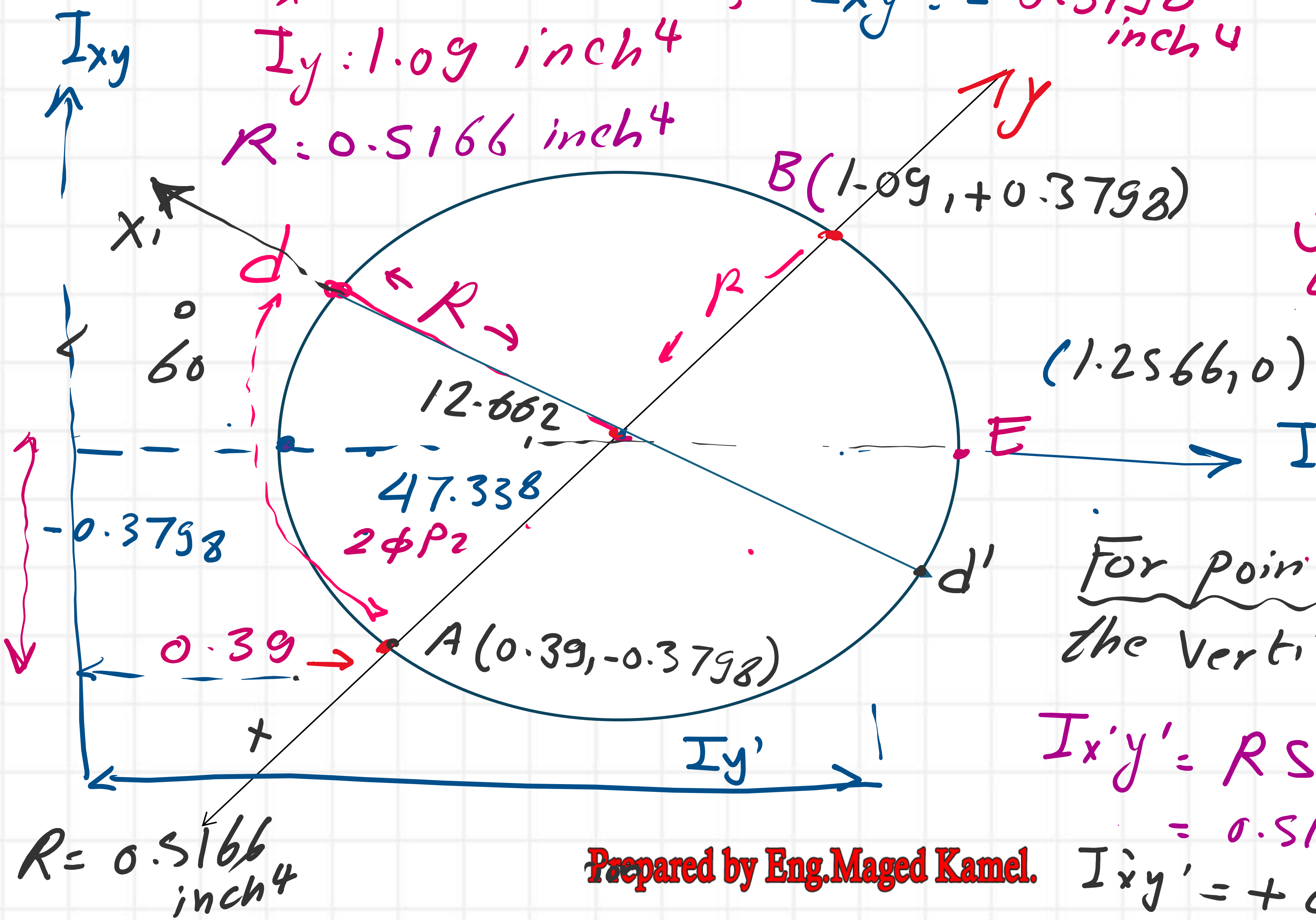
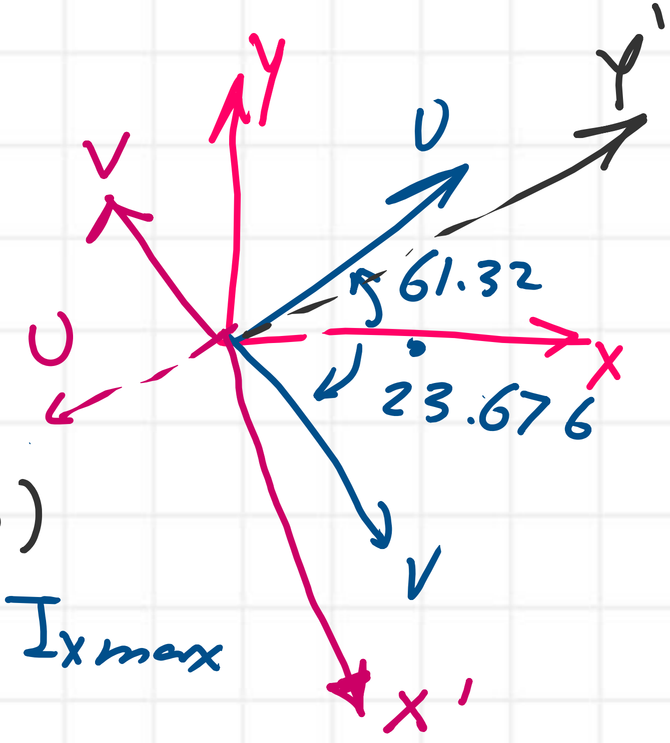
$I_x : 0.39 \text{ inch}^4$, $I_{xy} : -0.3798 \text{ inch}^4$

$I_y : 1.09 \text{ inch}^4$

$R : 0.5166 \text{ inch}^4$





For point d

the Vertical Cordinate

$I_{x'y'} = R \sin(12.662)$

$= 0.5166\,(0.2192)$

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$I_{x'y'} = +0.1132 \text{ inch}^4$

Prepared by Eng.Maged Kamel.