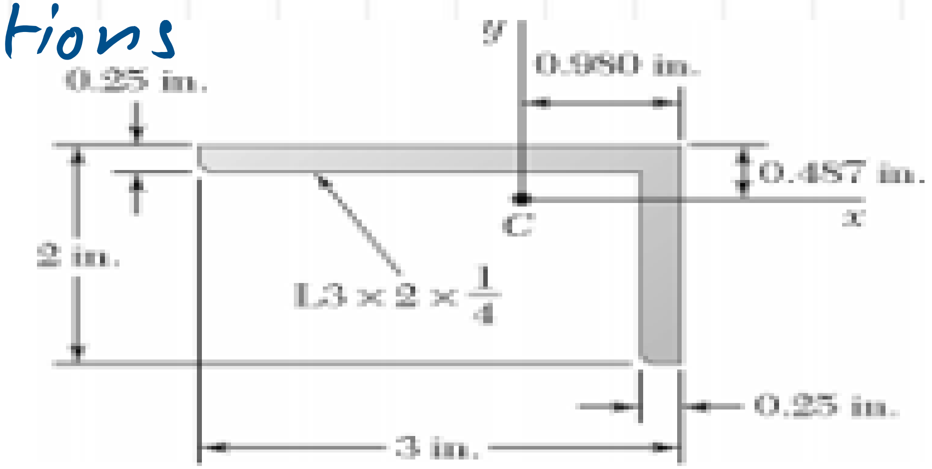
Vector mechanics For Engineers → BEER

Determine the principal moments of inertia for the given section and their directions



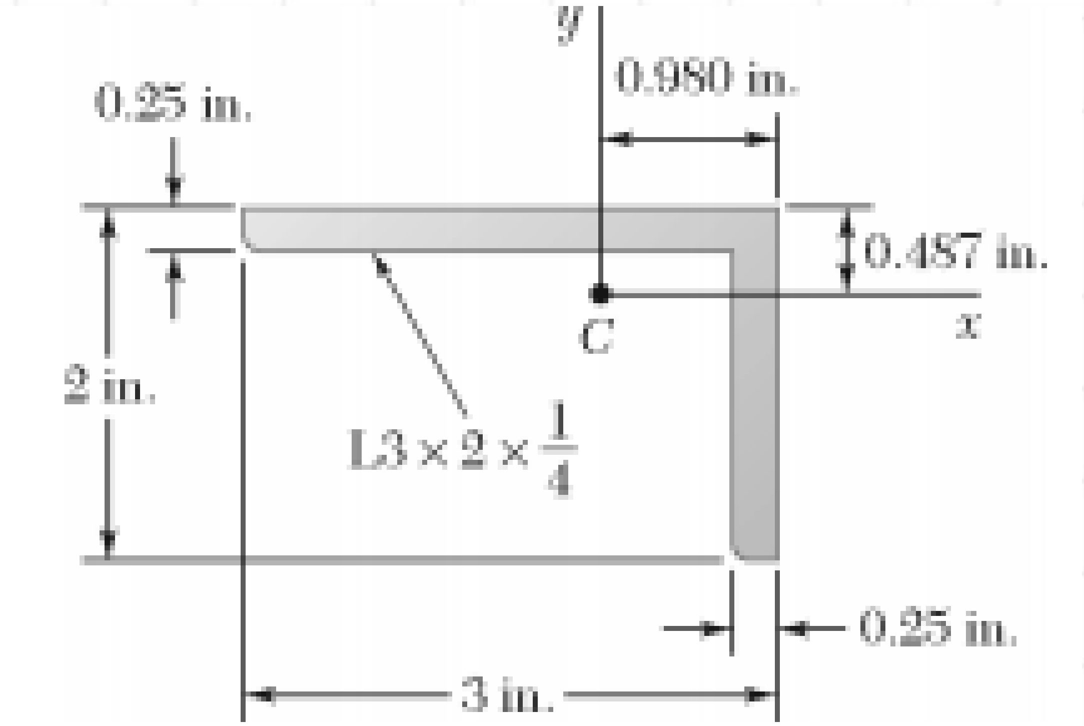
Solution

Use table 9.7 to get the necessary informations

For L $3'' \times 2'' \times \frac{1}{4}''$

Prepared by Eng.Maged Kamel.

Get the Centroidal principal moment of Inertia For the given L $3'' \times 2'' \times \frac{1}{4}''$



Solution

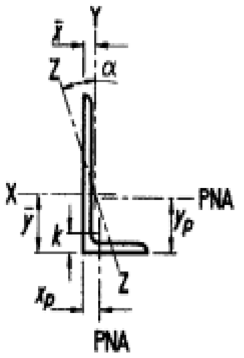


Table 1–7 (continued)

Angles

Properties

Shape	k	Wt.	Area, A	Axis X-X						Flexural-Torsional Properties		
				I	S	r	$\bar{y}$	Z	y_p	J	C_w	$\bar{r}_o$
	in.	lb/ft	in.^{2}	in.^{4}	in.^{3}	in.	in.	in.^{3}	in.	in.^{4}	in.^{6}	in.
L3×2×1/2	13/16	7.70	2.25	1.92	1.00	0.922	1.08	1.78	0.736	0.192	0.0908	1.39
×3/8	11/16	5.90	1.73	1.54	0.779	0.937	1.03	1.39	0.668	0.0855	0.0413	1.42
×5/16	5/8	5.00	1.46	1.32	0.662	0.945	1.01	1.19	0.633	0.0510	0.0248	1.43
×1/4	9/16	4.10	1.19	1.09	0.541	0.953	0.980	0.969	0.596	0.0270	0.0132	1.45
×3/16	1/2	3.07	0.902	0.847	0.414	0.961	0.952	0.743	0.556	0.0119	0.00576	1.46

From table 1-7

$A = 1.19$ inch^2

$\bar{x} = 0.980$ → in table it is $\bar{y}$ →

$\bar{Y} = 0.487$

Prepared by Eng.Maged Kamel.

$I_x = 1.09$ inch^4 → I_y in our Problem

$I_y > I_x$

I_{xy} -ve

Table 1–7 (continued)
Angles
Properties

L3-L2

Shape	Axis Y-Y						Axis Z-Z				Q_s
	I	S	r	$\bar{x}$	Z	x_p	I	S	r	Tan α	$F_y = 36$ ksi
	in.^{4}	in.^{3}	in.	in.	in.^{3}	in.	in.^{4}	in.^{3}	in.		
L3×2×1/2	0.667	0.470	0.543	0.580	0.887	0.377	0.409	0.266	0.425	0.413	1.00
×3/8	0.539	0.368	0.555	0.535	0.679	0.291	0.318	0.209	0.426	0.426	1.00
×5/16	0.467	0.314	0.562	0.511	0.572	0.247	0.271	0.179	0.428	0.432	1.00
×1/4	0.390	0.258	0.569	0.487	0.463	0.200	0.223	0.149	0.431	0.437	1.00

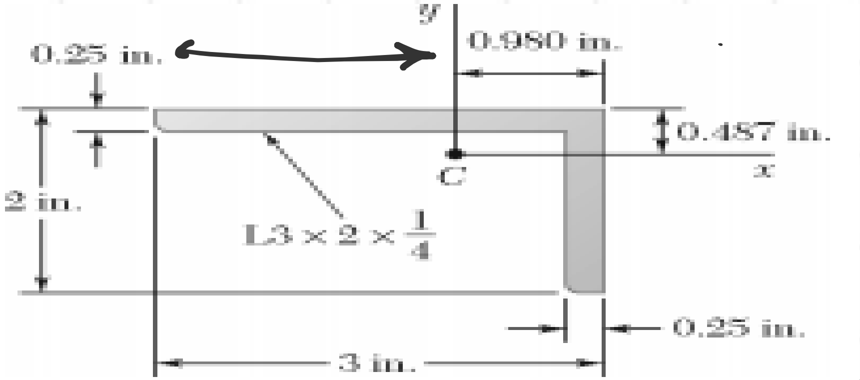
from table

$I_y = 0.39$ inch^4

$\Downarrow$

For our problem

$= I_x = 0.39$ inch^4



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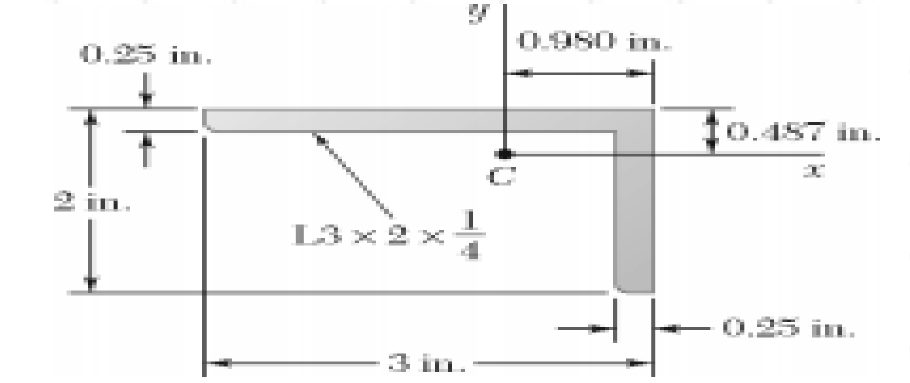
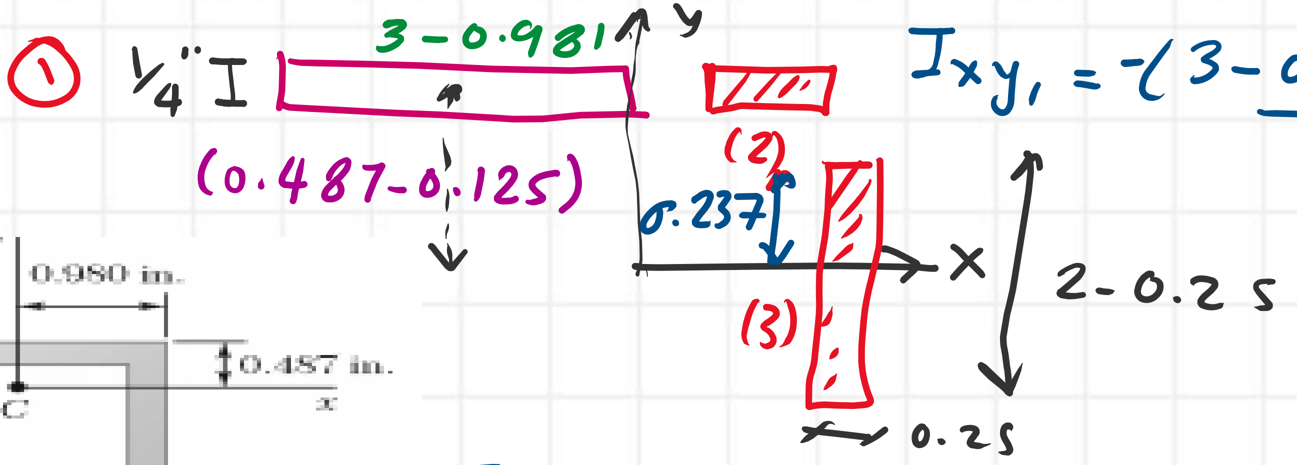
Estimate I_{xy}





$$I_{xy_1} = -\left(\frac{3-0.98}{2}\right)^2\left(\frac{1}{4}\right)(0.362)$$

$$I_{xy_2} = 0.98\left(\frac{1}{4}\right)(0.487-0.125)\left(\frac{0.98}{2}\right)$$

$$I_{xy_3} = \left(2-\frac{1}{4}\right)\left(\frac{1}{4}\right)(-)\left(0.98-\frac{1}{8}\right)\left(\frac{1.75}{2}-0.237\right)$$

$=$

$I_{xy_1} = -0.1846$ inch^4

$I_{xy_2} = +0.0435$ inch^4

$I_{xy_3} = -0.2386$ inch^4

$\Sigma I_{xy} = -0.3798$ inch^4

$I_x = 0.39$ inch^4

$I_y = 1.09$ inch^4

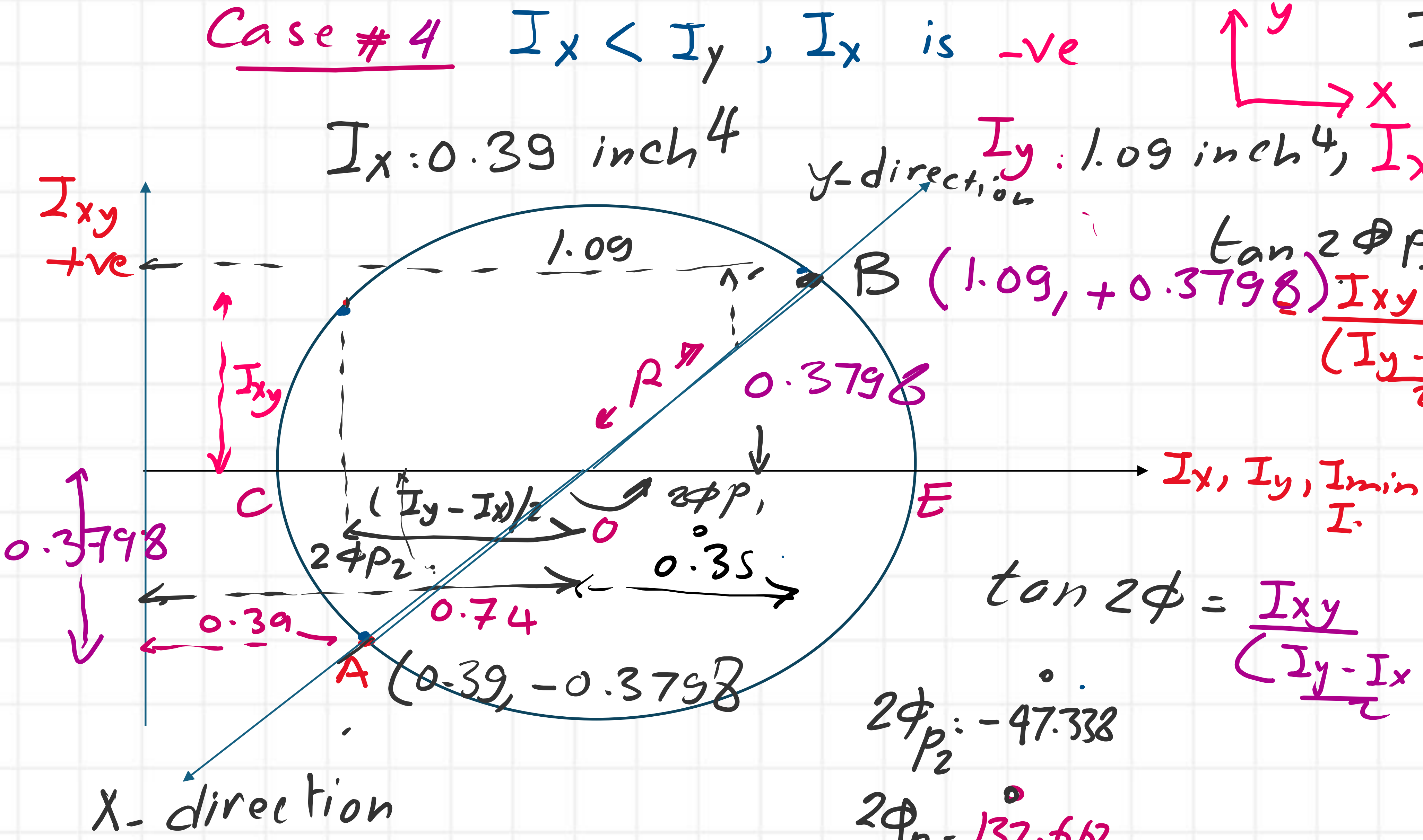
$I_{xy} = -0.3798$ inch^4

Prepared by Eng.Maged Kamel.

Case # 4 $I_x < I_y$, I_x is -ve

$I_p = 1.48$ inch^4

$I_x : 0.39$ inch^4 , $I_y : 1.09$ inch^4, $I_{xy} = -0.3798$ inch^4



$$\tan 2\phi p = \frac{I_{xy}}{\frac{(I_y - I_x)}{2}} = \frac{-0.3798}{\frac{(1.09 - 0.39)}{2}} = -1.0851$$

$$\tan 2\phi = \frac{I_{xy}}{\frac{(I_y - I_x)}{2}} = \frac{-0.3798}{\frac{1}{2}(1.09 - 0.39)} = -1.0851$$

$2\phi_{p_2} : -47.338^\circ$

$2\phi_{p_1} = 132.662^\circ$

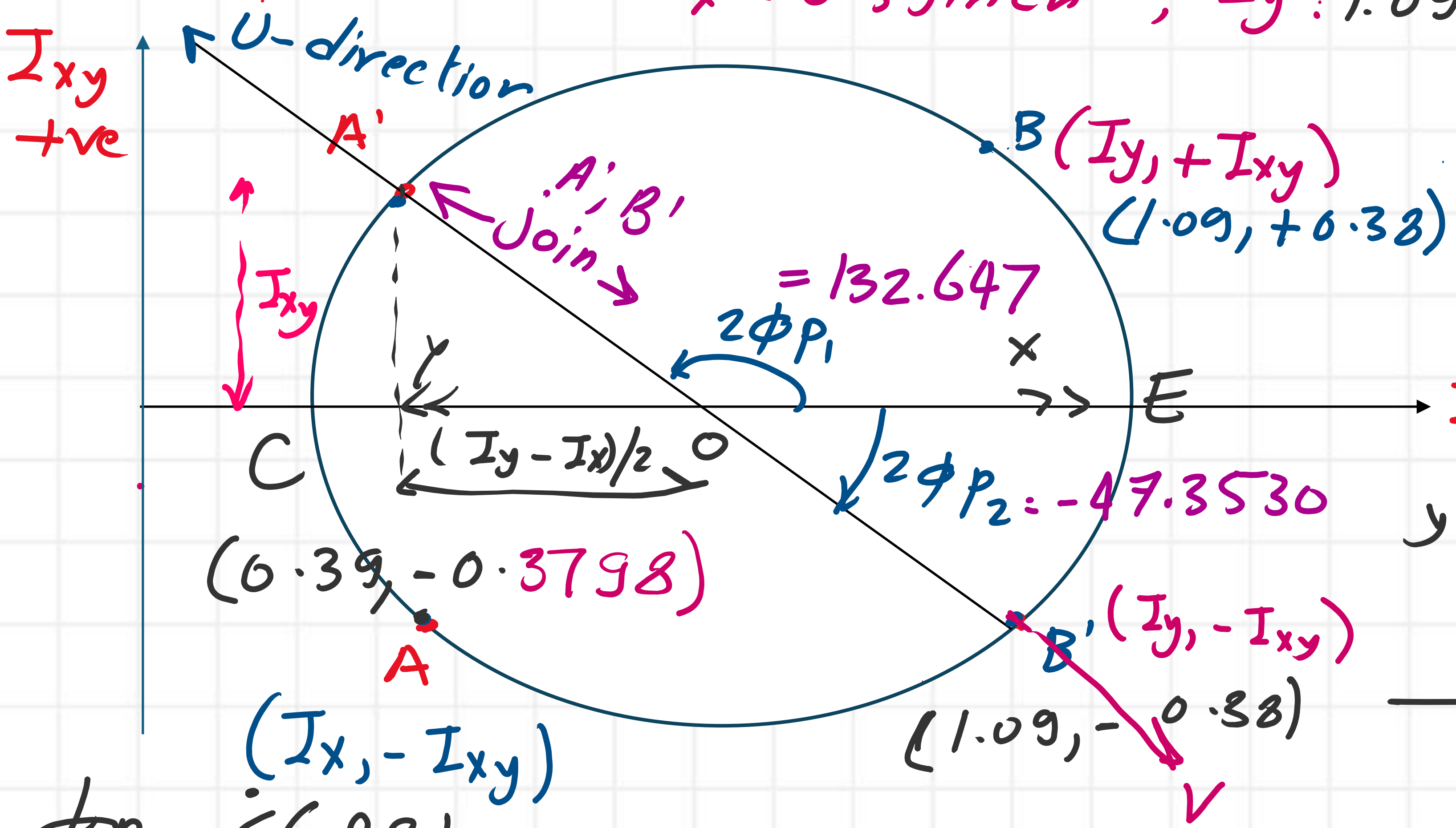
Prepared by Eng.Maged Kamel.

Case # 4

$I_x < I_y$, I_{xy} is -ve

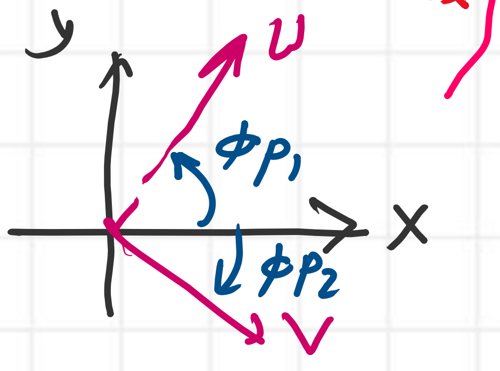
Use A', B'

$I_x = 0.39\ inch^4$, $I_y : 1.09\ inch^4$, $I_{xy} : -0.3798\ inch^4$



$2\phi p_1 : 132.662°$

$2\phi p_2 : 47.3530$



$\phi p_1 = 66.831°$

$\phi p_2 = -23.169°$

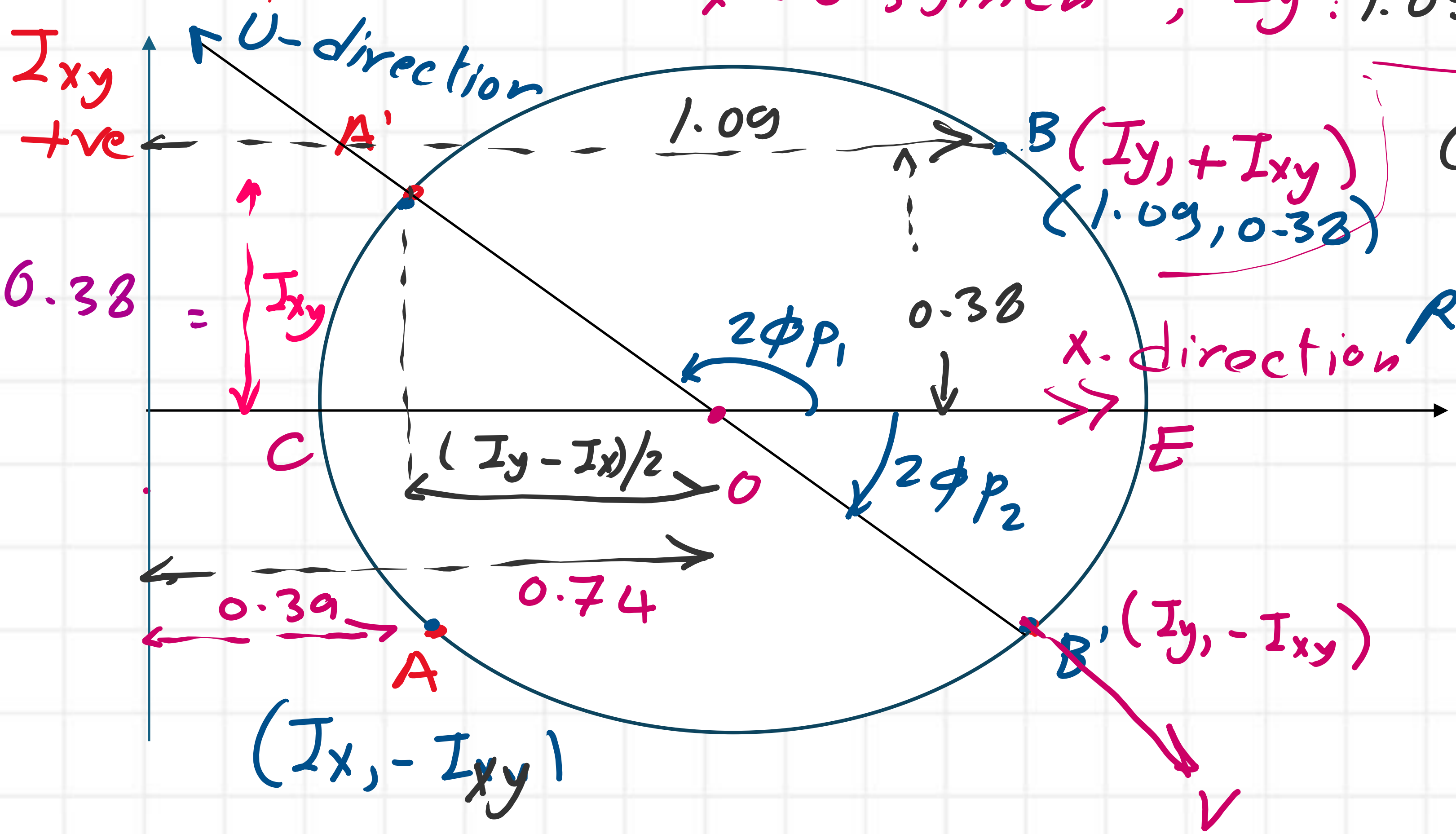
Prepared by Eng.Maged Kamel.

Case # 4 $I_x < I_y$, I_x is -ve

$I_p = 1.48$ inch4

Use A', B'

$I_x = 0.39$ inch4 , $I_y : 1.09$ inch4, $I_{xy} = -0.3798$ inch4



I_x, I_y, I_{min}
I_{max}

Center point
(0.74, 0)

$R : 0.5165$ inch4

$I_{min} = 0.74 - 0.5165$
0.2235 inch4

$I_{max} = 0.74 + 0.5165 = 1.2565$ inch4

$I_{min} + I_{max} = 1.48$ inch4 $= I_P$

Prepared by Eng.Maged Kamel.

$2\phi P_1 : 132.662$

$\cos 132.662 :- 0.6776$

$2\theta = 2\phi P_1$

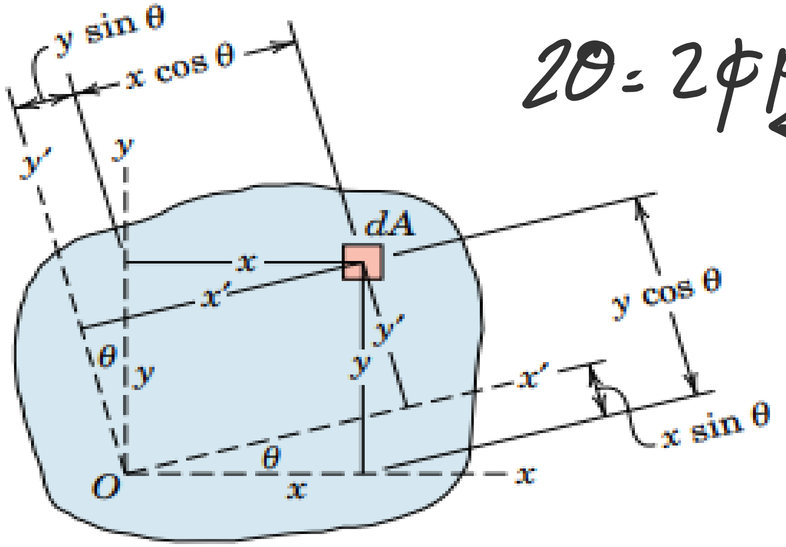


Figure A/6

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta \rightarrow I_U$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta \rightarrow = I_V$$

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta = \text{Zero}$$

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x}$$

$I_x : 0.39$ inch⁴

$I_y : 1.09$ inch⁴

$I_{xy} = -0.3798$ inch⁴

$\sin(132.662) = 0.7354$

$$I_{x'} = \frac{(0.39) + 1.09}{2} + \left(\frac{-0.70}{2}\right)(-0.6776) - (-0.3798)(0.7354)$$

$$= 0.74 + 0.2372 + 0.2793 = 1.2565 \text{ inch}^4$$

Prepared by Eng.Maged Kamel.

$2\phi P_1 : 132.662$

$\cos 132.662 :- 0.6776$

$\theta = 2\phi P_1$

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta \rightarrow I_U$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta \rightarrow = I_V$$

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta = \text{Zero}$$

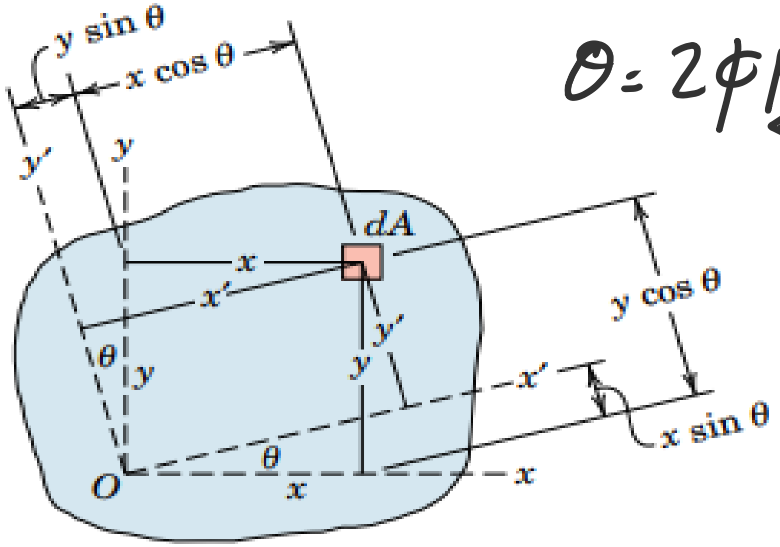


Figure A/6

$I_x : 0.39$ inch^4

$I_y : 1.09$ inch^4

$I_{xy} := -0.3798$ inch^4

$I_p = 1.48$ inch^4

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x}$$

$\sin(132.662) = 0.7354$

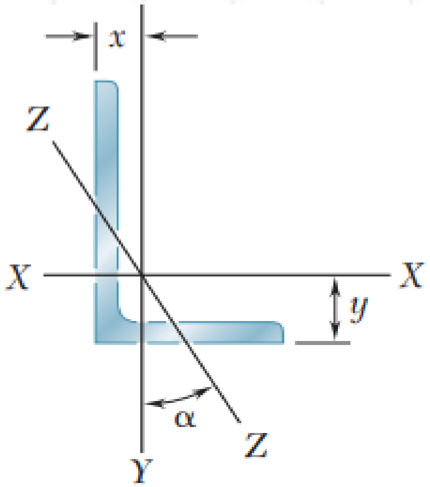
$$I_{y'} = \frac{1}{2}(0.39 + 1.09) - \left(\frac{-0.70}{2}\right)(-0.6776) + (-0.3798)(0.7354)$$

$$= 0.74 - 0.2372 - 0.2793 = 0.2235 \text{ inch}^4$$

Prepared by Eng.Maged Kamel.

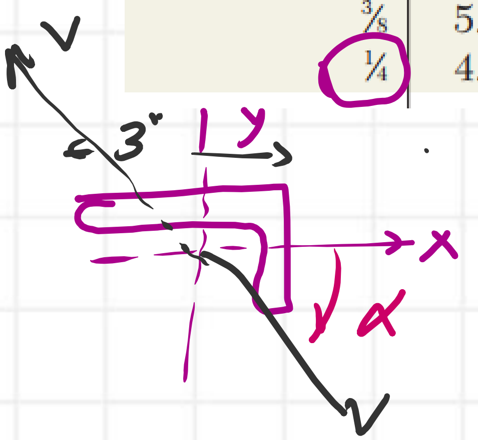
APPENDIX C **Properties of Rolled-Steel Shapes**
(U.S. Customary Units)

Angles
Unequal Legs



$I_{min} = A r_z^2$

Size and Thickness, in.	Weight per Foot, lb/ft	Area, in^2	Axis X-X				Axis Y-Y				Axis Z-Z	
			I_x, in^4	S_x, in^3	r_x, in.	y, in.	I_y, in^4	S_y, in^3	r_y, in.	x, in.	r_z, in.	tan α
L3 × 2 × ½	7.70	2.25	1.92	1.00	0.922	1.08	0.667	0.470	0.543	0.580	0.425	0.413
⅜	5.90	1.73	1.54	0.779	0.937	1.03	0.539	0.368	0.555	0.535	0.426	0.426
¼	4.10	1.19	1.09	0.541	0.953	0.980	0.390	0.258	0.569	0.487	0.431	0.437



From Estimation

$I_V : 0.2235$ inch^4

$\alpha : 23.669^\circ$

$I_V : 0.431^2 (1.19)$

$= 0.22106$ inch4

$\alpha : \tan^{-1} 0.437 : 23.605^\circ$

Prepared by Eng.Maged Kamel.

Prepared by Eng.Maged Kamel.