Synthetic Division

To get the zero for a function, **sometimes we are guessing the values for x , the so-called** rational roots test.
Rational roots test is a handy way of obtaining a list of useful first guesses when you are trying to find the zeroes (roots) of a polynomial.

Example: solve the function y=$x^2 - 5x + 6$ $= f = a_2 x^2 + a_1 x - a_0$

$a_n = 1$ & $a_0 \rightarrow$ Constant

Solution: we examine the leading coefficient (X^2), we have +1 constant +6, what will be the possible values of x

1,-1, 2,-2,+3,-3,+6,-6. for constants , $\pm\frac{1,6}{1,1}, \pm\frac{2,3}{1,1}$

$6 \left\{ \begin{array}{l} (1)(6) \\ (-1)(-6) \\ (2)(3) \\ (-2)(-3) \\ (1)(6) \\ (-1)(-6) \end{array} \right.$

Next step, we let x+3=0 as first trial x=-3

$$\begin{array}{r|rrr} -3 & 1 & -5 & +6 \\ & & -3 & +24 \\ \hline & 1 & -8 & 30 \end{array}$$

$f(-3) = 9 + 15 + 6 = 30$

1-We write coefficients only without x^2 or x

2- Coefficient of x^2 carrying down

3-multiply -3*+1.

4- add (-3)+(-5)=-8

5-multiply (-8)*(-3)=24

6-add +24 to +6=+30.

Continue for roots finding.

Example: solve the function $y=x^2-5x+6$

	x^2	x	
+2	1	-5	+6
		+2	-6
	1	-3	0

$r=0$

+3	1	-3	0
		+3	0
	1	0	remainder = 0

$\Rightarrow$ there is a root

2^{nd} trial :

Next step we let X-2=0 i.e x=2

$y=$ (x-2)*(x-3)=0 the result shows that we have two roots x1=2, x2=3

https://www.mathportal.org/calculators/polynomials-solvers/polynomial-graphing-calculator.php

$P(x) = x^2 - 5x + 6$

$x = 0 \rightarrow y = 6$

$x = 2 \quad y = 0$

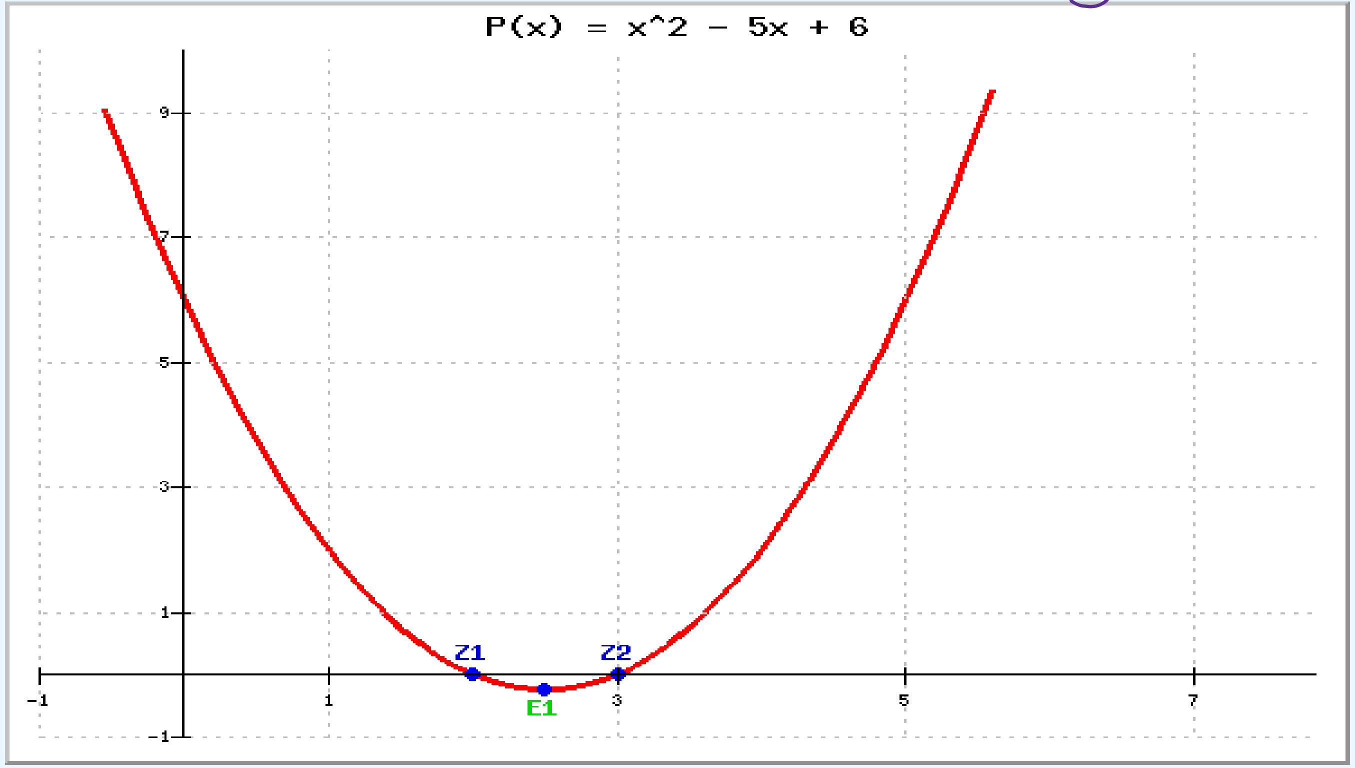
$x = 3 \quad y = 0$

There are two roots that make

$f(x) = 0$

$x = 2$

$x = 3$



Prepared by Eng.Maged Kamel.

Example: Divide $5x^4+6x^3-9x^2-7x+6$ by $(x+2)$ by synthetic division

Solution Consider $x+2=0$

$x=-2$

We put at the Left

	x^4	x^3	x^2	x	c
-2	5	6	-9	-7	$+6$
		-10			
	5	-4			

Steps $1 \to 3$

Step-1 : add 5 to 0 = 5

Step-2 : Multiply $5(-2)=-10$

Step-3 : add (-10) to $6 \Rightarrow -4$

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Divide $5x^4+6x^3-9x^2-7x+6 \,/\, (x+2)$ Contd

-2	5	6	-9	-7	$+6$
		-10	$+8$	$+2$	$+10$
	5	-4	-1	-5	$+16$

Steps from 4 → 8

Step 4

Multiply $(-4 \times -2) = +8$

step 5 add $+8+(-9) = -1$

step 6 Multiply (-1) by $(-2) = +2$

step 7 add $+2+(-7) = -5$

step 8 Multiply (-5) by $(-2) = +10$

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Divide : $\dfrac{5x^4+6x^3-9x^2-7+6}{(x+2)}$

$$\begin{array}{r|rrrrr} -2 & 5 & 6 & -9 & -7 & +6 \\ & & -10 & +8 & +2 & +10 \\ \hline & 5 & -4 & -1 & -5 & +16 \end{array}$$

Step-9

Step 9 : Add 6 to 10 $\rightarrow$ 16

remainder = 16

Rewrite as

$$5x^3-4x-x-5+\frac{16}{(x+2)}$$

No root

[remainder is not Zero]

*Quotient with a remainder of 16

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Example: Divide: $5x^4+6x^3-9x^2-7x+6$ by $(x+2)$

Try Long division

Solution

Coefficients are placed, For $x+2=0$

$$
\begin{array}{r|l}
 & 5x^3 \\
x+2 & 5x^4 + 6x^3 - 9x^2 - 7x + 6 \\
 & -5x^4 - 10x^3 \\
\hline
 & 0 \quad -4x^3 - 9x^2 - 7x + 6
\end{array}
$$

step 1: $\frac{5x^4}{x} = 5x^3$

Step 2: Multiply $(5x^3)(x+2) = 5x^4 + 10x^3$

step 3: Subtract From the Line above

$-(5x^4+10x^3) + 5x^4 + 6x^3 = -4x^3$

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Example: $5x^4 + 6x^3 - 9x^2 - 7x + 6 \,/\, (x+2)$

by Long division Contd

$$
\begin{array}{r|l}
 & 5x^3 - 4x^2 \\
x+2 & 5x^4 + 6x^3 - 9x^2 - 7x + 6 \\
 & -5x^4 - 10x^3 \\
\hline
 & 0 \quad -4x^3 - 9x^2 - 7x + 6 \\
 & \quad +4x^3 + 8x^2 \\
\hline
 & \quad\quad -x^2 - 7x + 6
\end{array}
$$

From previous slide

Step 5: divide

$$-\frac{4x^3}{x} = -4x^2$$

step 6: Multiply

$$(-4x^2)(x+2) = -4x^3 - 8x^2$$

step 7: subtract from the line above

$+4x^3 + 8x^2$

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Example:

Divide $5x^4+6x^3-9x^2-7x+6$ by $(x+2)$ by Long division

Contd

Solution: Coefficients are placed, For $x+2=0$

$$
\begin{array}{r|llllll}
 & 5x^3 & -4x^2 & -x & & & \\
\hline
x+2 & 5x^4 & +6x^3 & -9x^2 & -7x & +6 \\
 & -5x^4 & -10x^3 & & & \\
\hline
 & 0 & -4x^3 & -9x^2 & -7x & +6 \\
 & & +4x^3 & +8x^2 & & \\
\hline
 & & & -x^2 & -7x & +6
\end{array}
$$

Step 8: $\frac{\text{Divide}}{x} \rightarrow \frac{-x^2}{x} = -x$

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Example:

Divide $5x^4 + 6x^3 - 9x^2 - 7x + 6$

by $(x+2)$ by Long division

Solution: Coefficients are placed, For $x + 2 = 0$

$$
\begin{array}{r|l}
 & 5x^3 - 4x^2 - x \\
x+2 & 5x^4 + 6x^3 - 9x^2 - 7x + 6 \\
 & -5x^4 - 10x^3 \\
\hline
 & 0 \quad -4x^3 - 9x^2 - 7x + 6 \\
 & +4x^3 + 8x^2 \\
\hline
 & 0 \quad -x^2 - 7x + 6 \\
 & + x^2 + 2x^2 \\
\hline
 & -5x + 6
\end{array}
$$

Step 9:

Multiply $(-x)(x+2)$

$= -x^2 - 2x$

Step 10: Subtract

$= +x^2 + 2x^2$

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Example:

Divide $5x^4+6x^3-9x^2-7x+6$

by $(x+2)$ by Long division

Solution Coefficients are placed, For $x+2=0$

$$
\begin{array}{r|l}
 & 5x^3-4x^2-x \\
\hline
x+2 & 5x^4+6x^3-9x^2-7x+6 \\
 & -5x^4-10x^3 \\
\hline
 & 0\quad -4x^3-9x^2-7x+6 \\
 & +4x^3+8x^2 \\
\hline
 & 0\quad -x^2-7x+6 \\
 & +x^2+2x \\
\hline
 & -5x+6
\end{array}
$$

Step 11

Divide $\frac{-5x}{x}=-5$

step 12: Multipl

$-5(x+2)=-5x-10$

step 13: subtract

$+5x+10$

Prepared by Eng.Maged Kamel.

$$5x^3 - 4x^2 - x - 5$$ Quotient

Divesory $x+2$

$$5x^4 + 6x^3 - 9x^2 - 7x + 6$$ ← Dividend

$$-5x^4 + 10x^3$$

$$0 \quad -4x^3 - 9x^2 - 7x + 6$$

$$+4x^3 + 8x^2$$

$$0 \quad -x^2 - 7x + 6$$

$$+x^2 + 2x$$

$$-5x + 6$$

$$+5x + 10$$

$$0 + 16$$ → Remainder

Same result is obtained

$$\frac{5x^4 + 6x^3 - 9x^2 - 7x + 6}{x+2} = (5x^3 - 4x^2 - x - 5) + \frac{16}{(x+2)}$$

***Quotient with a remainder of 16**

Prepared by Eng.Maged Kamel.