6.5 INTERPOLATION USING A SINGLE POLYNOMIAL

Interpolation is a procedure in which a mathematical formula is used to represent a given set of data points, such that the formula gives the exact value at all the data points and an estimated value ***between*** the points. This section shows how this is done by using a single polynomial, regardless of the number of points. As was mentioned in the previous section, for any number of points n there is a polynomial of order $n-1$ that passes through all of the points. For two points the polynomial is of first order (a straight line connecting the points). For three points the polynomial is of second order (a parabola that connects the points), and so on. This is illustrated in Fig. 6-11 which shows how first, second, third, and fourth-order polynomials connect two, three, four, and five points, respectively.

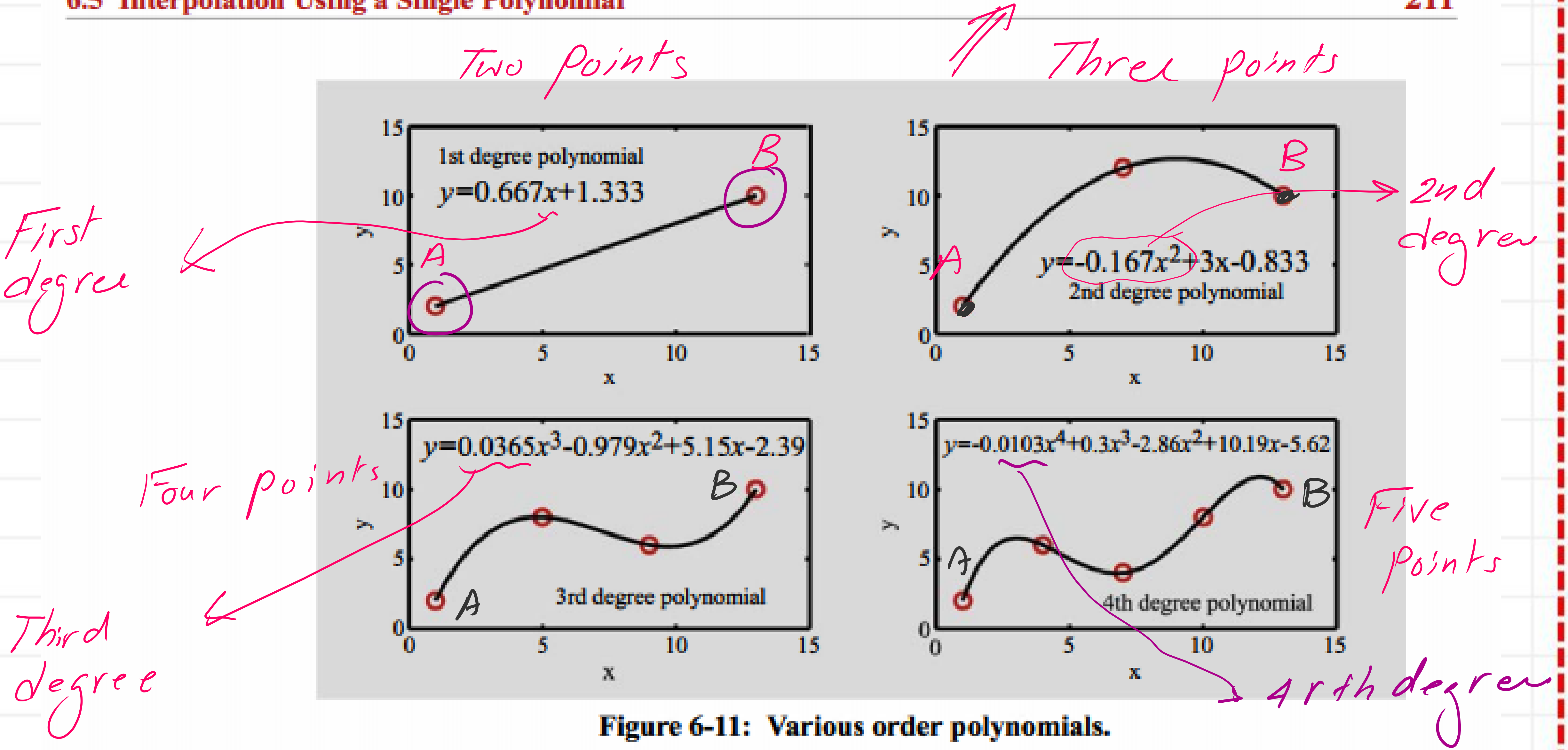

Figure 6-11: Various order polynomials.

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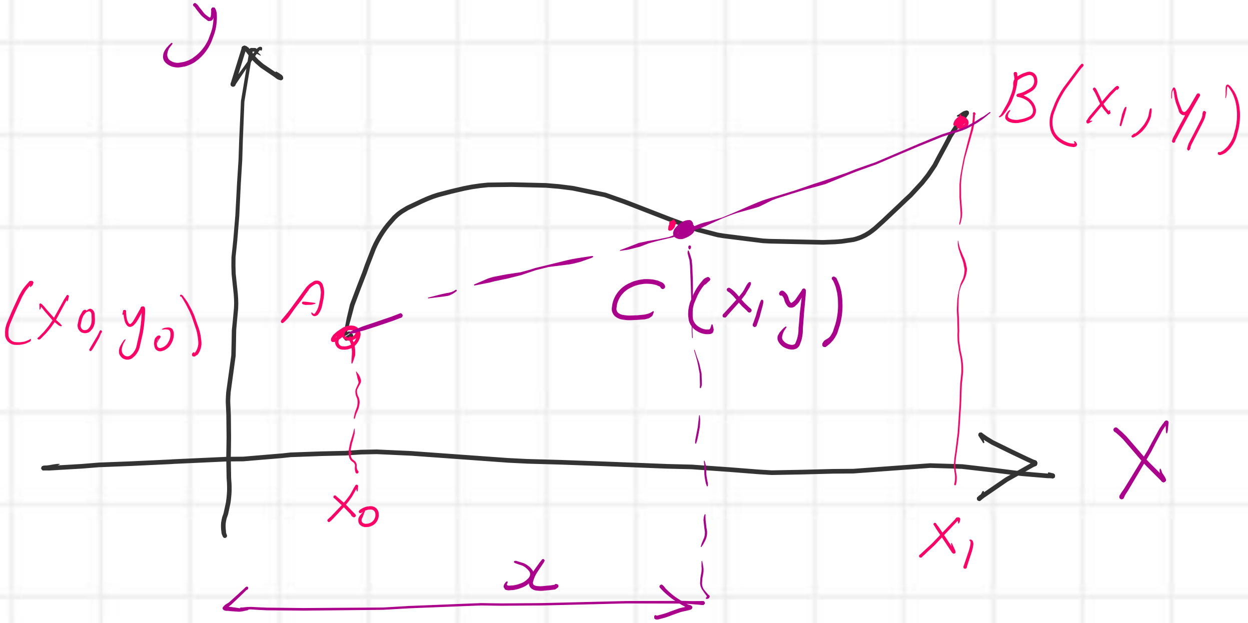
Once the polynomial is determined, it can be used for estimating the y values between the known points simply by substituting for the x coordinate in the polynomial. Interpolation with a single polynomial gives good results for a small number of points. For a large number of points the order of the polynomial is high, and although the polynomial passes through all the points, it might deviate significantly between the points. This was shown in Fig. 6-10 for a polynomial of tenth degree and is shown later in Fig. 6-17, where a 15th-order polynomial is used for interpolation of a set of 16 data points. Consequently, interpolation with a single polynomial might not be appropriate for a large number of points. For a large number of points, better interpolation can be done by using piecewise (spline) interpolation (covered in Section 6.6) in which different lower-order polynomials are used for interpolation between different points of the same set of data points.

What is Interpolation?

It is the process of finding and evaluating a function whose graph goes through the given points.
For example: If we have a table of data, that includes the coordinates, x, y, without giving any further information regarding the function f(x), and if we were asked to find the y value for a certain point within the table, in this case, we are going to use Numerical Analysis to create an interpolating function that satisfies the y values as given by the table. Sketch for three points namely A, B, C.



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The used Function is polynomial one, that gives a lot of advantages, for instance, it is easy to differentiate, integrate, added, or multiplied .can be written as for 2nd degree.

$$P(x) = a_0 + a_1 . X + a_2 . x^2 + a_3 X^3 + \ldots + a_{n-1} . x^{n-1} + a_n . x^n$$

For the first- degree polynomial, it can be written as:

$P(x) = a_0 + a_1 . X$ First Two terms .

For the second- degree polynomial, it can be written as:

$P(x) = a_0 + a_1 . X + a_2 . x^2$ First three terms

For the third- degree polynomial, it can be written as:

$P(x) = a_0 + a_1 . X + a_2 . x^2 + a_3 X^3$ Use the First Four terms

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From the previous discussion the polynomial degree giving n+1 coefficient, for instance P2(x) has 3 coefficients. Where the a's are called the coefficients.

Terminology: Where is the leading coefficient? Where is the leading term?

For example if :

$$f(x) = a_n . X^n$$

a_n is the leading coefficient

$(a_n * X^n)$ is the leading term, with the highest power.

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Example if :

$$f(x) = 5 * X^4 + 2 * X^3 - x + 4$$

$f(x) = a_0 + a_1 X + a_2 X^2 + a_3 X^3 + a_4 x^4$

Rewritten as

$a_4 X^4 + a_3 X^3 + a_2 X^2 + a_1 X$

a_4 = 5

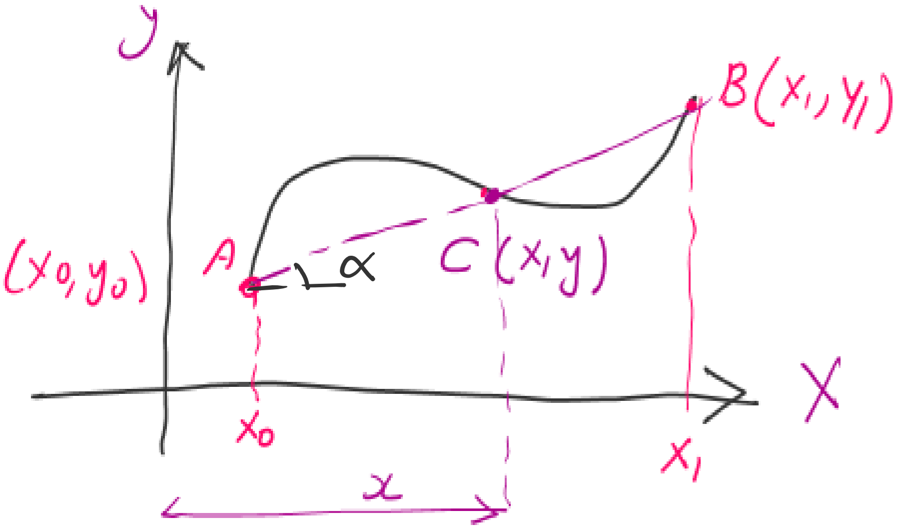
Then the leading term is $5 * X^4$

Then the leading term is The coefficient of X^4, which is the highest power.

Linear Interpolation.

For the curve AB ,it is required to make linear interpolation ,a line between points AB is drawn, the general equation is

$f(x)= y =a_0+a_1x$ *Equation I*



To get expression and values for both a_0 & a_1

Two equation thus can be written, using points A,B as follows:

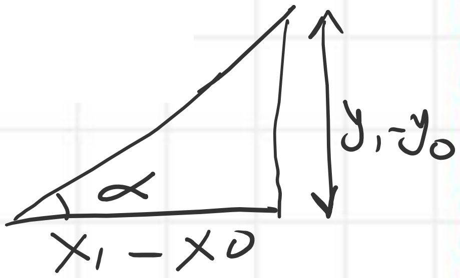
$P_0(x_0) =y_0=a_0+a_{1.}x_0$ *Equation II*

$P_1(x_1) = y_1=a_0+a_{1.}x_1$ *Equation III*

Subtracting y_1-y_0, thus we will have $y_1 - y_0 =a_1(x_1-x_0)$

$y_1 - y_0 = (a_0 + a_1x_1 - a_0 - a_1x_0)$

Then $a_1 = \frac{y_1-y_0}{x_1-x_0}$



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Back to Equation II then

$$y_0 = a_0 + (y_1 - yo)/(x_1 - x_0) \cdot X_0$$

For the value of a_0

$$a_0 = y_0 - \left(\frac{(x_0).(y_1 - y_0)}{x - xo}\right)$$

Rewrite equation I

$$P(x) = y = y_0 - \frac{y_1 - y_0}{x - xo} \cdot (x_0) + \frac{y_1 - y_0}{x_1 - x_0} . X$$

2nd point y value

First Point y Value

Finally, we have

$$P(x) = y = y_0 + \frac{y_1 - y_0}{x_1 - xo} \cdot (X - x_0)$$

Which is called first divided difference.

First point y value

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Example-1 For y =$\sqrt{x}$ the given data points are (1,1)& (9,3)

Use linear interpolation, then estimate the $\sqrt{5}$, knowing that the actual value is 2.23606, estimate the error value.

$$P(x) = y = y_0 + \frac{y_1 - y_0}{x_1 - xo} \cdot (X - x_0)$$

Solution:

For x_0 as a starting point which is=1.0, y_0=1 as given

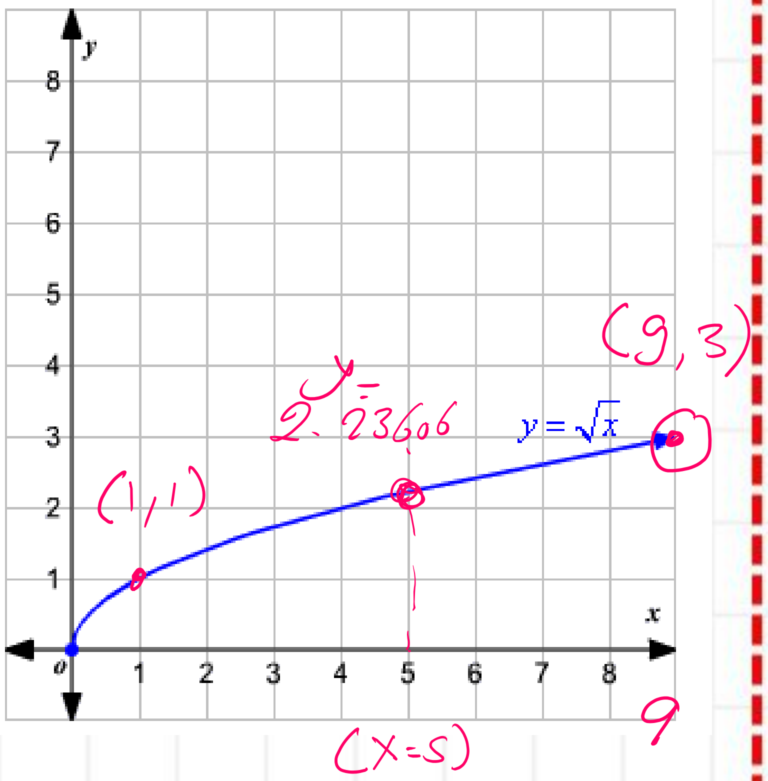
For x_1 as a second point which is x_1 =9, y_1=3 as given

$$P(x) = y = 1 + \frac{3-1}{9-1} \cdot (X - 1)$$

$$P(x) = y = 1 + \frac{2}{8} \cdot (X - 1)$$

$$P(x) = y = 1 + \frac{1}{4} \cdot x - \frac{1}{4}$$

$$y = 1 + \frac{1}{4} \cdot x - \frac{1}{4} = \frac{3}{4} + \frac{1}{4}\ x$$



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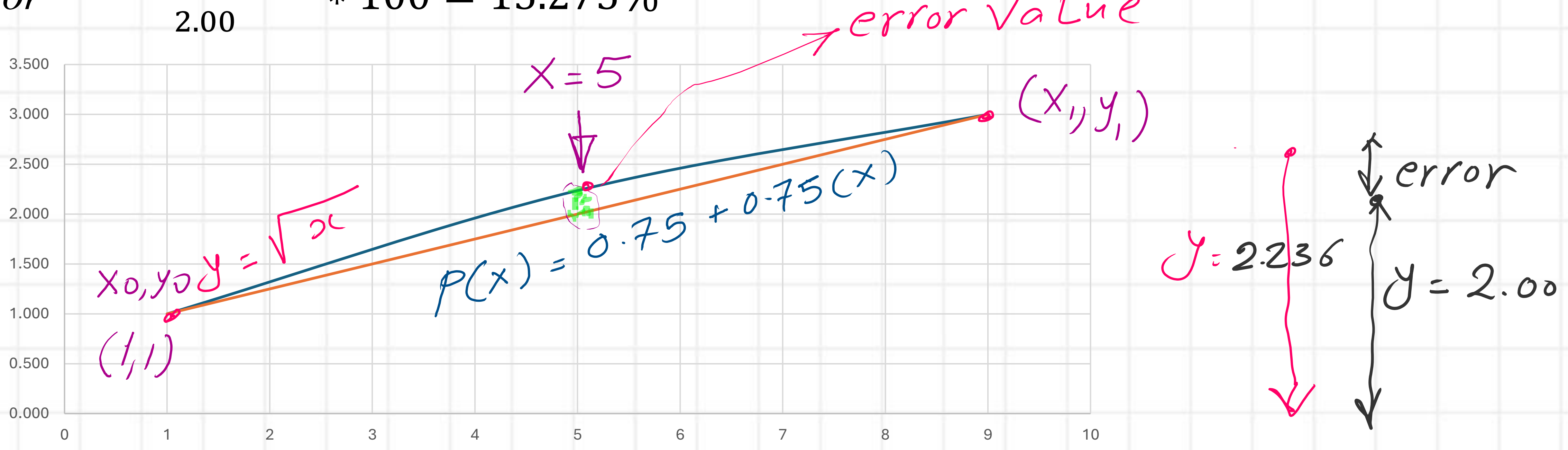
Substitute for x=5

Then $y_5 = \frac{3}{4} + \frac{1}{4}.5 = 2.00$

Since the real value is 2.2360 which is sqrt(5)

$$Error = \frac{abs(2-2.2360)}{2.00} * 100 = 15.275\%$$



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