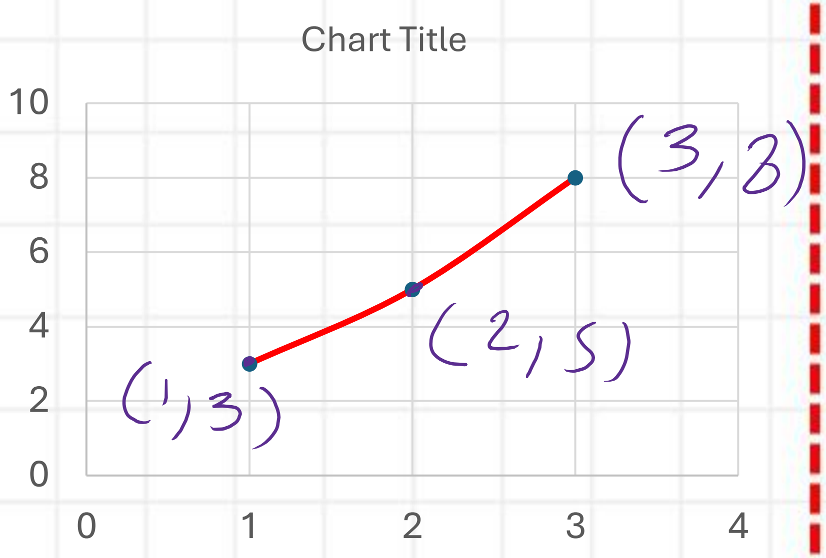
Solved example-1

For the given table, use Newton divided difference to *derive an expression* of a *polynomial* and get the value of f(2.7)

X	1	2	3
f(x)	3	5	8



Solution:

$x_0 = 1$

$y_0 = 3 \rightarrow b_0 = 3$

For n+1 points=3 we have 3 b's

$x_1 : 2$

$y_1 : 5$

$$P(x) = \underbrace{y_0}_{b_0} + \underbrace{f[x_0, x_1]}_{b_1}(x - x_0) + \underbrace{f[x_0, x_1, x_2]}_{b_2}(x - x_0)(x - x_1)$$

find b_1 : $\dfrac{(y_1 - y_0)}{x_1 - x_0} = \dfrac{(5 - 3)}{(2 - 1)} = \dfrac{2}{1} = 2$

Prepared by Eng.Maged Kamel.

Solved example-1

For the given table, use Newton divided difference to d*erive an expression* of a *polynomial* and get the value of f(2.7).

	x_0	x_1	x_2
X	1	2	3
f(x)	3	5	8
	y_0	y_1	y_2

$y_0 = 3$ | $x_0 = 1$

$y_1 = 5$ | $x_1 = 2$

$y_2 = 8$ | $x_2 = 3$

Solution:

For n+1 points=3 we have 3 b's b_0, b_1, b_2

$$P(x) = y_0 + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

$$b_2 = f[x_0, x_1, x_2] = \frac{\frac{y_2 - y_1}{x_2 - x_1} - \frac{y_1 - y_0}{x_1 - x_0}}{x_2 - x_1} = \frac{\frac{8-5}{3-2} - \left(\frac{5-3}{2-1}\right)}{(3-1)}$$

$$b_2 = \frac{3-2}{2} = \frac{1}{2}$$

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$$P(x) = y_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$\left.\begin{aligned} b_0 &= 3 = y_0 \\ b_1 &= +2 \\ b_2 &= 1/2 \end{aligned}\right\} \qquad \begin{aligned} x_0 &= 1 \\ x_1 &= 2 \end{aligned}$$

$$P(x) = 3 + 2(x - \overset{x_0}{1}) + \frac{1}{2}(x - \overset{x_0}{1})(x - \overset{x_1}{2})$$

$$P(x) = 3 + 2x - 2 + \frac{1}{2}(x^2 - 3x + 2)$$

$$P(x) = x^2\left(\frac{1}{2}\right) + x(2 - 1.50) + (3 - 2 + 1)$$

$$P(x) = \frac{x^2}{2} + \frac{x}{2} + 2 = \frac{1}{2}(x^2 + x + 4)$$

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Data Points		First Divided	2nd Divided difference
$x_0 = 1$	$y_0 = 3$		
		$\frac{5-3}{2-1} = 2$ (b_1)	
$x_1 = 2$	$y_1 = 5$		$\frac{(3-2)}{(3-1)} = \frac{1}{2}$ (b_2)
		$\frac{8-5}{3-2} = 3$	
$x_2 = 3$	$y_2 = 8$		
		b_1	b_2

$$P(x) = 3 + 2(x-1) + \frac{1}{2}(x-1)(x-2)$$

$$P(x) = \frac{x^2}{2} + \frac{x}{2} + 2 \qquad \text{Check}$$

	x_0	x_1	x_2
X	1	2	3
f(x)	3	5	8
	y_0	y_1	y_2

Check $P(1) = \frac{(1)^2}{2} + \frac{1}{2} + 2 = \frac{1+1+4}{2} = 3 = y_0$

$$P(2) = \frac{2^2}{2} + \frac{2}{2} + 2 = \frac{4+2+4}{2} = 5 = y_1$$

$$P(3) = \frac{(3)^2}{2} + \frac{3}{2} + 2 = \frac{9+3+4}{2} = 8 \to y_2$$

$$P(2.7) = \frac{2.7^2}{2} + \frac{2.7}{2} + 2 = 3.645 + 1.35 + 2$$

$$= 6.995$$

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Solved example-2

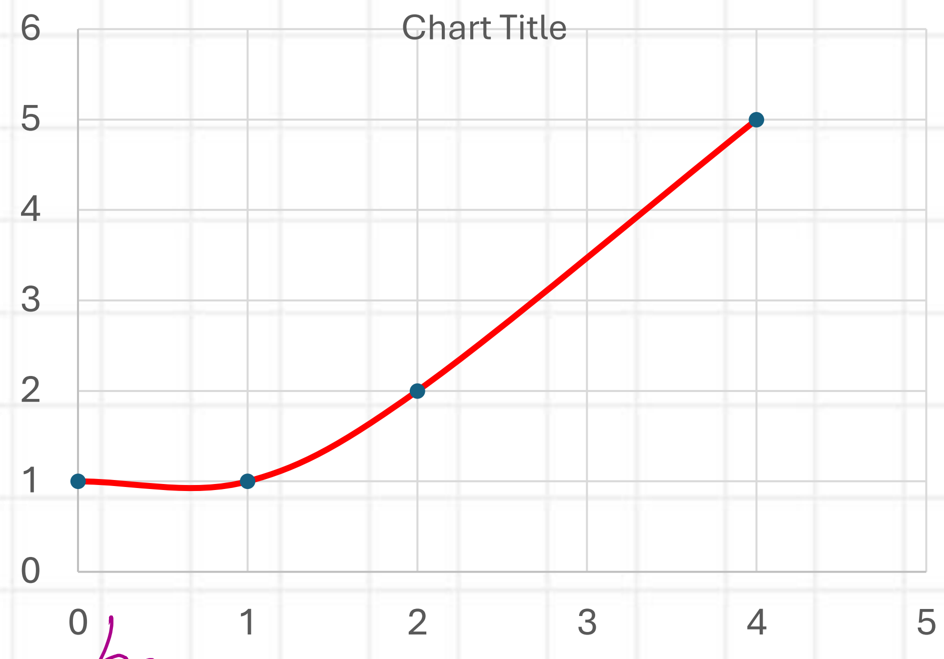
For the given table, use newton divided difference to derive an expression of a polynomial.

x_0 x_1 x_2 x_3

X	0	1	2	4
Y	1	1	2	5

y_0 y_1 y_2 y_3

Solution:



For n+1 points=4 we have 4 b's

b_0 b_1 b_2

$$P(x) = y_0 + f[x_0, x_1](x - x_0) + f[(x_0, x_1, x_2)](x - x_0)(x - x_1) + f[(x_0, x_1, x_2, x_3)](x - x_0)(x - x_1)(x - x_2)$$

b_3

$b_0 = y_0 = 1$

For $b_1 = f[x_0, x_1] = \dfrac{(y_1 - y_0)}{(x_1 - x_0)} = \dfrac{1-1}{1-0} = 0$

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Solved example-2

For the given table, use newton divided difference to derive an expression of a polynomial.

X	0	1	2	4
Y	1	1	2	5

$x_0 = 0$ | $y_0 = 1$
$x_1 = 1$ | $y_1 = 1$
$x_2 = 2$ | $y_2 = 2$
$x_3 = 4$ | $y_3 = 5$

Solution:

$b_2 = f[x_0, x_1, x_2] = \dfrac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$

For n+1 points=4 we have 4 b's

$$P(x) = y_0 + \overset{b_1}{f[x_0, x_1]}(x - x_0) + \overset{b_2}{f[(x_0, x_1, x_2)]}(x - x_0)(x - x_1) + f[(x_0, x_1, x_2, x_3)](x - x_0)(x - x_1)(x - x_2)$$

$b_2 = \dfrac{\dfrac{(2) - (1)}{2 - 1} - \dfrac{(1 - 1)}{1 - 0}}{2 - 0} = \dfrac{1/1 - 0}{2} = +\dfrac{1}{2}$

Table for Newton Divided difference

x	$f(x)$	First divided differences	Second divided differences	Third divided differences
x_0	$f[x_0]$ b_0			
		$f[x_0, x_1] = \dfrac{f[x_1] - f[x_0]}{x_1 - x_0}$ b_1		
x_1	$f[x_1]$		$f[x_0, x_1, x_2] = \dfrac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$ b_2	
		$f[x_1, x_2] = \dfrac{f[x_2] - f[x_1]}{x_2 - x_1}$		$f[x_0, x_1, x_2, x_3] = \dfrac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0}$ b_3
x_2	$f[x_2]$		$f[x_1, x_2, x_3] = \dfrac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1}$	
		$f[x_2, x_3] = \dfrac{f[x_3] - f[x_2]}{x_3 - x_2}$		$f[x_1, x_2, x_3, x_4] = \dfrac{f[x_2, x_3, x_4] - f[x_1, x_2, x_3]}{x_4 - x_1}$
x_3	$f[x_3]$		$f[x_2, x_3, x_4] = \dfrac{f[x_3, x_4] - f[x_2, x_3]}{x_4 - x_2}$	
		$f[x_3, x_4] = \dfrac{f[x_4] - f[x_3]}{x_4 - x_3}$		$f[x_2, x_3, x_4, x_5] = \dfrac{f[x_3, x_4, x_5] - f[x_2, x_3, x_4]}{x_5 - x_2}$
x_4	$f[x_4]$		$f[x_3, x_4, x_5] = \dfrac{f[x_4, x_5] - f[x_3, x_4]}{x_5 - x_3}$	
		$f[x_4, x_5] = \dfrac{f[x_5] - f[x_4]}{x_5 - x_4}$		
x_5	$f[x_5]$			

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Solved example-2

For the given table, use newton divided difference to derive an expression of a polynomial.

	x_0	x_1	x_2	x_3
X	**0**	**1**	**2**	**4**
Y	1	1	2	5
	y_0	y_1	y_2	y_3

$b_3 : F[x_0, x_1, x_2, x_3]$

$F[x_0, x_1, x_2, x_3]$

$$\frac{\frac{f(x_3)-f(x_2)}{x_3-x_2} - \frac{f(x_2)-f(x_1)}{x_2-x_1}}{x_3-x_1} = \frac{\left(\frac{5-2}{4-2}\right) - \left(\frac{2-1}{2-1}\right)}{4-1} = \frac{\frac{3}{2}-1}{3} = \frac{1}{6}$$

$F[$

$$= \frac{\left(\frac{f(x_2)-f(x_1)}{x_2-1} - \frac{f(x_1)-f(x_0)}{x_1-x_0}\right)}{x_2-x_0} = \frac{\left(\frac{2-1}{2-1}\right) - \left(\frac{1-1}{1-0}\right)}{2-0} = \frac{1}{2}$$

Solved example-2

For the given table, use newton divided difference to derive an expression of a polynomial.

	x_0	x_1	x_2	x_3
X	**0**	**1**	**2**	**4**
Y	1	1	2	5
	y_0	y_1	y_2	y_3

$b_0 = 1$

$b_1 =$

$$b_3 = \frac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0} = \frac{1/6 - 1/2}{4-0}$$

$$b_3 = \frac{\frac{1}{6} - \frac{1}{2}}{4} = \frac{\frac{1-3}{6}}{4} = \frac{-2}{24} = -1/12$$

	x_i	y_i (b_0)	First divided differences	2nd divided differences	Third divided differences
$x_0 =$	0	1			
			$\frac{1-1}{1-0} = 0$ (b_1)		
$x_1 =$	1	1		$b_2 = \frac{1-0}{2-0} = 1/2$	
			$\frac{2-1}{2-1} = 1$		$b_3 = -1/12$
$x_2 =$	2	2		$\frac{\frac{3}{2}-1}{4-1} = \frac{1}{6}$	$\frac{\frac{1}{6}-\frac{1}{2}}{4-0} = \frac{\frac{1-3}{6}}{4} = \frac{-1}{12}$
			$\frac{5-2}{4-2} = \frac{3}{2}$		
$x_3 =$	4	5			

$b_0 = 1$, $b_1 = 0$, $b_2 = +\frac{1}{2}$, $b_3 = -\frac{1}{12}$

Using table

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$$\underset{}{P(x)} = \overset{b_0}{y_0} + \overset{b_1}{f[x_0, x_1]}(x - x_0) + \overset{b_2}{f[(x_0, x_1, x_2)]}(x - x_0)(x - x_1) + \underset{b_3}{f[(x_0, x_1, x_2, x_3)]}(x - x_0)(x - x_1)(x - x_2)$$

$b_0 = y_0 = 1$ & $b_1 = 0$ & $b_2 = +1/2$ & $b_3 = -1/12$

$$\underbrace{P(x) = 1 + 0 + \left(\frac{1}{2}\right)(x - x_0)(x - x_1) - \frac{1}{12}(x - x_0)(x - x_1)(x - x_2)}$$

Check $P(x = \underset{x_0}{0}) = 1 + 0 + 0 - 0 = +1$

$$\underset{=1}{P(x = x_1)} = 1 + 0 + \frac{1}{2}(1 - 0)(1 - 1) - \frac{1}{12}(1 - 0)(1 - 1)(1 - 2)$$

$$\underbrace{= 1 + 0 + 0 + 0 = +1}$$

$$P(x = x_2 = 2) = 1 + 0 + \frac{1}{2}(2 - 0)(2 - 1) - \frac{1}{12}(2 - 0)(2 - 1)(2 - 2)$$

$$= 2$$

$x_0 = 0$ & $x_1 = 1$ & $x_2 = 2$ & $x_3 = 4$
$y_0 = 1$ & $y_1 = 1$ & $y_2 = 2$ & $y_3 = 5$

$$P(x) = 1 + 0 + \frac{1}{2}(x - x_0)(x - x_1) - \frac{1}{12}(x - x_0)(x - x_1)(x - x_2)$$

$$P(x_3 = 4) = 1 + 0 + \frac{1}{2}(4 - 0)(4 - 1) - \frac{1}{12}(4 - 0)(4 - 1)(4 - 2)$$

$$= 1 + 6 - \frac{1}{12}(4)(3)(2) = 7 - 2 = 5$$

All Points are matched

$$P(x) = 1 + \frac{1}{2}(x - x_0) - \frac{1}{12}(x - x_0)(x - x_1)(x - x_2)$$