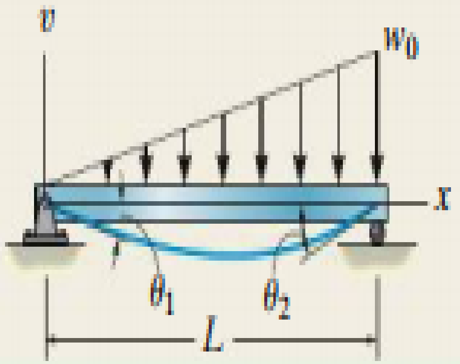
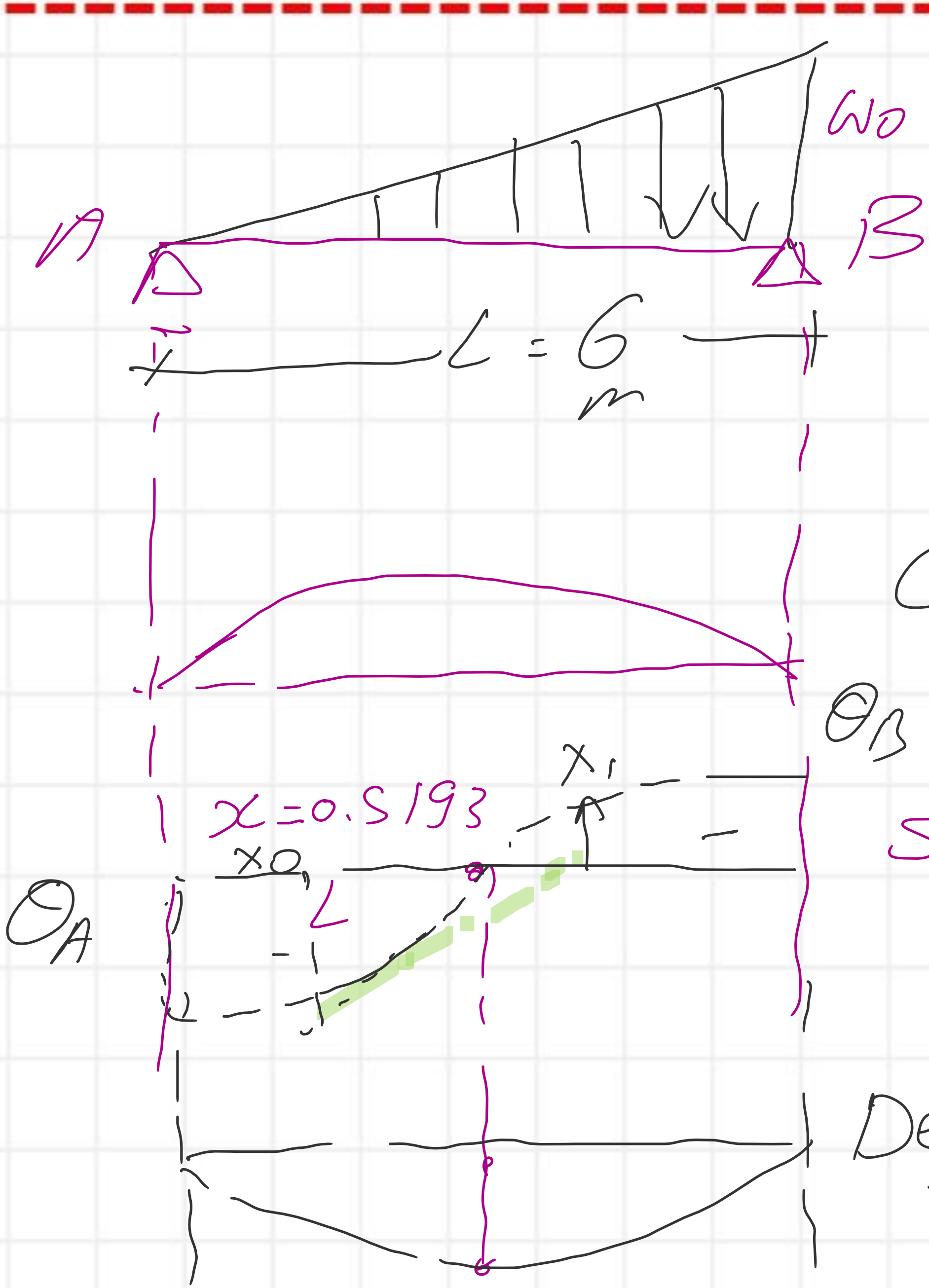
Hibbeller - Mechanics of MaterialS

Simply Supported Beam Slopes and Deflections

BEAM	SLOPE	DEFLECTION	ELASTIC CURVE
	$\theta_1 = \frac{-7w_0L^3}{360EI}$ $\theta_2 = \frac{w_0L^3}{45EI}$	$v_{max} = -0.00652\frac{w_0L^4}{EI}$ at $x = 0.5193L$	$v = \frac{-w_0x}{360EIL}(3x^4 - 10L^2x^2 + 7L^4)$

Vmax at
$x = 0.5193L$

Prepared by Eng.Maged Kamel.



$2\ \frac{kN}{m}$

$EI = 8(10^4)\ kN.m^2$

$L = 6m$ Span

x_{max}: analytical solution $\rightarrow 0.5193L$

$x_{max} = 3.1158$

Curvature:

$$EIy'' = M(x) = \frac{w_0}{6}\left(Lx - \frac{x^3}{L}\right)$$

Slope:

$$EIy' = EI\nu' = \frac{w_0 L x^2}{12} - \frac{w_0}{24L}x^4 - \frac{7}{360} w_0 L^3$$

Deflection

$$EI\nu = \frac{-w_0 x}{360L}\left(3x^4 - 10x^2L^2 + 7L^4\right)$$

To get the Point of max. deflection

Slope: $EIy'' = EIv' = \frac{w_0 L}{12} x^2 - \frac{w_0}{24L} x^4 - \frac{7 w_0 L^3}{360}$ $v' = 0$

Deflection

$$EIv = \frac{-w_0 x}{360L}\left(3x^4 - 10x^2L^2 + 7L^4\right)$$

One we get x value we can find the deflection

slope $\frac{EIv'}{EIv''}$

Curvature

$EI = 8(10^4)\ KN.m^2$

Newton - Raphson Equation

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

For Newton - Raphson

Expression is $f(x_i) = x_0 - \frac{f(x_0)}{f'(x_0)}$

we need to get function that hits x - axis at zero point

Modification

$f(x_i) = x_0 - \frac{f'(x_0)}{f''(x_0)}$

$f'(x_0) \rightarrow$ slope

$f''(x_0) \rightarrow$ Curvature

Our starting point is $x_0 = 2$

$\rightarrow$ $f'(2) \rightarrow$ slope

$f''(2) \rightarrow$ Curvature

We have $w_0 = 2\ \frac{kN}{m}$ & $L = 6\ m$

slope Equation y' or v' =

$EI = 8(10^4)$ $kN.m^2$

$$v' = \frac{1}{EI}\left(\frac{w_0 L x_0}{12} - \frac{w_0 x_0^4}{24L} - \frac{7w_0 L^3}{360}\right)$$

if you prefer divide the Expression

$$y'(x_0 = 2) \text{ or } v'(2) = \frac{1}{8(10^4)}\left[\frac{2(6)(2)}{12} - \frac{2(2)^4}{24(6)} - \frac{7(2)(6)^4}{360}\right]$$

$$y'(x_0 = 2) = -5.77775E\text{-}05$$

$y''(x_0 = 2)$ curvature $\frac{M}{EI}$

$$y''(x_0 = 2) \rightarrow \frac{1}{EI}\left(\frac{w_0 L}{6} x_0 - \frac{w_0 x_0^3}{6L}\right)$$

$$\rightarrow \frac{1}{8(10^4)}\left[\frac{2(6)(2)}{6} - \frac{2(2)^3}{(6)(6)}\right] = v''$$

$= 4.44444E\text{-}05$

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$$x_1 = x_0 - \frac{f'(x_0)}{f''(x_0)} \rightarrow \frac{\text{Slope}}{\text{Curvature}} \qquad x_0 = 2$$

$$x_1 = 2 - \frac{(-5.7777(10^{-5}))}{4.44444(10^{-5})} = 2 + 1.30 = 3.30 \text{ m}$$

$\Downarrow$

Same result

if we didnot divide Curvature by EI $\Rightarrow$ $x_0 = 2$

$$x_1 = 2 - \frac{(-4.6222)}{3.5555}$$

$$EIy'' = M(x) = \frac{w_0}{6}\left(Lx - \frac{x^3}{L}\right) \quad (+3.5555) = 3.30 \text{ m}$$

Slope

$x_0 = 2$

$$EIy' = EIV' = \frac{w_0 L}{12}x^2 - \frac{w_0}{24L}x^4 - \frac{7}{360}w_0 L^3 \rightarrow (-4.6222)$$

$$x_2 = x_1 - \frac{f'(x_1)}{f''(x_1)} \quad \begin{matrix} \rightarrow \text{Slope} \\ \text{Curvature} \end{matrix}$$

$\boxed{X_1 = 3.30}$ m

$$\frac{1}{EI}\left(w_0 \frac{Lx^2}{12} - \frac{w_0 x^4}{24L} - \frac{7w_0 L^3}{360}\right)$$

$$f'(x_1 = 3.30\,m) = \frac{1}{8(10^4)}\left[\frac{2(6)(3.3)^2}{12} - \frac{2(3.3)^4}{24(6)} - \frac{7(2)(6)^3}{360}\right]$$

$$= 1.05361(10^{-5})$$

$x = 3.30$ m

$$f''(x_1 = 3.30\,m) = \frac{1}{EI}\left[\frac{w_0 Lx}{6} - \frac{w_0 x^3}{6L}\right]$$

$$f''(3.30) = 5.75438(10^{-5})$$

$$X_2 = 3.30 - \left(\frac{(1.05361)(10^{-5})}{5.75438(10^{-5})}\right) = 3.30 - 0.1831$$

$$= 3.1169\ m$$

$x_2 = 3.1169\ m$

again get x_3, Slope Elastic curve

NEwton - Raphson Table for x_i

Newton raphson

X	f(x)-slope	f(curvature	ratio	X
2	-0.0000578	0.000044445	-1.29998	3.3
3.3	0.0000105	5.75438E-05	0.1831	3.1169
3.1169	0.0000001	5.68938E-05	0.00092	3.116
3.116	0.0000000	0.00005689	0.00002	3.116
3.116	0.0000000	0.00005689	0.00002	3.116

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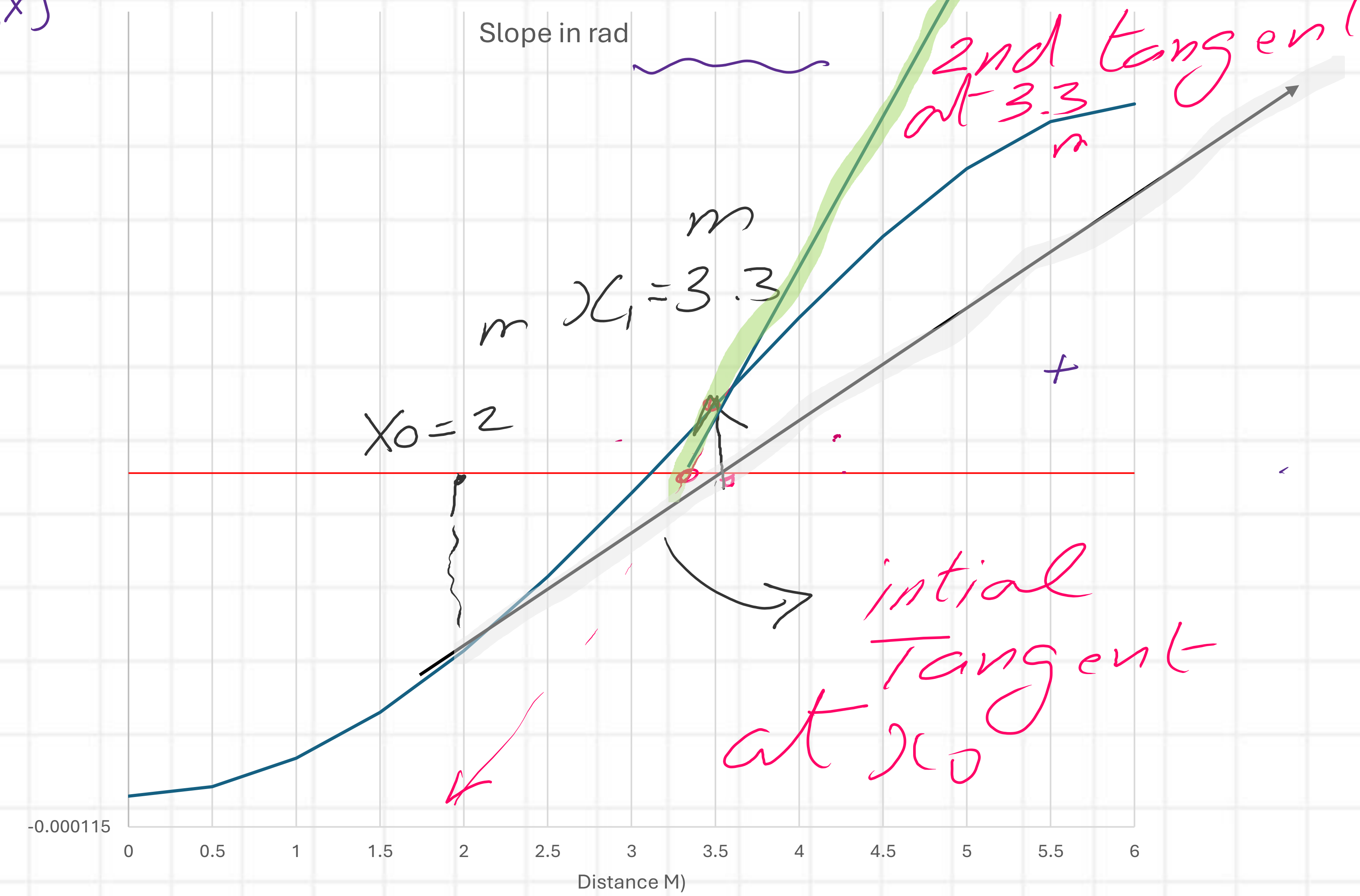
$$X_1 = X_0 - \frac{f'(x)}{f''(x)}$$ slope

2 → 3.3 → 3.1159 m

Distance x (m)	curvature y"	slope in rad
0	0	-0.000105
0.5	1.241E-05	-0.000102
1	2.431E-05	-0.000093
1.5	3.516E-05	-0.000078
2	4.444E-05	-0.00005778
2.5	5.165E-05	-0.000034
3	5.625E-05	-0.000007
3.116	5.689E-05	0.000000
3.5	5.773E-05	0.000022
4	5.556E-05	0.000051
4.5	4.922E-05	0.000077
5	3.819E-05	0.000099
5.5	2.196E-05	0.000114
6	0	0.000120



$x_2 = 3.1169$ m

if we select $x_0 = 3.00$ m the iteration table will be

→ 3.116 m

X	f(x)-slope	f(curvature	ratio	X	Absolute
3	-0.000007	0.00005625	-0.11667	3.1167	0.0389
3.1167	0.000000	5.68932E-05	0.00072	3.116	0.0002
3.116	0.000000	5.68898E-05	0.00002	3.116	0
3.116	0.000000	5.68898E-05	0.00002	3.116	0
3.116	0.000000	5.68898E-05	0.00002	3.116	0

while for $x_0 = 4$ m, the iteration table will be

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X	f(x)-slope	f(curvature	ratio	X	Absolute
4	0.000051	5.55556E-05	0.91	3.09	0.2275
3.09	-0.000001	5.67614E-05	-0.02601	3.116	0.0084
3.116	0.000000	5.68898E-05	0.00002	3.116	0
3.116	0.000000	5.68898E-05	0.00002	3.116	0
3.116	0.000000	5.68898E-05	0.00002	3.116	0

$x_2 = 3.116$ m

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if we want to practice by using

Modified _ Newton _ Raphson

slope curvature

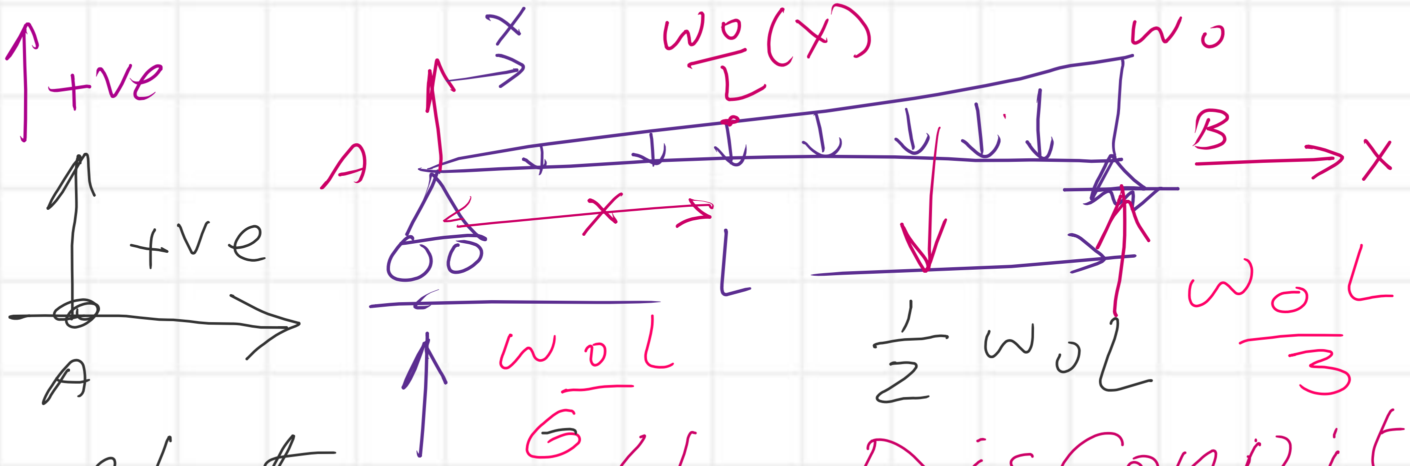
$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

curv slope shear

$EIy'''' = P(x) = \frac{w_0 L}{6} - \frac{w_0}{2L}(x)^2$ use x_0

$f'(x_i) = \frac{2(6)}{6} - \frac{2(2)^2}{2(6)} = 1.666$ KN

$EIy'' = M(x) = \frac{w_0 L x}{6} - \frac{w_0}{6L}x^3 = \frac{2(6)(2)}{6} - \frac{2(2)^3}{36} = 3.555$ kN.m



+ ↑ Loads

Cases 2&6

What are the DisConnuity function ?

$$w(x) = \frac{w_o L}{6}\langle x-0\rangle^{-1} - \frac{w_o}{L}\langle x-0\rangle^{1} + \frac{w_o L}{3}\langle x-L\rangle^{-1}$$

ignored

$$V(x) = \frac{w_o L}{6}\langle x-0\rangle^{0} - \frac{w_o}{L}\frac{\langle x-0\rangle^{2}}{2}$$

$$EIy''' = V$$

$$EIy'' = EIv' = M(x) = \frac{w_o L}{6}\langle x-0\rangle^{1} - \frac{w_o}{2L}\frac{\langle x-0\rangle^{3}}{3}$$

$x_0 = 2\,m$, $w_0 = 2\,k/m$, $L = 6\,m$

$EIv' = \left(\frac{w_0 L x_0^2}{12} - \frac{w_0 x_0^4}{24L} - \frac{7 w_0 L^3}{360}\right)$

$EIv' = \left(\frac{2(6)(2)^2}{12} - \frac{2(2)^4}{24(6)} - \frac{7(2)(6)^3}{360}\right)$

$= \left(4 - \frac{2}{9} - 8.40\right)$

-4.6222

$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

slope, Cur

Cur, slop, shear

$X_1 = 2 - \frac{(-4.6222)(3.555)}{(3.555)^2 - (-4.6222)(1.666)}$

$x_1 = 2 + \frac{16.4345}{12.8049} = 2.8739\,m$

$x_0 = 2m \longrightarrow x_3 = 3.116m$

Modified Newton raphson

	X	f(x)-slope	f'(curvature	Numerator	f''- shear	Denomin	ratio	X	Absolute
x_0	2	-4.6222	3.5556	-16.4347	1.3333	18.8051	-0.8739	2.8739	0.437
x_1	2.8739	-1.0881	4.4291	-4.8193	0.6234	20.2952	-0.2375	3.1114	0.0826
x_2	3.1114	-0.0208	4.5494	-0.0946	0.3865	20.7051	-0.0046	3.116	0.0015
x_3	3.116	0.0001	4.5512	0.0005	0.3818	20.7134	0	3.116	0
x_4	3.116	0.0001	4.5512	0.0005	0.3818	20.7134	0	3.116	0

Modified - Newton - Raphson method