Newton Divided Quadratic interpolation Function.

An attempt to use the Second divided difference and link that expression to the Vandermonde expression by arranging the following equations.

A new form for Quadratic expression was adopted.

Our n+1=3 points namely (x_0, x_1, x_2) and their y coordinates are (y_0, y_1, y_2)

$Q(x) = b_0+b_1(x-x_0) +b_2(x-x_0)(x-x_1)$ → new form

Which can be further expanded as

$Q(x) = b_0+b_1x- b_1x_0 +b_2(x^2-x_0x-xx_1+x_0x_1)$

Recalling our polynomial

$P_2(x) = a_0+a_1x+a_2x^2$ → Vandermonde

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Which can be further expanded as

$Q(x) = b_0+b_1x- b_1x_0 +b_2(x^2-x_0x-xx_1+x_0x_1)$ ← one term for x^2, 3ree terms for x, Constant one term

Recalling our polynomial

$P_2(x) = a_0+a_1x+a_2x^2$ ←

These two equations are equivalent

So items of x^2 : $a2\ x^2= b2\ x^2$ so $a_2=b_2$

For items of x : $a_1\ x= (b_1-b_2x_0- b_2x_1)x$

So $a_1= b_1-b_2x_0-b_2x_1$

For $x^2 \rightarrow$ Vandermonde : a_2x^2

$x^2 \rightarrow Q(x)$: b_2x^2

$a_2=b_2$

For x Vandermonde $\Rightarrow a_1(x)$

$Q(x) \Rightarrow b_1x - b_2x_0x - b_2xx_1$

For the constant term : $a_0 = (b_0 - b_1 x_0 + b_2 x_0 x_1)$

Back to our equation ,and consider $x=x_0$

$Q(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1)$ $\Rightarrow$ $P(x) = a_0 + a_1 x + a_2 x^2$

Then

$Q(x_0) = y_0 = b_0 + b_1(x_0-x_0) + b_2(x-x_0)(x-x_1)b = b_0 + 0 + 0 = b_0$

$b_0 = y_0$

Then $Q(x) = y_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1)$

Then for $x=x_1$

$Q(x_1) = y_1 = y_0 + b_1(x_1-x_0) + 0 = y_1$

Then $b_1 = \left(\frac{y_1 - y_0)}{(x_1 - x_0)}\right)$ Which is the First divided difference $f[x_0, x_1]$

Substitute in $Q(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1)$ our original equation

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$$P(x) = a_0 + a_1 x + a_2 x^2$$

$$Q(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

for $x = x_0 \rightarrow y_0 = a_0 + a_1 x_0 + a_2 x_0^2$

$y = y_0$ $\quad Q(x) = y_0 = b_0 + b_1 \underset{0}{(0)} + b_2 \underset{0}{(0)}$

$b_0 = y_0$. For $x = x_1$

$$y_1 = P(x_1) = a_0 + a_1 x_1 + a_2 x^2$$

$$y_1 = Q(x_1) = b_0 + b_1(x_1 - x_0) + b_2(0)$$

$$y_1 = y_0 + b_1(x_1 - x_0)$$

$a_2 = b_2$

$$b_1 = \frac{(y_1 - y_0)}{(x_1 - x_0)}$$ First divided differenc

$$Q(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$Q(x) = b_0 + \underbrace{\frac{(y_1 - y_0)}{(x_1 - x_0)}}_{\boxed{b_1}}(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$b_0 = y_0$$

for $x = x_2$

$y = y_2$

$$y_2 = y_0 + \frac{(y_1 - y_0)}{(x_1 - x_0)}(x_2 - x_0) + b_2(x_2 - x_0)(x_2 - x_1)$$

Find b_2 Value

$$b_2(x_2 - \cancel{x_0})(x_2 - x_1) = \left[y_2 - y_0\right] - \frac{(y_1 - y_0)}{x_1 - x_0}(x_2 - x_0)$$

$$\frac{(x_2 \cancel{-} x_0)}{(x_2 - x_0)}\left[y_2 - y_0\right] - (x_2 \cancel{-} x_0)\frac{(y_1 - y_0)}{(x_1 - x_0)}$$

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Check for $x=x_2$ and substitute by the values of both b_0, b_1

$$Q(x_2) = y_2 = y_0 + \left(\frac{y_1 - y_0}{x_1 - x_0}\right) * (x_2 - x_0) + b_2 (x_2 - x_0) * (x_2 - x_1)$$

b_1

Finally, *our* b_2 can be estimated from adjusting the equation

$$y_2 - y_0 - \left(\frac{y_1 - y_0}{x_1 - x_0}\right) * (x_2 - x_0) = b_2 (x_2 - x_0) * (x_2 - x_1)$$

Add $+y_1 - y_1$

$$(y_2 - y_1) + (y_1 - y_0) - \left(\frac{y_1 - y_0}{x_1 - x_0}\right) * (x_2 - x_0) = b_2 (x_2 - x_0)(x_2 - x_1)$$

Change from $(x_1 - x_2) = -(x_2 - x_1$

$$b_2 = \frac{\left(\frac{y_2 - y_1}{x_2 - x_1}\right) - \left(\frac{y_1 - y_0}{x_2 - x_1}\right) * \left(\frac{x_2 - x_1}{x_1 - x_0}\right)}{(x2 - x0)}$$

$$b_2 = \frac{\left(\frac{y_2 - y_1}{x_2 - x_1}\right) - \left(\frac{y_1 - y_0}{1}\right) * \left(\frac{1}{x_1 - x_0}\right)}{(x2 - x0)}$$

$\longrightarrow$ b_2 value

Which is the Second divided difference $f[x_0, x_1, x_2]$

The final expression for the quadratic polynomial.

$Q(x) = b_0+b_1(x-x_0) + b_2(x-x_0)(x-x_1)$ $\rightarrow$ $P(x) = a_0 + a_1 x + a_2 x^2$

our original equation will be

$$Q(x) = \overbrace{y_0}^{b_0} + \overbrace{\left(\frac{y_1-y_0}{x_1-x_0}\right)}^{b_1}(x-x0) + \overbrace{\frac{\left(\frac{y_2-y_1}{x_2-x_1}\right)-\left(\frac{y_1-y_0}{1}\right)*\left(\frac{1}{x_1-x_0}\right)}{(x2-x0)}}^{b_2}(x-x_0)(x-x_1)$$

For x_0, x_1, x_2
y_0, y_1, y_2

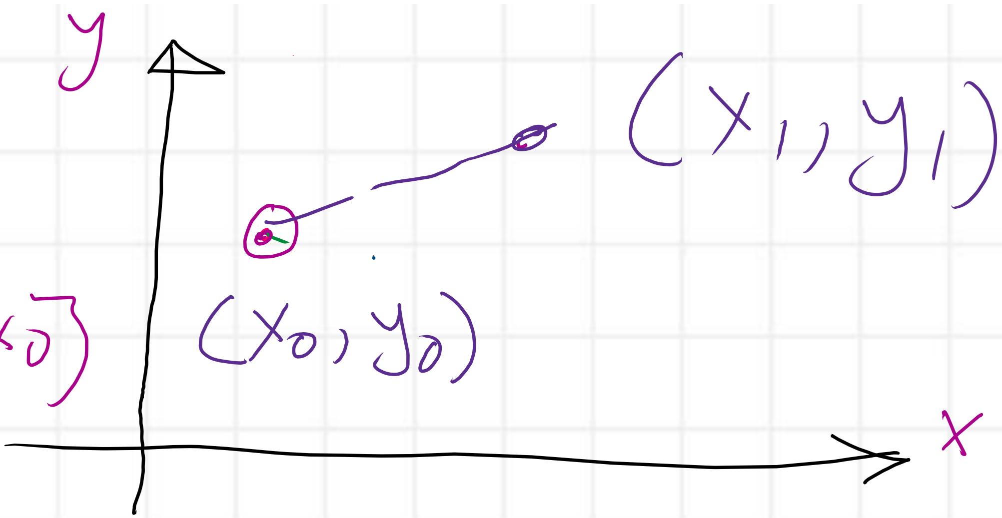
Other form will be

$$Q(x) = y_0 + f[x_0, x_1]\,(x-x0) + f[x_0, x_1, x_2)]\,(x-x_0)(x-x_1)$$

For a Linear function

$x_2 = 0$, $y_2 = 0$

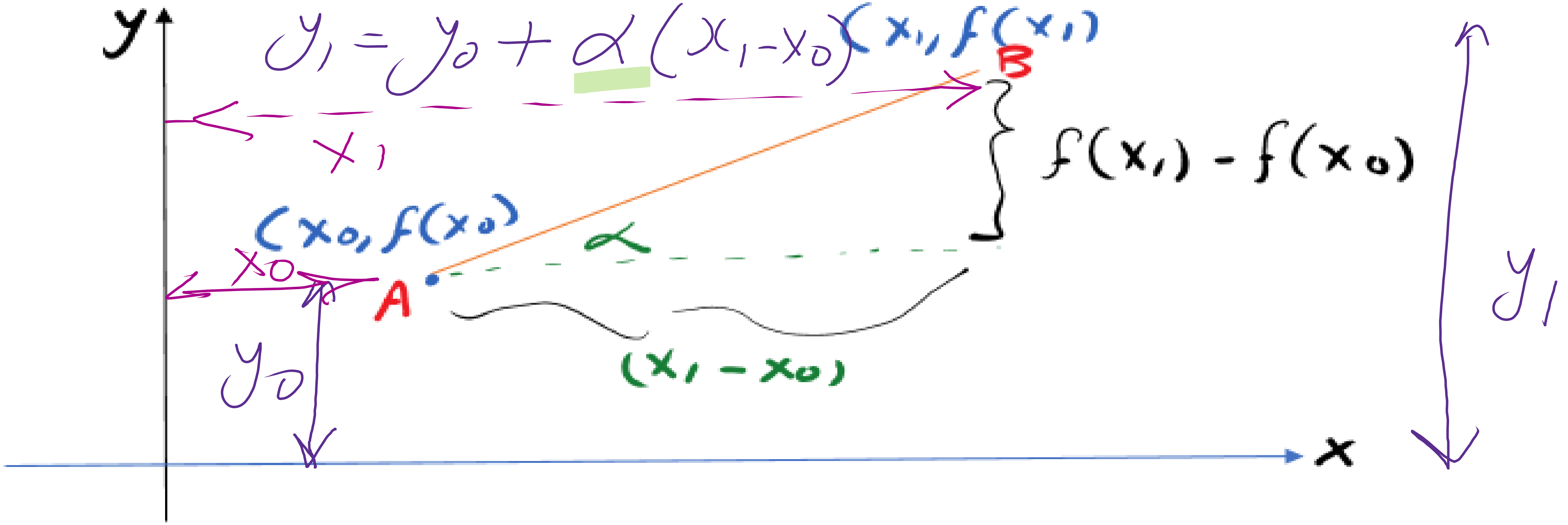
$P(x) = y_0 + f[x_0, x_1](x-x_0)$



$$y_1 = P(x_1) = y_0 + f(x_0, x_1)(x_1 - x_0)$$

First divided difference

$$y_1 = y_0 + \alpha(x_1 - x_0)$$



For the line AB , the first difference is the slope of the line=Rise /run=f(x_1)-f(x_0)/$(x_1$-$x_0)$

Can be written as

$f[x_0, x_1] = (f(x_1)-f(x_0))/(x_1-x_0) = (f(x_0)-f(x_1))/(x_0-x_1)$

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x_0, y_0 if not used but starting from x_1, y_1 $\Rightarrow f(x) = y_1 + f(x_1, x_2)(x - x_1) + f(x_1, x_2, x_3)(x - x_1)(x - x_2)$

Used in our Expression

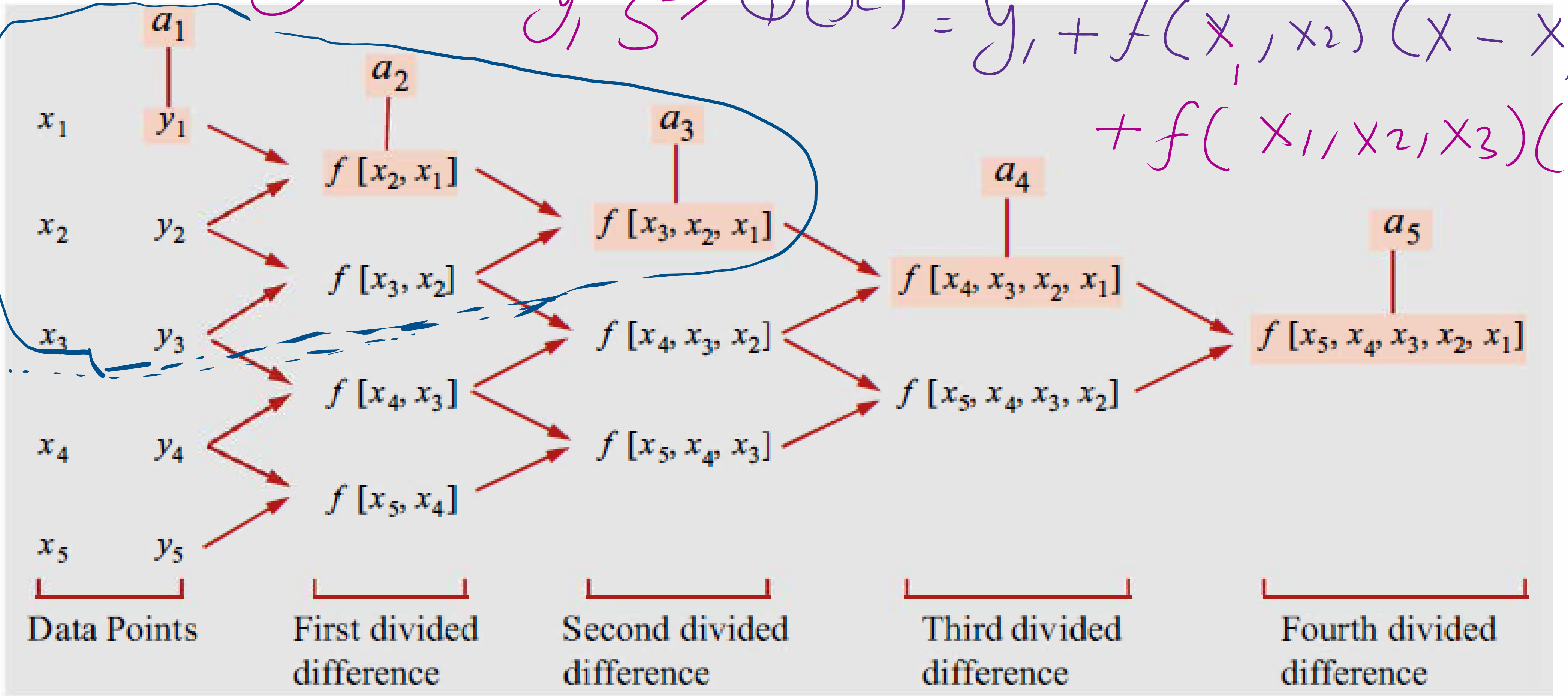

Figure 6-16: Table of divided differences for five data points.

When Using a or $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$

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