DAVID C Lay

In Exercises 1 and 2, determine which matrices are in reduced echelon form and which others are only in echelon form.

David. C - Lay

Linear algebra and its application

1. a. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Solution:

Part a: Matrix is a reduced echelon form

We have $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $\Rightarrow$ we have three pivots: 1's at the first row, 2nd row, third row

For Columns 2, 3, there are Zeros above the Leading ones. and below

Prepared by Eng.Maged Kamel.

In Exercises 1 and 2, determine which matrices are in reduced echelon form and which others are only in echelon form.

1. a. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Solution:

Part (b): Matrix is a reduced - echelon form

We have $\begin{bmatrix} \boxed{1} & 0 & 1 & 0 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\Rightarrow$ We have two pivots 1's

For Columns 1, 2, there are Zeros above the Leading ones.
and below

row - 3 No Leading entry.

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$$c.\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad d.\ \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

Part c:

$$\begin{bmatrix} \boxed{1} & 0 & 0 & 0 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

} not in Echelon form

rows 1,2 only Two leadin entries

row 3 should be at the bottom of the matrix

→ all zeros to be at the bottom row

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$$\text{c. } \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{d. } \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

Part d:

not Zero

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & \boxed{4} \end{bmatrix}$$

First leading 1
2nd leading 2 should be = 1
a_{12} not $\neq 0$ for RREF

Matrix is an Echelon form but no RREF

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Exercise - 2

In Exercises 1 and 2, determine which matrices are in reduced echelon form and which others are only in echelon form.

Zero element

d. $\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

NoT echelon form

a_{11} : is a Zero Entry

Linear Algebra With Applications 8th Edition
by Steve Leon (Author)

APPLICATION 1 Traffic Flow

In the downtown section of a certain city, two sets of one-way streets intersect as shown in Figure 1.2.2. The average hourly volume of traffic entering and leaving this section during rush hour is given in the diagram. Determine the amount of traffic between each of the four intersections.

Chapter : 1-2

solution

Every node A incoming outgoing in balance

Chapter 1 Matrices and Systems of Equations

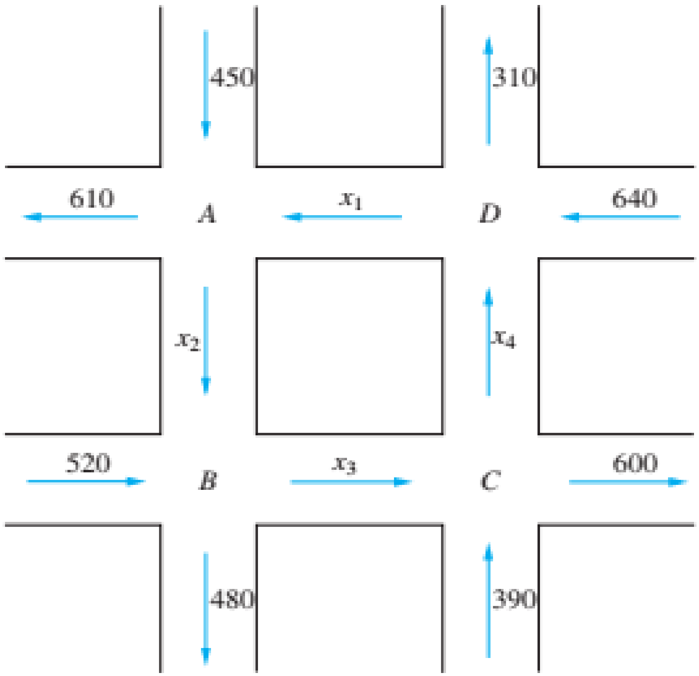

Figure 1.2.2.

Joint

A : $x_1 + 450 = x_2 + 610$

B : $x_2 + 520 = x_3 + 480$

C : $x_3 + 390 = x_4 + 600$

D : $x_4 + 640 = 310 + x_1$

Prepared by Eng.Maged Kamel.

Rearrange the previous four equations $\rightarrow$ 4 variables

$$x_1 - x_2 + 0x_3 + 0x_4 = 160 \quad \text{I}$$

$$0x_1 + x_2 - x_3 + 0x_4 = -40 \quad \text{II}$$

$$0x_1 + 0x_2 + x_3 - x_4 = 210 \quad \text{III}$$

$$-x_1 + 0x_2 + 0x_3 + x_4 = -330 \quad \text{IV}$$

$$\left[\begin{array}{cccc|c} \boxed{1} & -1 & 0 & 0 & 160 \\ 0 & 1 & -1 & 0 & -40 \\ 0 & 0 & 1 & -1 & 210 \\ -1 & 0 & 0 & 1 & -330 \end{array}\right] \rightarrow +R_1 + R_4 \rightarrow R_4$$

$$\left[\begin{array}{cccc|c} \boxed{1} & -1 & 0 & 0 & 160 \\ 0 & \boxed{1} & -1 & 0 & -40 \\ 0 & 0 & 1 & -1 & 210 \\ 0 & -1 & 0 & 1 & -170 \end{array}\right]$$

Zero

REF

$\searrow \dfrac{R_4 + R_2}{R_4}$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 160 \\ 0 & 1 & -1 & 0 & -40 \\ 0 & 0 & 1 & -1 & 210 \\ 0 & 0 & -1 & 1 & -210 \end{array}\right] \rightarrow R_3 + R_4 \rightarrow R_4$$

Prepared by Eng.Maged Kamel.

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 160 \\ 0 & 1 & -1 & 0 & -40 \\ 0 & 0 & 1 & -1 & 210 \\ 0 & 0 & -1 & 1 & -210 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 160 \\ 0 & 1 & -1 & 0 & -40 \\ 0 & 0 & 1 & -1 & 210 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

to be 0

from last row → Consistent many solution

Basic variables are columns 1, 2, 3   We have three pivot column

x_4 → free variable: it can take any value

↳ RREF

$$\xrightarrow{R_2+R_3 \to R_2} \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 160 \\ 0 & 1 & 0 & -1 & 170 \\ 0 & 0 & 1 & -1 & 210 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_1+R_2 \to R_1} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 330 \\ 0 & 1 & 0 & -1 & 170 \\ 0 & 0 & 1 & -1 & 210 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

zero

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$$x_1 - x_4 = 330$$
$$x_2 - x_4 = 170$$
$$x_3 - x_4 = 210$$

Rearrange

$$x_1 = x_4 + 330$$
$$x_2 = x_4 + 170$$
$$x_3 = x_4 + 210$$
x_4 : free variable

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