Determine the principal centroidal moments of inertia for the unequal angle shown in Fig. 15-31.

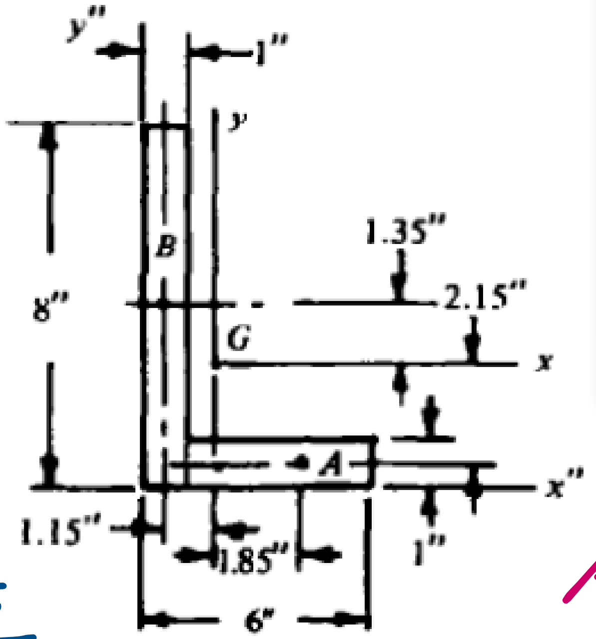
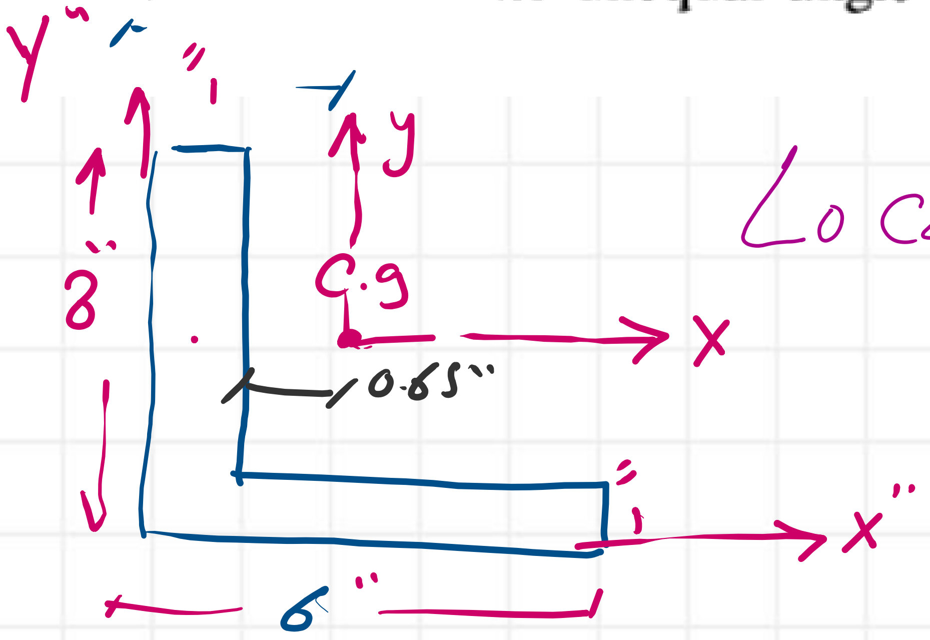
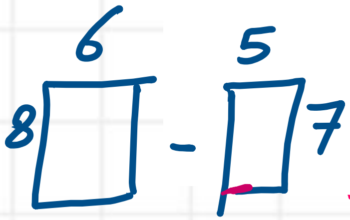


Fig. 15-31



Locate c.g

$\bar{y} = 2.65''$



$$\text{Area} = 8 \times 6 - 5 \times 7 = 48 - 35 = 13 \text{ inch}^2$$

$$\bar{x} = \frac{48(3) - 35(2.50+1)}{13} = 1.65''$$

$$\text{or } \bar{x} = \frac{8(1)(\tfrac{1}{2}) + 5 \times 1 \times 3.50}{(8+5)} = 1.653'' = 1.65''$$

matches

Prepared by Eng.Maged Kamel.

Determine the principal centroidal moments of inertia for the unequal angle shown in Fig. 15-31.

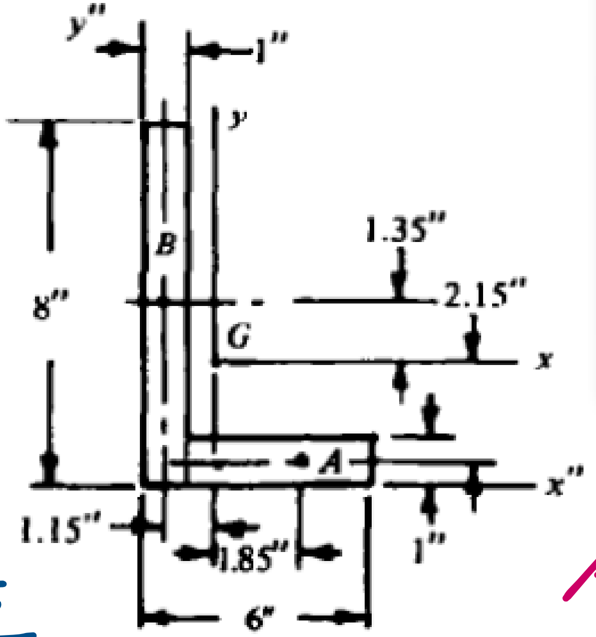
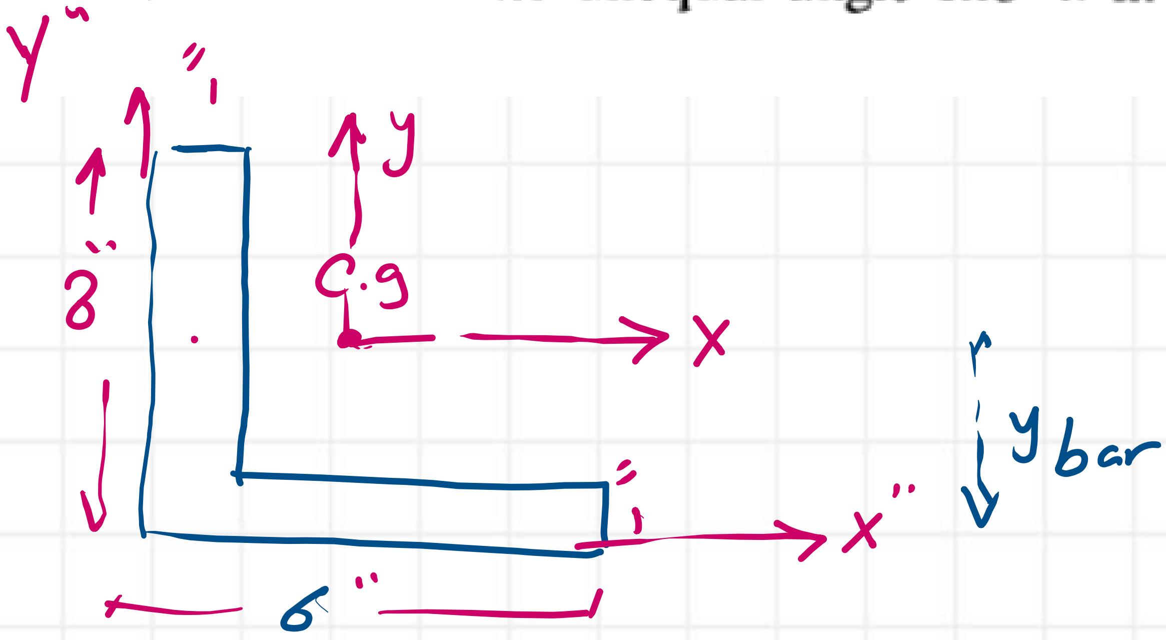
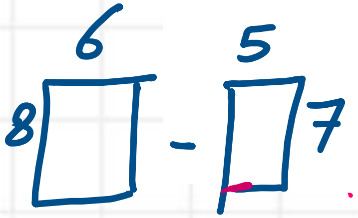


Fig. 15-31





$$\text{Area} = 8\times6 - 5\times7 = 48 - 35 = 13 \text{ inch}^2$$

$$y_{bar} = \frac{A_1(y_1) - A_2 y_2}{A_1 - A_2} = \frac{48(4) - 35(3.5+1)}{13} = 2.65''$$

$$\text{or} = \frac{8(1)(4) + 5\times1\times0.5}{8+5} = 2.65''$$

Prepared by Eng.Maged Kamel.

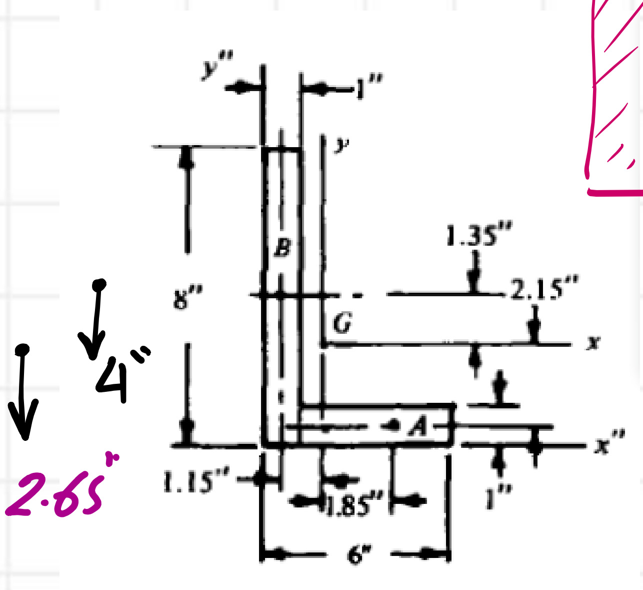
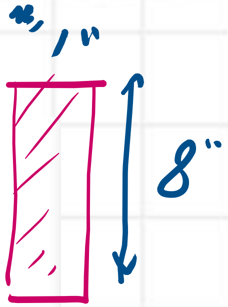
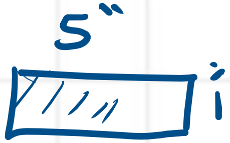


Fig. 15-31



$$I_{x\,c.g\;A-1} = \frac{1(8)^3}{12} = 42.66 \text{ inch}^4$$



$$I_{x\,c.g\;A-2} = \frac{5(1)^3}{12} = 0.417 \text{ inch}^4$$

Find $I_{x\,c.g}$

$$I_{x\,c.g} = I_{x\,c.g\,A_1} + I_{x\,c.g\,A\text{-}2} + A_1(4-y_b)^2 + A_2(2.65-0.5)^2$$

$$I_{x\,cg} = 42.66 + 0.417 + (8)(4-2.65)^2 + 5(2.15)^2$$

$$= 43.077 + 14.58 + 23.113 = 80.77 \text{ inch}^4 \simeq 80.8 \text{ inch}^4$$

Prepared by Eng.Maged Kamel.

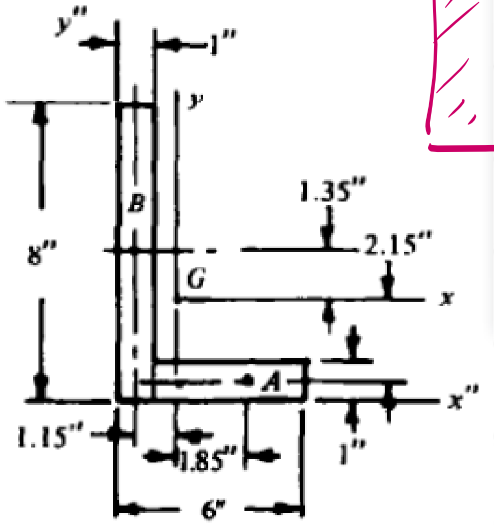
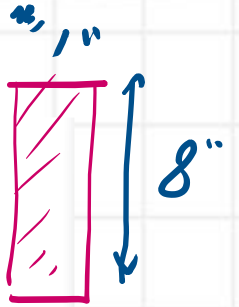
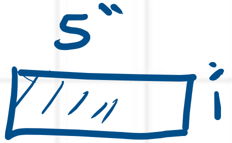


Fig. 15-31



$$I_{y_{c.g}\ A-1} = \frac{8(1)^3}{12} = \frac{8}{12} \text{ inch}^4$$



$$I_{y_{c.g}\ A-2} = \frac{1(5)^3}{12} = 10.42 \text{ inch}^4$$

Estimate I_y

$$I_{c.g} = I_{c.g\ A_1} + I_{c.g\ A-2} + A_1(x_b - 0.5)^2 + A_2(3.5 - 1.65)^2$$

$$I_{y\,cg} = \frac{8}{12} + 10.42 + 8(1.15)^2 + (5)(1.85)^2$$

$$= 11.09 + 10.58 + 17.11 = 38.78 \text{ inch}^4 \simeq 38.8 \text{ inch}^4$$

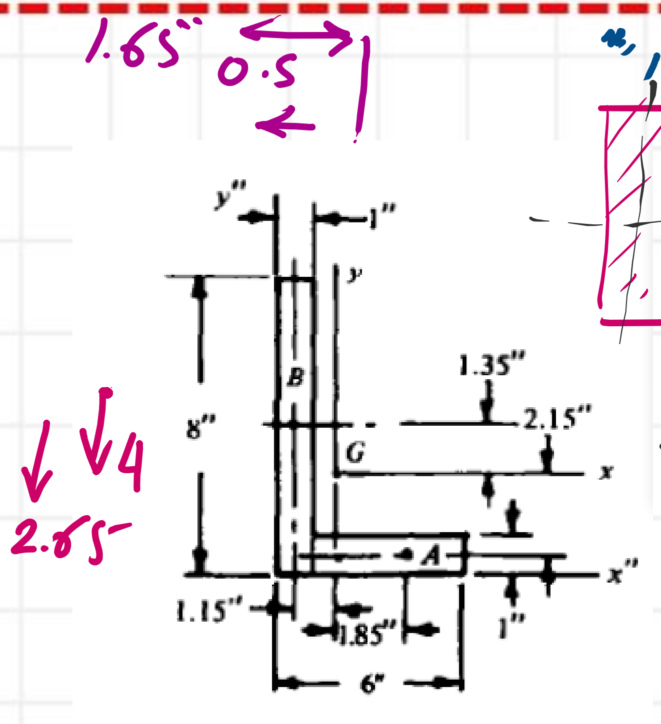
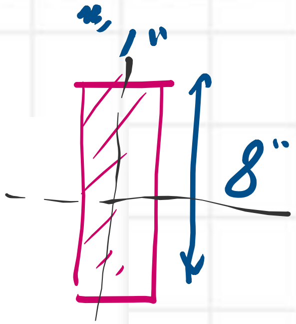
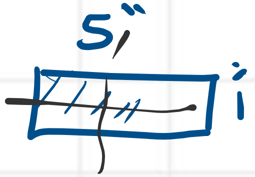


Fig. 15-31





$I_{xy_{c.g_1}} = 0$ inch4

$I_{xy_{c.g_2}} = 0$ inch4

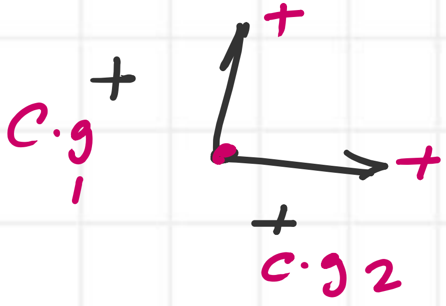
Estimate $I_{xy_{c.g}}$

Why Case 3?

$$I_{c.g} = I_{xy_{c.g\,A_1}} + I_{xy_{c.g\,A2}} + A_1(x_1 y_1) + A_2(x_2 y_2)$$

$$I_{xy_{c.g}} = 0 + 0 + (8)((-1.35)(1.15) - 5(3.5 - 1.65)(2.65 - 0.50)$$

$$= -12.42 - 19.89 = -32.31 \text{ inch}^4$$



$I_x > I_y$ —ve

I_{xy} -ve

I_x : 80.80 inch^4

I_y : 38.80 inch4

Prepared by Eng.Maged Kamel.

Draw Mohr's Circle : $I_{x\,c.g}$: 80.80 inch^4 , I_{xy} = -32.31 inch4

$I_{y\,c.g}$: 38.80 inch^4

$\frac{I_x + I_y}{2}$ = 59.80 inch4

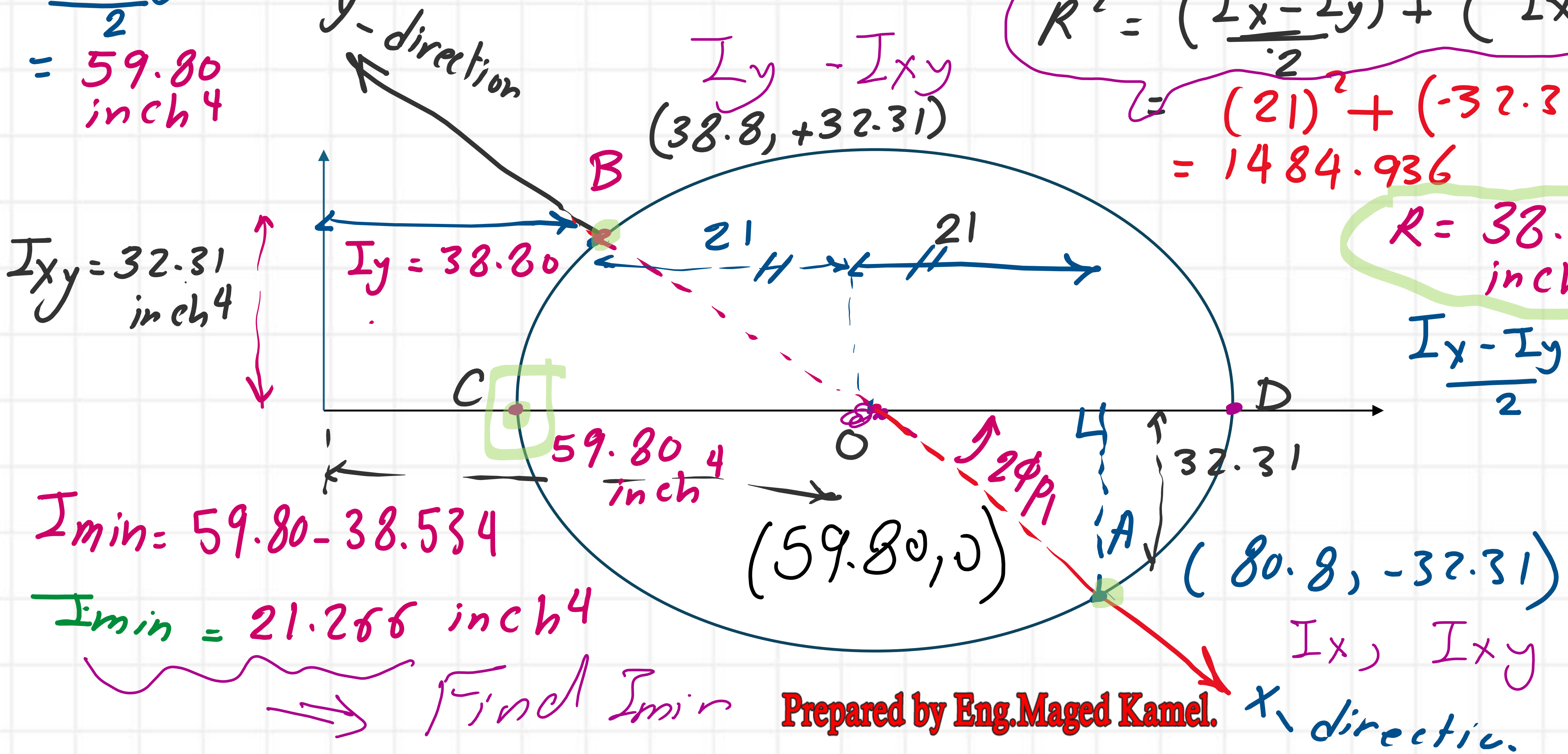
$R^2 = \left(\frac{I_x - I_y}{2}\right)^2 + (I_{xy})^2$

$= (21)^2 + (-32.31)^2$

= 1484.936

R = 38.534 inch4

$\frac{I_x - I_y}{2} = 21$



Imin = 59.80 - 38.534

Imin = 21.266 inch^4

Find Imin

Prepared by Eng.Maged Kamel.

Mohr's Circle of inertia : $I_{x\,c.g} = 80.80\ inch^4$, $I_{xy} = -32.31\ inch^4$

$I_{y\,c.g} = 38.80\ inch^4$

Find Imax

$\dfrac{I_x + I_y}{2} = 59.80\ inch^4$

C-Point

$I_{xy} = 32.31\ inch^4$

$R^2 = \left(\dfrac{I_x - I_y}{2}\right)^2 + (I_{xy})^2$

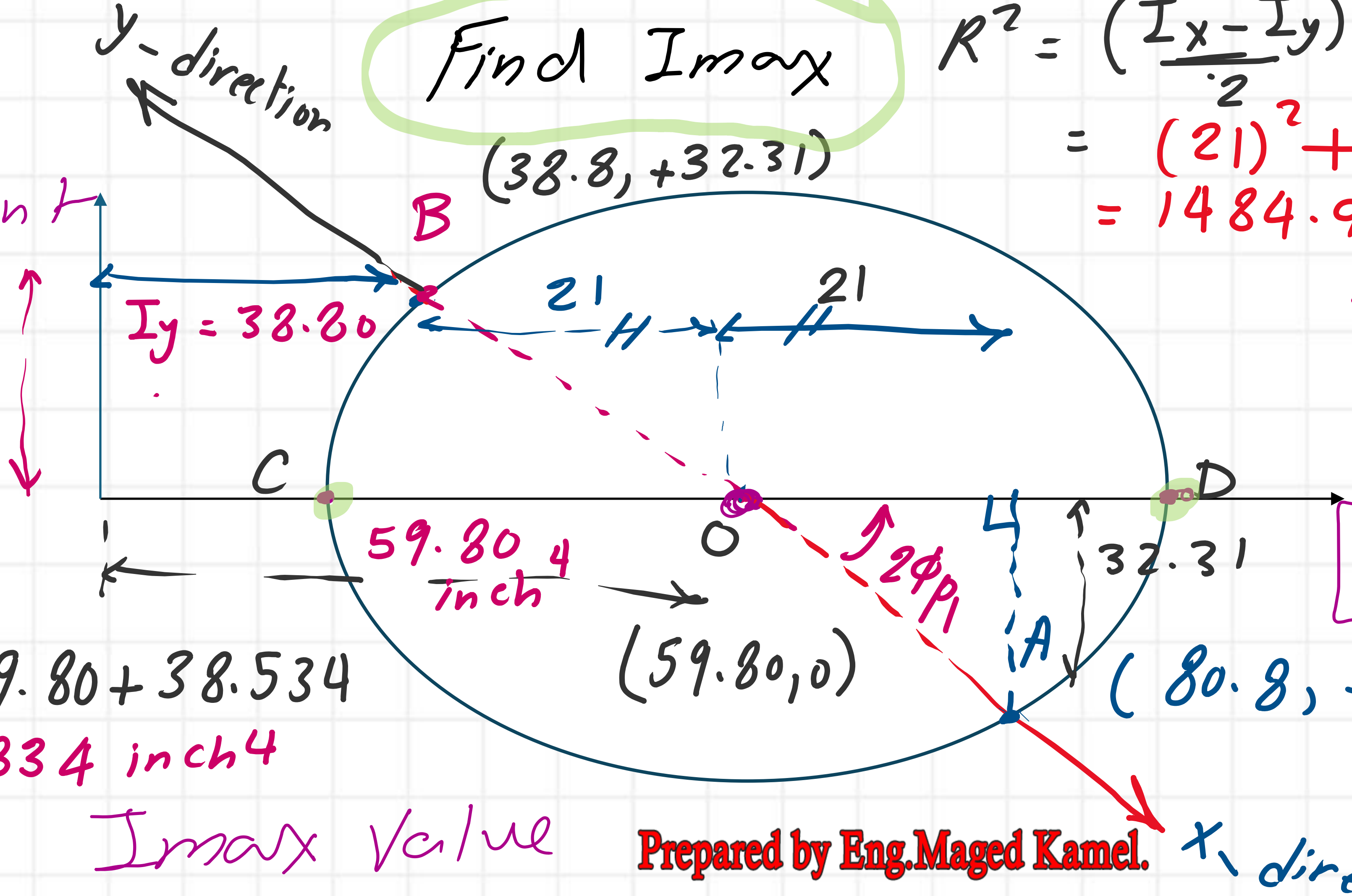
$= (21)^2 + (-32.31)^2$

$= 1484.936$

$R = 38.534\ inch^4$

$\dfrac{I_x - I_y}{2} = 21$

$[98.344, 0]$



$I_{max} = 59.80 + 38.534$

$= 98.334\ inch^4$

→ Imax Value

Prepared by Eng.Maged Kamel.

Case : 3 $I_x > I_y$, I_{xy} is -ve

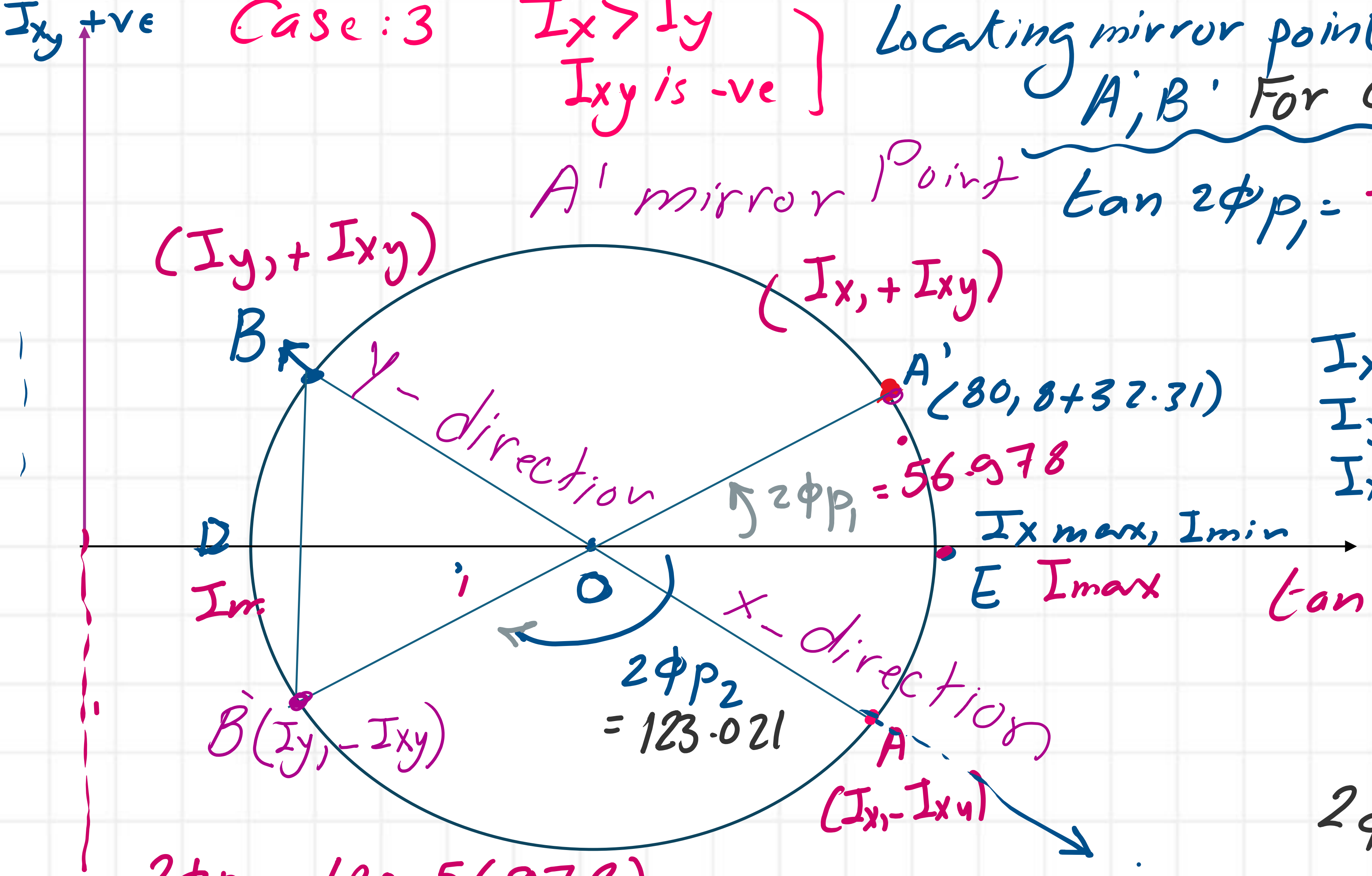
Locating mirror points A', B' for directions

$$\tan 2\phi p_1 = \frac{-I_{xy}}{\frac{1}{2}(I_x - I_y)}$$

I_x : 80.8 inch^4

I_y : 38.80 inch^4

I_{xy} : -32.31 inch^4



$$\tan 2\phi p_1 = \frac{-(-32.31)}{\frac{1}{2}(80.8 - 38.8)}$$

$$= 1.53857$$

$$2\phi p_1 = 56.978$$

$$2\phi p_2 = 180 - 56.978$$

$$= 123.021 \text{ clockwise}$$

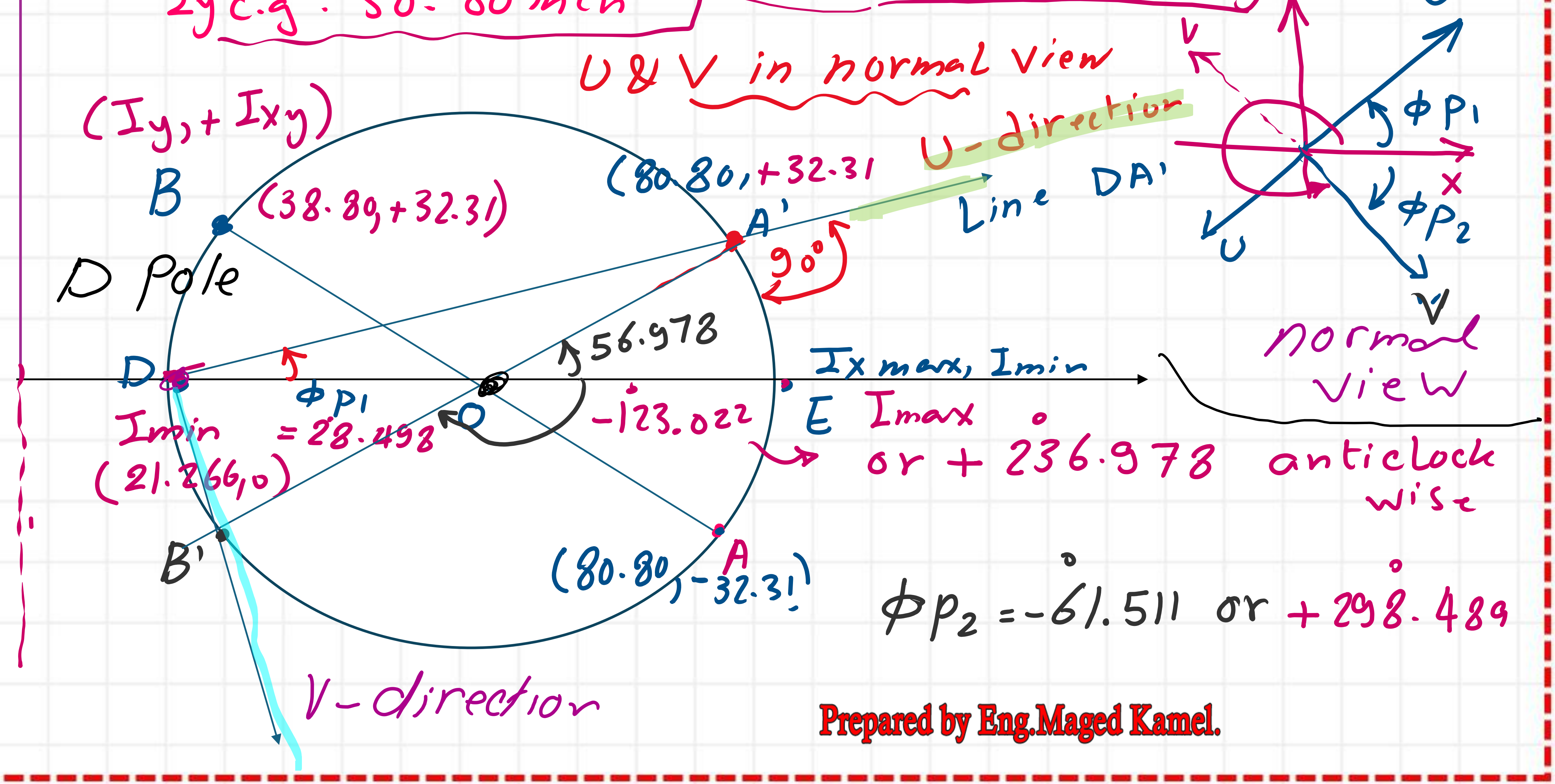
Prepared by Eng.Maged Kamel.

I_{xy} +ve : $I_{x\,c.g}$: 80.80 inch^4 , I_{xy} = −32.31 inch^4

$I_{y\,c.g}$: 38.80 inch^4

$2\phi p_1 = 56.978$
$2\phi p_2 = +236.978$

U & V in normal view



ϕp_2 = −61.511° or + 298.489°

$2\phi_{p_1} = 56.978$

Check Using general Expressi

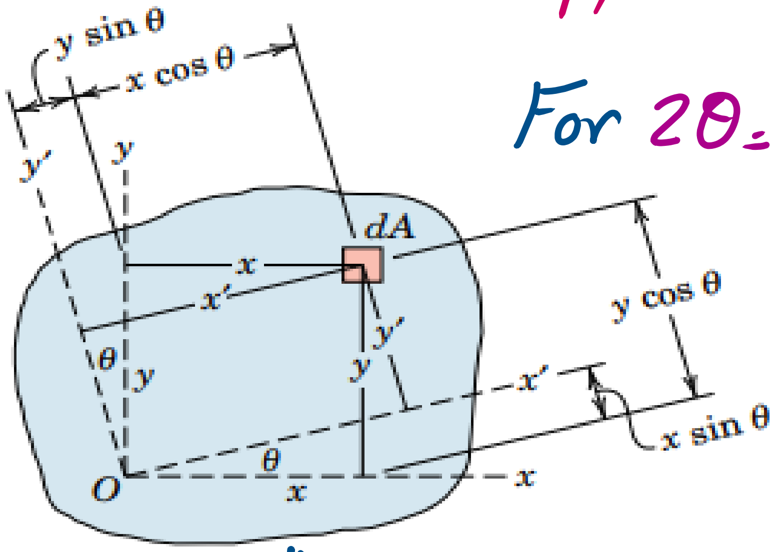


Figure A/6

For $2\theta = 2\phi_{p_1}$

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta \rightarrow I_{max}$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta \rightarrow = I_{min}$$

$2\phi_{p_1}$

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta = 0$$

$2\theta = 2\phi_{p_1}$

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x}$$

$I_x = 80.80$ inch^4

$I_y : 38.80$ inch^4

$I_{xy} : -32.31$ inch^4

$\cos(56.978) : 0.544961$

$\sin(56.978) = 0.83846$

$$I_{x'} = \frac{(80.80+38.80)}{2} + \frac{(80.80-38.80)}{2}(0.54496) - (-32.31)(0.83846)$$

$$= 59.80 + 11.444 + 27.0907$$

$$= 98.334 \text{ inch}^4 = I_{max}$$

Prepared by Eng.Maged Kamel.

$2\theta_{p_1} = 56.978$

check $I_{y'} = I_{min}$ when

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$2\theta = 2\theta_{p_1}$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

I_{min}

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta = \text{Zero}$$

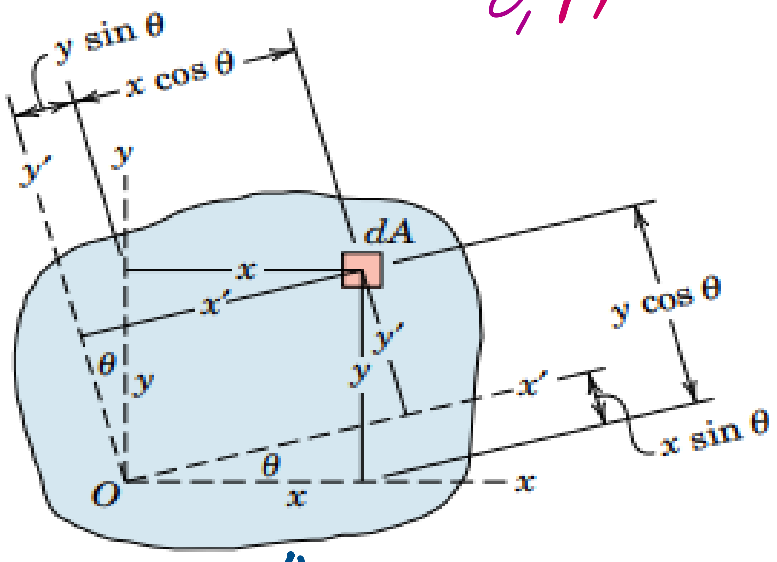


Figure A/6

$2\theta_{p_1}$

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x}$$

$I_x = 80.80$ inch

$I_y : 38.80$ inch^4

$I_{xy} : -32.31$ inch^4

$\cos(56.978) : 0.54496$

$\sin(56.978) = 0.83846$

$I_{y'} = \frac{(80.80+38.80)}{2} - \frac{(80.80-38.80)}{2}(0.54496) + (-32.31)(0.83846)$

$= 59.80 - 11.444 - 27.0907 = 21.266$ inch^4

$= I_{min}$

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$2\phi_{p_1} = 56.978$

Check Ip value

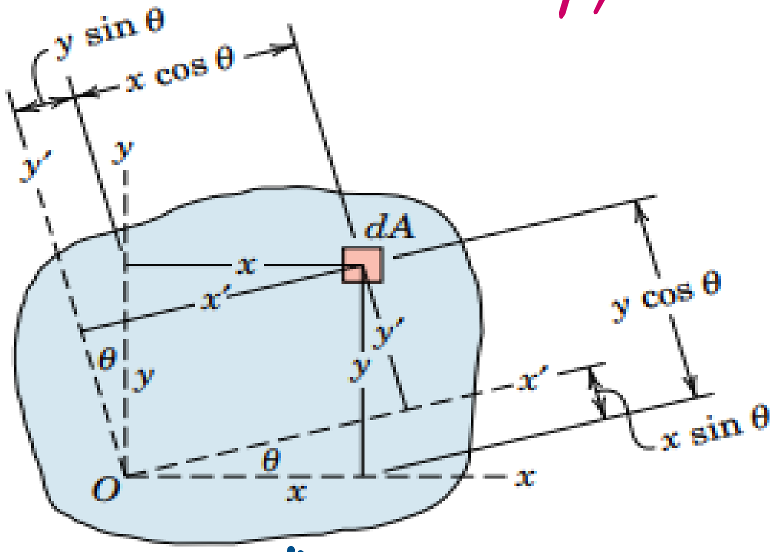


Figure A/6

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta \rightarrow = I_v$$

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta = \text{Zero}$$

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x}$$

$I_x = 80.80 \text{ inch}^4$

$I_y : 38.80 \text{ inch}^4$

$I_{xy} : -32.31 \text{ inch}^4$

$I_{max} : 98.334$

$I_{min} : 21.266 \text{ inch}^4$

$I_p = I_x + I_y = 80.80 + 38.80 = 119.60 \text{ inch}^4$

$I_p = \underbrace{I_u}_{max} + \underbrace{I_v}_{min} = 98.334 + 21.226 : 119.60 \text{ inch}^4$

Prepared by Eng.Maged Kamel.

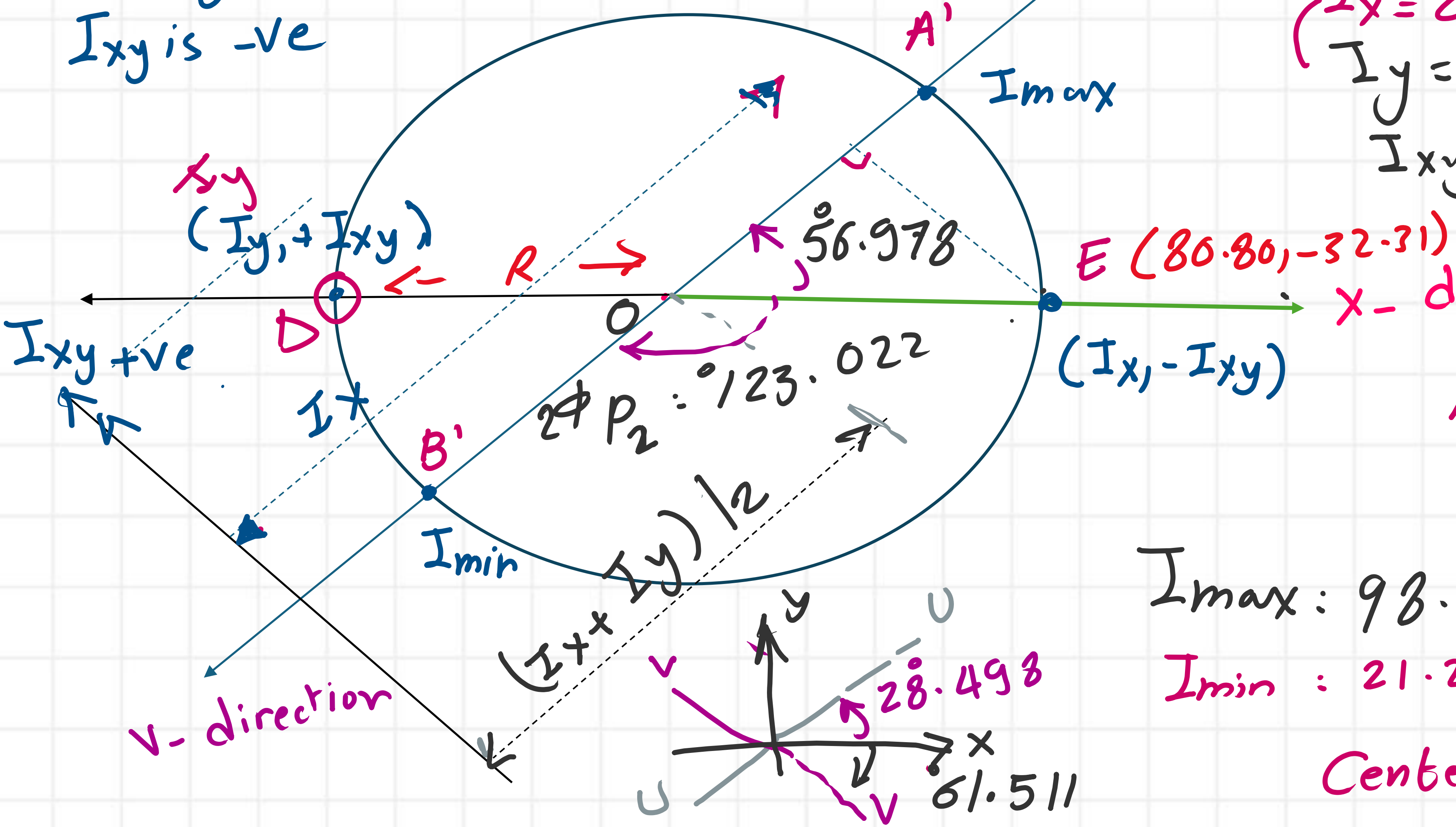
Case # 3

$I_x > I_y$

I_{xy} is -ve

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$



Ip: 119.60 $inch^4$

$I_x = 80.80\ inch^4$

$I_y = 38.80\ inch^4$

$I_{xy} = -32.31\ inch^4$

R: 38.534 $inch^4$

Imax: 98.334 $inch^4$

Imin: 21.226 $inch^4$

Center point: (59.80,0)

L : 8" x 6" x 1"   From table check values   Imin : Iv.v

1-1 X-X
2-2 Y-Y

TABLE E-5(a) PROPERTIES OF ANGLE SECTIONS WITH UNEQUAL LEGS (L SHAPES) – USCS UNITS (ABRIDGED LIST)

Designation	Weight per foot	Area	Axis 1-1				Axis 2-2				Axis 3-3	
			I	S	r	d	I	S	r	c	r_{min}	tan α
in.	lb	in.^{2}	in.^{4}	in.^{3}	in.	in.	in.^{4}	in.^{3}	in.	in.	in.	
L 8 × 6 × 1	44.2	13.0	80.9	15.1	2.49	2.65	38.8	8.92	1.72	1.65	1.28	0.542
L 8 × 6 × 1/2	23.0	6.75	44.4	8.01	2.55	2.46	21.7	4.79	1.79	1.46	1.30	0.557

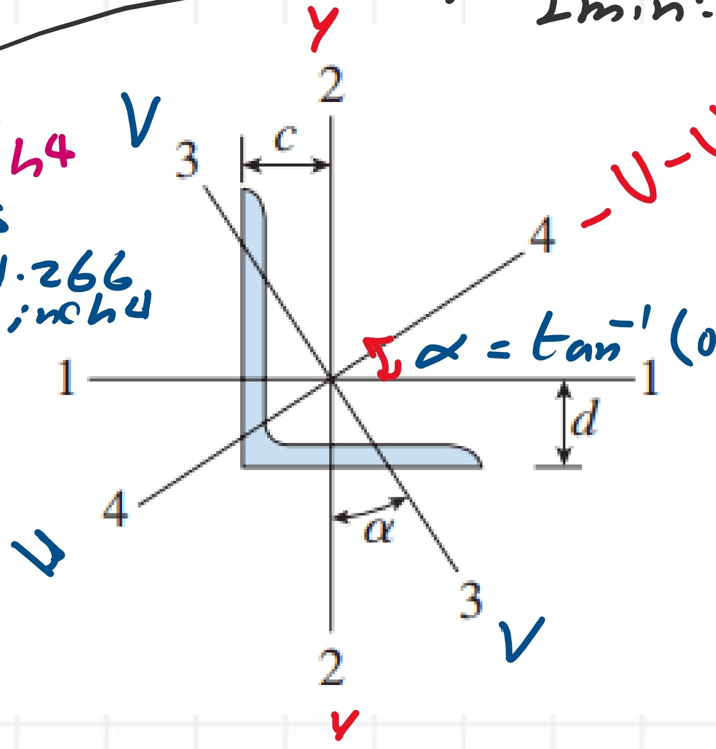
our Estimate

Imin: 21.266 inch4

3-3 is V-V min.

$I_V = r_{min}^2 (A) = 1.28^2 (13.00) = 21.299$ inch4

ours 21.266 inch4



$\alpha = \tan^{-1}(0.542) = 28.458$

Notes: 1. Axes 1-1 and 2-2 are centroidal axes parallel to the legs.
2. The distance c is measured from the centroid to the back of the legs.
3. For axes 1-1 and 2-2, the tabulated value of S is the smaller of the two se moduli for those axes.
4. Axes 3-3 and 4-4 are principal centroidal axes.
5. The moment of inertia for axis 3-3, which is the smaller of the two princi moments of inertia, can be found from the equation $I_{33} = Ar_{min}^2$.
6. The moment of inertia for axis 4-4, which is the larger of the two princip moments of inertia, can be found from the equation $I_{44} + I_{33} = I_{11} + I_{22}$.

Prepared by Eng.Maged Kam