Complements of the Set.

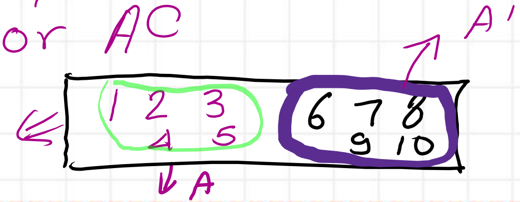
Example if $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3, 4, 5\}$. What Set is represented by A'?

A': ALL elements in U, but not in A is Called A' $\Rightarrow$ Complement of A.

$A' = \{6, 7, 8, 9, 10\}$ or A^C

BiG Box is U



Union and Intersection

The intersection of Two sets, is the Set all elements belong to Both sets

$\cap$ symbol

$P = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$Q = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$P \cap Q = \{2, 4, 6, 8, 10\}$

The Union of Two sets is the set that belongs to either or both Sets

$P \cup Q$

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 18, 20\}$

Another Example

In a Venn diagram, a set is represented as a region in the plane (for convenience, we use rectangles).

Venn-diagram $\Rightarrow$ $P = \{3, 6, 9, 12, 15, 18\}$

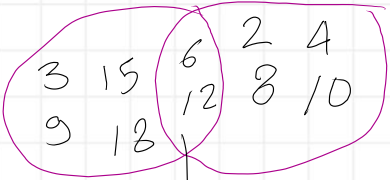
$Q = \{2, 4, 6, 8, 10, 12\}$

$P \cap Q = \{6, 12\}$

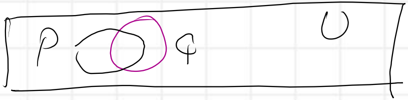
$6 \in P \cap Q$

$8 \notin P \cap Q$

$P \cup Q = \{3, 15, 6, 2, 4, 9, 18, 12, 8, 10\}$



intersection



Example # 10 -

LET = P : $\{3, 9, 27\}$

$Q = \{2, 3, 10, 18, 27, 28\}$

$R = \{2, 10, 28\}$

Find $P \cap Q$, $Q \cap R$, $P \cap R$.

Solution

$P \cap Q = \{3, 27\}$

$Q \cap R = \{2, 10, 28\}$

$P \cap R = \phi \Rightarrow$ Disjoint Set.

No Common Number

Cummutative, Associative and Distributive Properties of a Set

Cummutative

$A \cap B = B \cap A$
$A \cup B = B \cup A$
Can be reversed

Associative

$A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$

Distributive

$A \cup (B \cap C) = (A \cup B) \cap C$, $A \cap (B \cup C) = (A \cap B) \cup (C)$

Set Identity Law

① $A \cup A = A$

② $A \cap A = A$

③ $A \cup \phi = A$

④ $A \cap \phi = \phi$

4.2: Laws of Set Theory

◀ 4.1: Methods of Proof for Sets | 4.3: Minsets ▶

Al Doerr & Ken Levasseur
University of Massachusetts Lowell

4.2.1: Tables of Laws

Idempotent Laws	
(6) $A \cup A = A$	(6′) $A \cap A = A$

Null Laws	
(7) $A \cup U = U$	(7′) $A \cap \emptyset = \emptyset$

Prepared by Eng.Maged Kamel.

All The Subsets

For the set {a,b,c}:

- The empty set {} is a subset of {a,b,c}
- And these are subsets: {a}, {b} and {c}
- And these are also subsets: {a,b}, {a,c} and {b,c}
- And {a,b,c} is a subset of {a,b,c}

And altogether we get the **Power Set** of {a,b,c}:

$$P(S) = \{ \{\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\} \}$$

{a,b,c}

{ {}, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c} }

math is fun

$2^3 = 8$

$$|P(S)| = 2^n$$

$A = \{1, 2, 3, 4\} \rightarrow \{\phi\}$

$2^4 = 16$

{1}, {2}, {3}, {4} ⇒ One element

{1,2}, {1,3}, {1,4} ⇒ Two Elements

{2,3}, {2,4}

{3,4}

3ree Elements

{1,2,3,4}

{1,2,3}

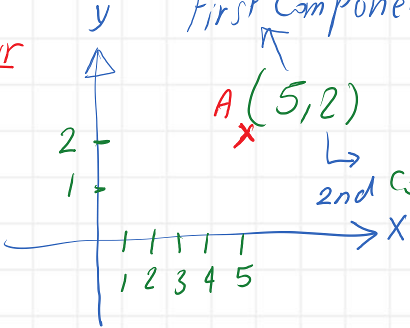
{1,2,4}

{1,3,4}

{2,3,4}

Ordered pair

First Component



$A(5,2)$

2nd Component

$\rightarrow 5$ $\uparrow 2$

Sequence is important

$\{2,5\} \neq \{5,2\}$

$\{a,b\}$, $\{c,d\}$ Two ordered pair

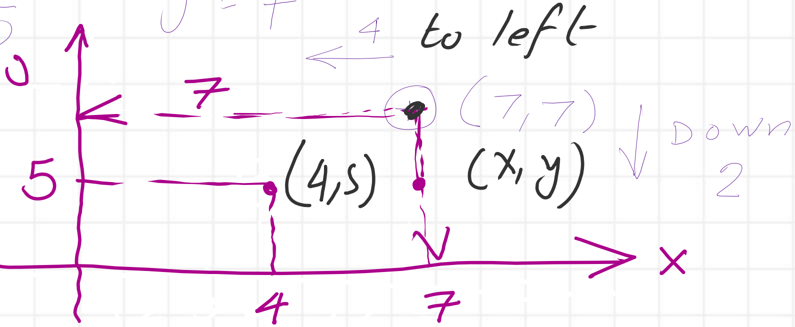
They are equal if $a = c$
$b = d$

Given: $\{x-3, y-2\} = \{4, 5\}$, Find x, y.

Solution

$x - 3 = 4 \qquad x = 7$

$y - 2 = 5 \qquad y = 7$



Cartesian Product of two Sets

https://www.ucl.ac.uk/~ucahmto/0005_2021/Ch2.S5.html

2.5.2 Cartesian products

Definition 2.5.1. The Cartesian product of two sets A and B, written $A \times B$, is the set of all ordered pairs in which the first element belongs to A and the second belongs to B:

$$A \times B = \{\langle a, b\rangle \ : a \in A, b \in B\}.$$

Notice that the size of $A \times B$ is the size of A times the size of B, that is, $|A \times B| = |A||B|$.

if $A = \{7, 2\}$ & $B = \{2, 4, 6\}$ Find $A \times B$

Solution We have six ordered pairs

$\{(7,2)$; $\{7,4\}$; $\{7,6\}$; $\{2,2\}$; $\{2,4\}$; $\{2,6\}$

$A \times B = 2 \times 3$ Refer to Table

Put numbers in Form of Tables

Given

Set $A = \{7, 2\}$ & set $B = \{2, 4, 6\}$

Find $A \times B$

	2	4	6
$a_i = 7$	$\{7, 2\}$	$\{7, 4\}$	$\{7, 6\}$
$a_j = 2$	$\{2, 2\}$	$\{2, 4\}$	$\{2, 6\}$

$\rightarrow$ 6 ordered pairs

$$\{(7,2), (7,4), (7,6), (2,2), (2,4), (2,6)\}$$

Example

If A, B are two sets and $A \times B$ Consists of 6 Elements, given three elements of $A \times B$ are $(2,5), (3,7), (4,7)$

Find $A \times B$ and $A \times A$

Solution $(2,5), (3,7), (4,7)$

$a_1, b_1 \quad a_2, b_2 \quad a_3, b_2$

$A = \{2, 3, 4\}$

$B = \{(5, 7)\}$

$A \times B = 2 \times 3 = 6$

$A \times A = 3 \times 3 = 9$

$\Rightarrow A \times B = \{(2,5), (2,7), (3,5), (3,7), (4,5), (4,7)\}$

Please refer to the next slide

Put numbers in Form of Tables

Set $A = \{2, 3, 4\}$ & $B = \{5, 7\}$

$A \times B$

	5	7
$a_i = 2$	$\{2, 5\}$	$\{2, 7\}$
$a_j = 3$	$\{3, 5\}$	$\{3, 7\}$
$a_k = 4$	$\{4, 5\}$	$\{4, 7\}$

Total

$\Rightarrow$ 6 $\leftarrow$ ordered pairs

given three ordered pairs

Put numbers in Form of Tables

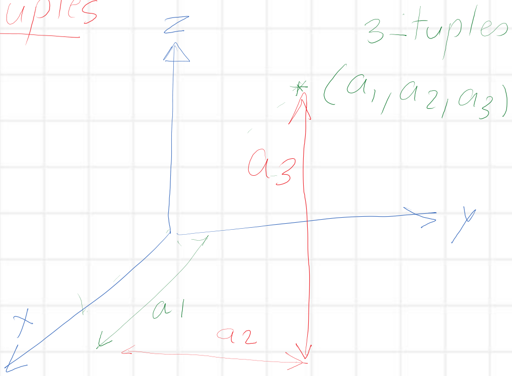
Set $A = \{2, 3, 4\}$ & $A = \{2, 3, 4\}$

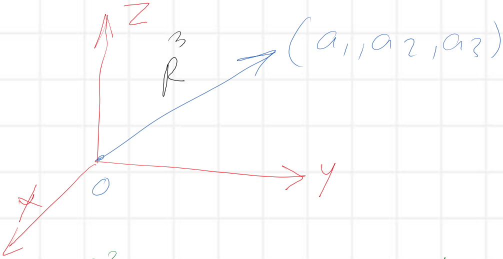
	2	3	4
$a_i = 2$	$\{2, 2\}$	$\{2, 3\}$	$\{2, 4\}$
$a_j = 3$	$\{3, 2\}$	$\{3, 3\}$	$\{3, 4\}$
$a_k = 4$	$\{4, 2\}$	$\{4, 3\}$	$\{4, 4\}$

$A \times A$

9 ordered Pairs

N-Tuples





R^3 : Set of all ordered n tuples of Real Numbers

R^1 : 1-Space : Set of all Real Numbers

R^2 : 2-Space : Set of all ordered Pairs of Real Numbers

Prepared by Eng.Maged Kamel.

Ron Larson

Elementary linear algebra

4.1 Page 156

William Rowan Hamilton (1805–1865)

Hamilton is considered to be Ireland's most famous mathematician. In 1828, he published an impressive

The properties of vector addition and scalar multiplication for vectors in R^n listed below are similar to those listed in Theorem 4.1 for vectors in R^2. Their proofs, based on the definitions of vector addition and scalar multiplication in R^n, are left as an exercise. (See Exercise 64.)

THEOREM 4.2 Properties of Vector Addition and Scalar Multiplication in R^n

Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in R^n, and let c and d be scalars.

1. $\mathbf{u} + \mathbf{v}$ is a vector in R^n. Closure under addition
2. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ Commutative property of addition
3. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$ Associative property of addition
4. $\mathbf{u} + \mathbf{0} = \mathbf{u}$ Additive identity property
5. $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$ Additive inverse property
6. $c\mathbf{u}$ is a vector in R^n. Closure under scalar multiplication
7. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$ Distributive property
8. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$ Distributive property
9. $c(d\mathbf{u}) = (cd)\mathbf{u}$ Associative property of multiplication
10. $1(\mathbf{u}) = \mathbf{u}$ Multiplicative identity property

Ron Larson

Elementary linear algebra

4.1 Page 155

Vectors in R^n

The discussion of vectors in the plane can now be extended to a discussion of vectors in n-space. A vector in n-space is represented by an **ordered n-tuple.** For instance, an ordered triple has the form (x_1, x_2, x_3), an ordered quadruple has the form (x_1, x_2, x_3, x_4), and a general ordered n-tuple has the form $(x_1, x_2, x_3, \ldots, x_n)$. The set of all n-tuples is called **n-space** and is denoted by R^n.

R^1 = 1-space = set of all real numbers

R^2 = 2-space = set of all ordered pairs of real numbers

R^3 = 3-space = set of all ordered triples of real numbers

R^4 = 4-space = set of all ordered quadruples of real numbers

$\vdots$

R^n = n-space = set of all ordered n-tuples of real numbers

The practice of using an ordered pair to represent either a point or a vector in R^2 continues in R^n. That is, an n-tuple $(x_1, x_2, x_3, \ldots, x_n)$ can be viewed as a **point** in R^n with the x_i's as its coordinates or as a **vector**

$$\mathbf{x} = (x_1, x_2, x_3, \ldots, x_n) \qquad \text{Vector in } R^n$$

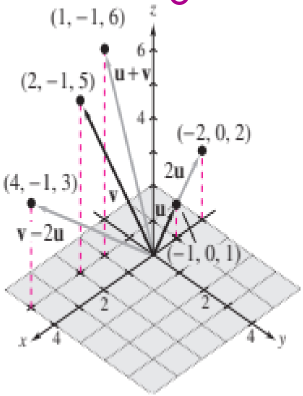


Figure 4.6

$n = 3$ $R^3 = $ 3-space (x_1, x_2, x_3) (x_1, x_2, x_3)

= set of all ordered triple of real numbers

$n = 4$

$R^4 = $ 4-space (x_1, x_2, x_3, x_4)

= set of all ordered quadruple of real numbers

(x_1, x_2, x_3, x_4)

$\overset{U}{(2, -1, 5, 0)}, \overset{V}{(4, 3, 1, -1)}, \overset{W}{(-6, 2, 0, 3)}$

For one operation out of 10 operations

$U + V + W = (U + V) + W$

$(2 + 4 - 6), (-1 + 3 + 2), (5 + 1 + 0), (0 - 1 + 3)) = (0, 4, 6, 2)$

Pythagorean quadruple

Refer to

$a^2 + b^2 + c^2 = d^2$

$\overset{a}{(1}, \overset{b}{2}, \overset{c}{2}, \overset{d}{3)}$ $d^2 = 1^2 + 2^2 + 2^2 = 9 = 3^2$

Ordered Pair

$\{2,5\} \neq \{5,2\}$

$\{a,b\}$, $\{c,d\}$ Two ordered Pair

They are equal if $a = c$
$b = d$

Prepared by Eng.Maged Kamel.