Objective of lecture.

- Difference between Numerical and Analytical methods.
- Methods used to get root of function , comparing between Analytical and Numerical.
- Bisecting method for root finding.
- Method of False position for root finding.

Prepared by Eng.Maged Kamel.

Numerical Analysis

Numerical analysis involves the study of all the methods of computing numerical data.

Difference between Numerical and Analytical method.

- Numerical methods provide approximation to the problem in question, while analytical method give exact solutions with more time consuming.

- Numerical method give solution at certain point only, like finite difference ,finite element, these methods are considered Numerical since they give results at certain points.

Prepared by Eng.Maged Kamel.

1.1 BACKGROUND

Numerical methods are mathematical techniques used for solving mathematical problems that cannot be solved or are difficult to solve analytically. An analytical solution is an exact answer in the form of a mathematical expression in terms of the variables associated with the problem that is being solved. A numerical solution is an approximate numerical value (a number) for the solution. Although numerical solutions are an approximation, they can be very accurate. In many numerical methods, the calculations are executed in an iterative manner until a desired accuracy is achieved.

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Definition of Quadratic Function	A function f is a **quadratic function** if $$f(x) = ax^2 + bx + c,$$ where a, b, and c are real numbers with $a \neq 0$.

Discriminant

If $b = c = 0$ in the preceding definition, then $f(x) = ax^2$, and the graph is a parabola with vertex at the origin. If $b = 0$ and $c \neq 0$, then

$$f(x) = ax^2 + c,$$

Determining the Number of Real Solutions to a Quadratic Equation

Let a, b and c be real numbers with $a \neq 0$.

- If $b^2 - 4ac < 0$, the equation $ax^2 + bx + c = 0$ has no real solutions.
- If $b^2 - 4ac = 0$, the equation $ax^2 + bx + c = 0$ has one real solution.[1]
- If $b^2 - 4ac > 0$, the equation $ax^2 + bx + c = 0$ has two real solutions.

The discriminant **tells you the number of solutions you will have and whether or not the solutions are real numbers.** Discriminant > 0: You have two solutions that are real numbers. Discriminant = 0: You have one solution that is a real number. Discriminant < 0: You have two solutions that are not real numbers.

Precalculus Cynthia young

DEFINITION **Polynomial Function**

Let n be a nonnegative integer, and let $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ be real numbers with $a_n \neq 0$. The function

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$$

Constant

is called a **polynomial function of x with degree n**. The coefficient a_n is called the **leading coefficient**, and a_0 is the constant.

a_n : Leading

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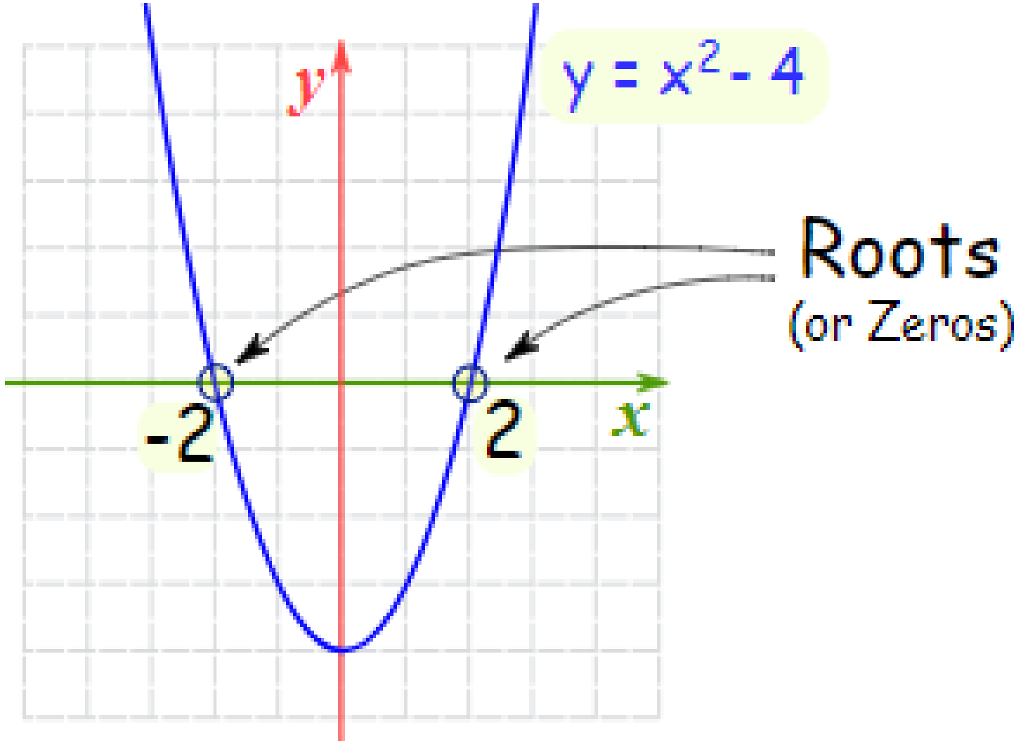
Finding the +ve root of function

What is the root of a function?

The roots of function ,or zeros, are the points where the function crosses the x axis ,where the value of f(x)=0



We let f(x)=y=0
(x+2)*(x-2)=0
So x=-2 or x=+2 are the zeros
To check substitute in the value of x

Y=(2)^2-4= 4-4=0
Y=(-2)^2-4=4-4=0

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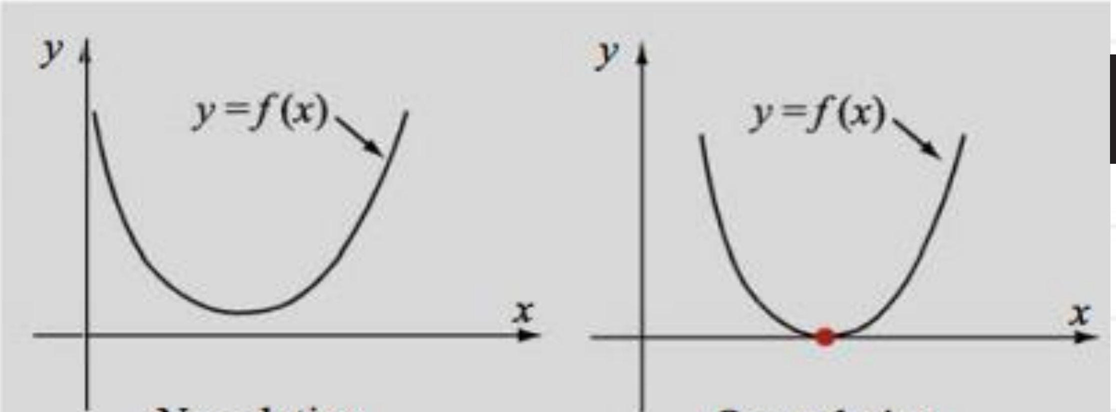
Chapter - 3

source Numerical methods For engineers

A solution to the equation (also called a ***root*** of the equation) is a numerical value of x that satisfies the equation. Graphically, as shown in Fig. 3-1, the solution is the point where the function $f(x)$ crosses or touches the x-axis. An equation might have no solution or can have one or several (possibly many) roots.

When the equation is simple, the value of x can be determined analytically. This is the case when x can be written explicitly by applying mathematical operations, or when a known formula (such as the for-



No solution One solution

Fig - 3

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Source : Numerical methods For engineers

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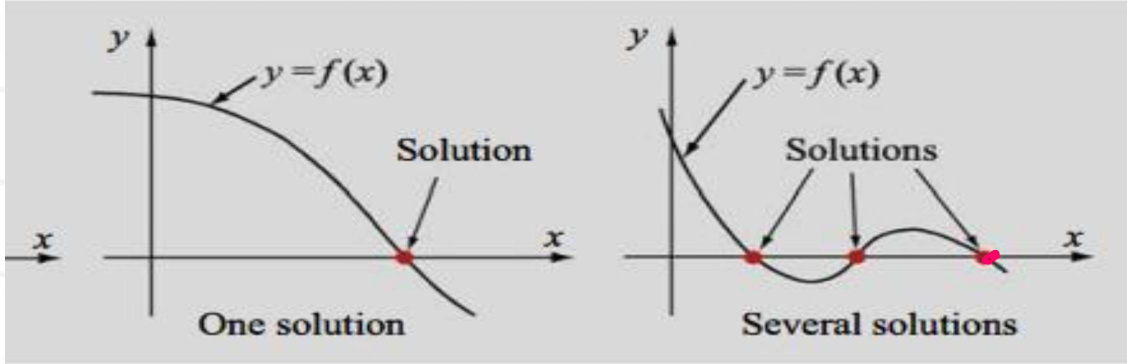

Figure 3-1: Illustration of equations with no, one, or several solutions.

4.4.1 The Remainder Theorem and the Factor Theorem

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Algebra and Trignometric

The zeros of a polynomial function assist us in finding the x-intercepts of the graph of a polynomial function. How do we find the zeros of a polynomial function? For polynomial functions of degree 2, we have the quadratic formula, which allows us to find the two zeros. For polynomial functions whose degree is greater than 2, much more work is required.* In this section, we focus our attention on finding the *real* zeros of a polynomial function. Later, in Section 4.5, we expand our discussion to *complex* zeros of polynomial functions.

In this section, we start by listing possible rational zeros. As you will see, there are sometimes many possibilities. We can then narrow the search using Descartes' rule of signs, which tells us possible combinations of positive and negative real zeros. We can narrow the search even further with the upper and lower bound rules. Once we have tested possible values and determined a zero, we will employ synthetic division to divide the polynomial by the linear factor associated with the zero. We will continue the process until we have factored the polynomial function into a product of either linear factors or irreducible quadratic factors. Last, we will discuss how to find irrational real zeros using the intermediate value theorem.

If we divide the polynomial function $f(x) = x^3 - 2x^2 + x - 3$ by $x - 2$ using synthetic division, we find the remainder is -1.

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 1 & -3 \\ & & 2 & 0 & 2 \\ \hline & 1 & 0 & 1 & -1 \end{array}$$

Notice that if we evaluate the function at $x = 2$, the result is -1. This leads us to the *remainder theorem*.

$f(2) = -1$

REMAINDER THEOREM

If a polynomial $P(x)$ is divided by $x - a$, then the remainder is $r = P(a)$.

$x - a$ is not a factor of $P(x)$

The remainder theorem tells you that polynomial division can be used to evaluate a polynomial function at a particular point.

FACTOR THEOREM

If $P(a) = 0$, then $x - a$ is a factor of $P(x)$. Conversely, if $x - a$ is a factor of $P(x)$, then $P(a) = 0$.

if we substitute by $X = a$ in the polynominal $P(X)$

and $P(X) = 0$

Then $(X - a)$ is a factor of $P(X)$

EXAMPLE 2 Using the Factor Theorem to Factor a Polynomial

Determine whether $x + 2$ is a factor of $P(x) = x^3 - 2x^2 - 5x + 6$. If so, factor $P(x)$ completely.

Solution:

By the factor theorem, $x + 2$ is a factor of $P(x) = x^3 - 2x^2 - 5x + 6$ if $P(-2) = 0$. By the remainder theorem, if we divide $P(x) = x^3 - 2x^2 - 5x + 6$ by $x + 2$, then the remainder is equal to $P(-2)$.

$X + 2 \Rightarrow X = -2$

$P(x=-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$

$= -8 - 8 + 10 + 6 = -16 + 16 = 0$

$(X = -2)$

$$\begin{array}{r|rrrr} & x^3 & x^2 & x & \\ -2 & 1 & -2 & -5 & +6 \\ & & -2 & +8 & -6 \\ \hline x^2 & 1 & -4 & 3 & 0 \end{array}$$

$\Rightarrow (X-2)(X^2 - 4X + 3)$

$(x-2)(X-3)(X-1)$

$x^2 - 4x + 3$ Check $x = 3$

$$\begin{array}{c|ccc} & 1 & -4 & +3 \\ 3 & & 3 & -3 \\ \hline & 1 & -1 & 0 \\ & x & & \end{array}$$

$(x-3)(x-1)$