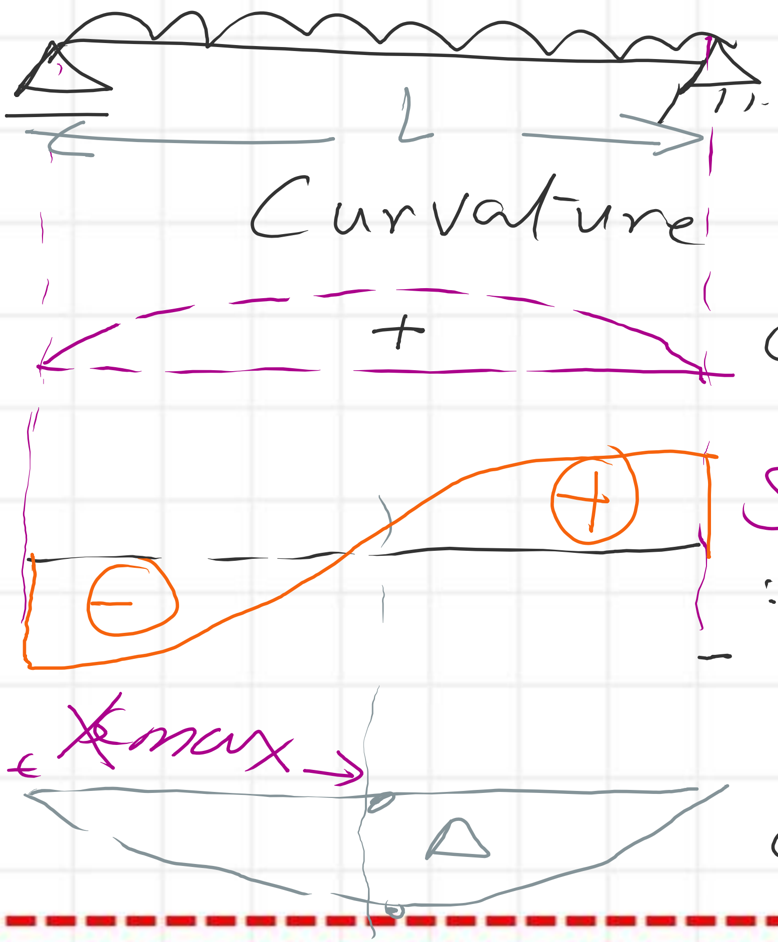
Find x distance to point of maximum deflection

https://www.geeksforgeeks.org/engineering-mathematics/newton-raphson-method/

ω (UL)



Double integration

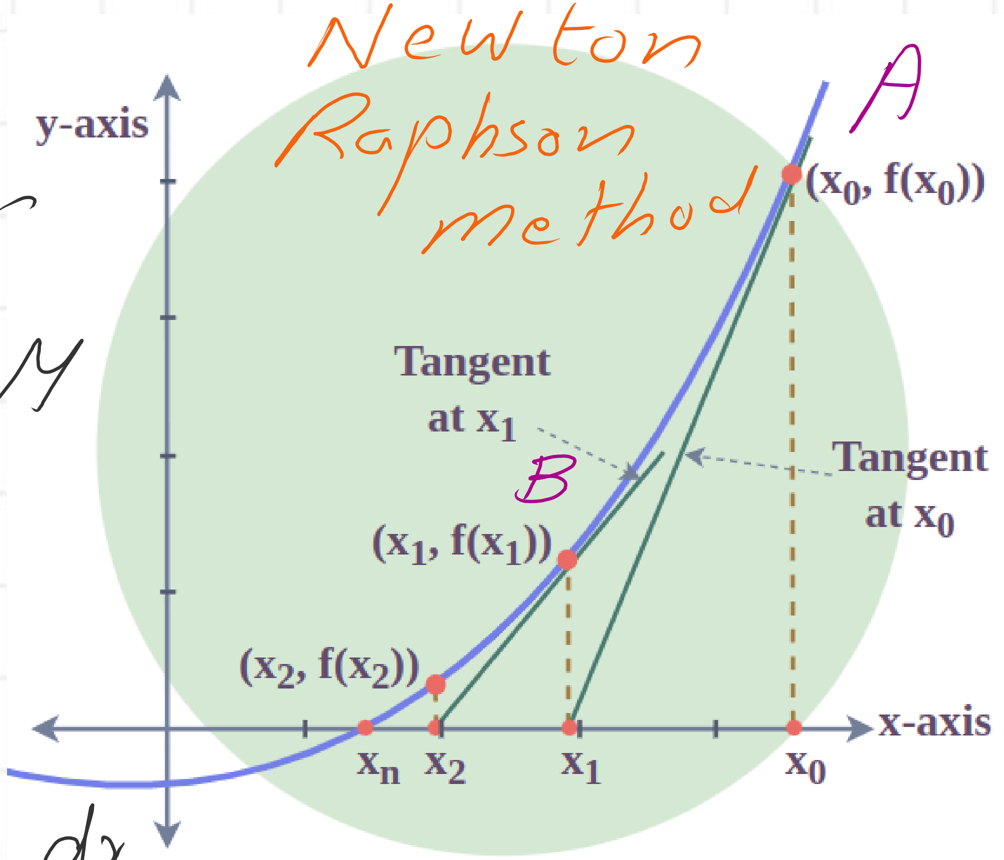
$EIy'' = M$

$y'' = \frac{M}{EI}$

Slope

$\therefore y' = \int \frac{M}{EI}\, dx$

$v(x)$ or y $= \iint \frac{M}{EZ}\, dx$

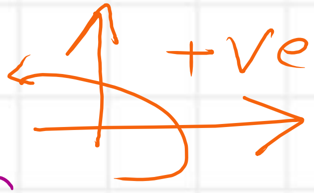


A: intial point: $(x_0, f(x_0))$

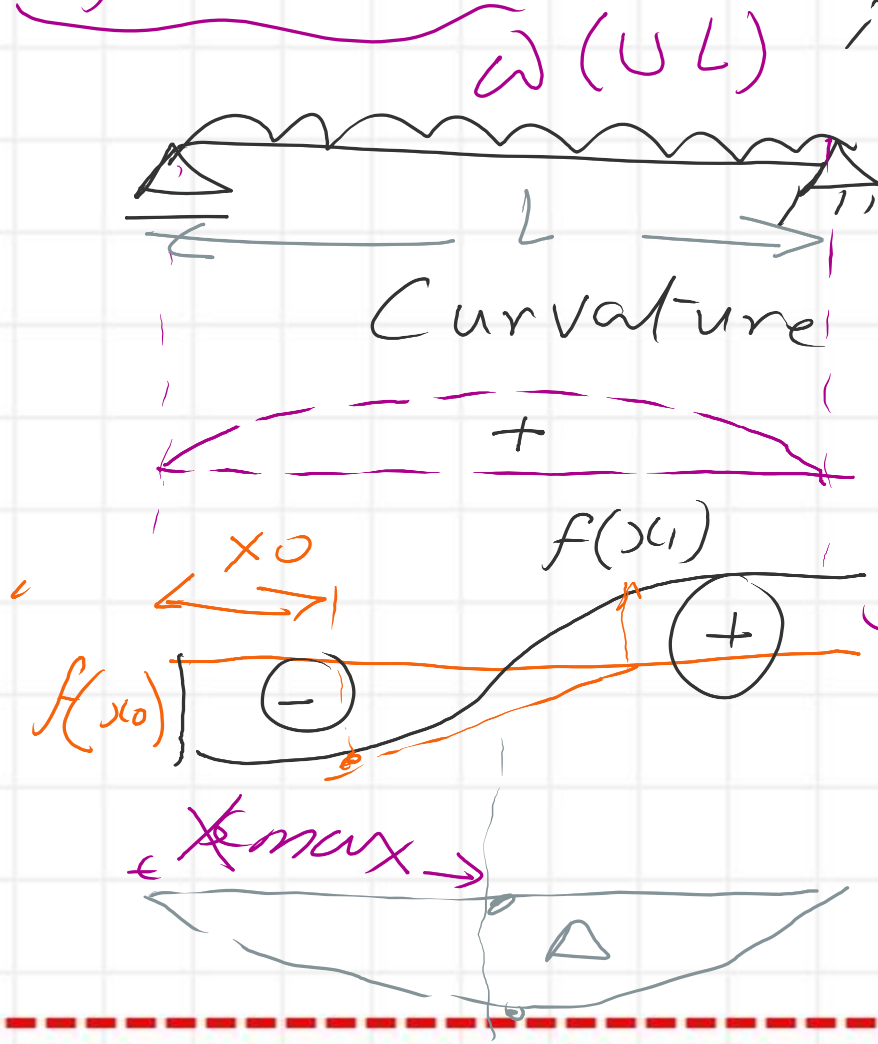
B: new point: $(x_1, f(x_1))$

$x_1 = x_0 - f(x_0)/f'(x_0)$

Find x distance to point of maximum deflection



Double integration



$$EIy'' = M$$

Curvature

$$y'' = \frac{M}{EI}$$

Slope

$$y' = \int \frac{M}{EI}\, dx$$

$$v(x) \text{ or } y = \iint \frac{M}{EI}\, dx$$

Newton Raphson method

$$\frac{\cancel{EI}\,y'}{\cancel{EI}\,y''}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

$$x_1 = x_0 - \frac{\text{slope } y'}{\text{Curvature } y''}$$

Root is the point of zero slope

X-max - numerically

$$\frac{d\,EIy''}{dx} = \frac{dM}{dx} = \varphi(x)$$

$$\frac{d^3y}{dx^3} = \varphi(x)$$

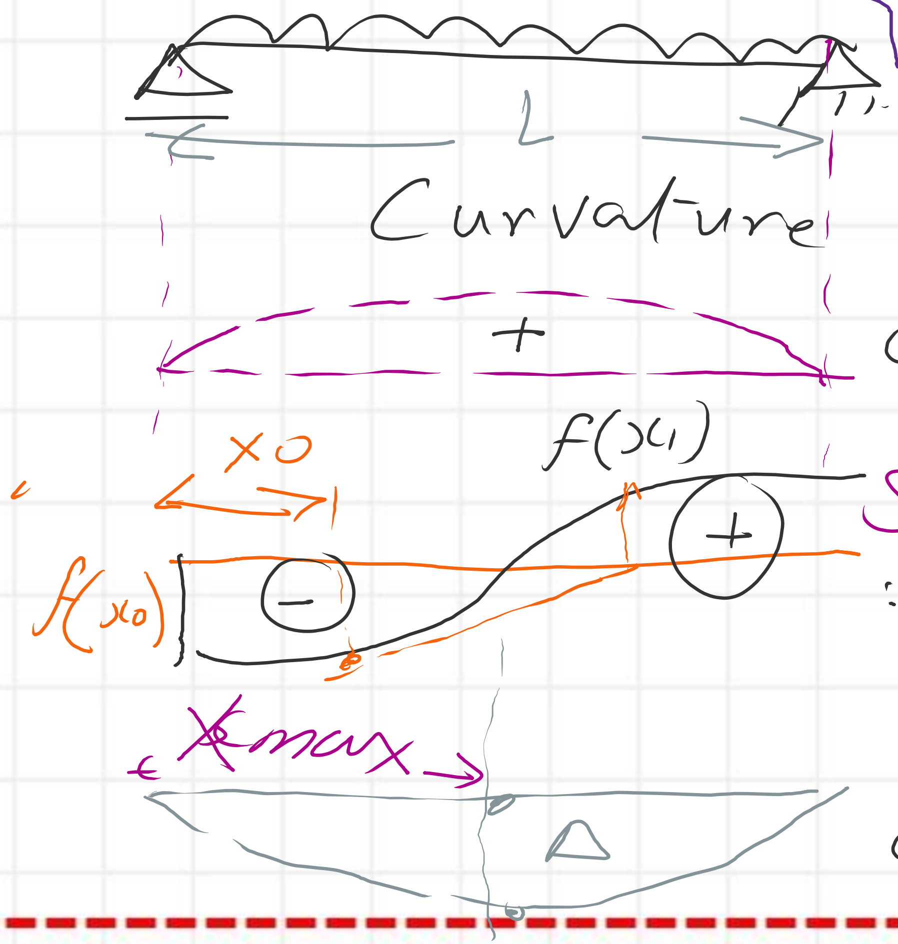
$\omega\ (UL)$

+ve

Modified Newton-Raphson

$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

Double integration



$EIy'' = M$

$y'' = \frac{M}{EI}$

slope

$y' = \int \frac{M}{EI}\,dx$

$V(x)$ or $y = \iint \frac{M}{EI}\,dx$

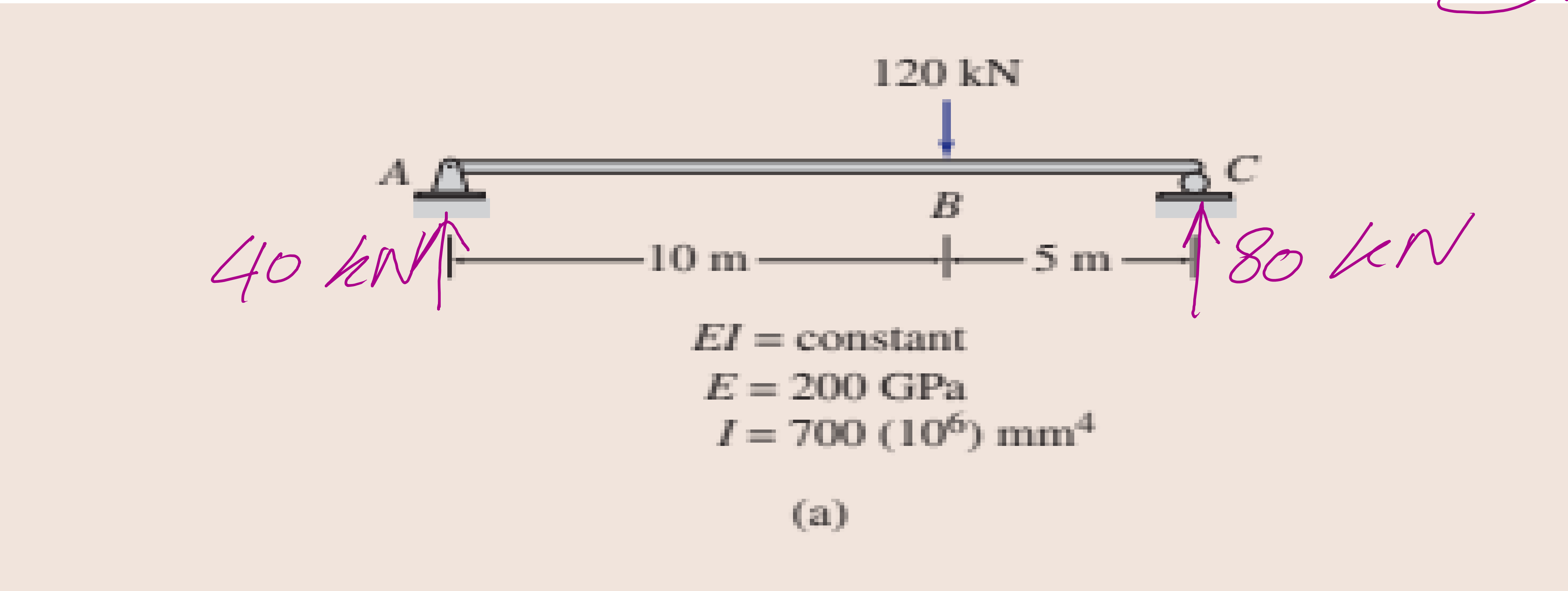
$f(x_i)$: slope $y'(x)$

$f'(x_i)$: Curvature $y''(x)$

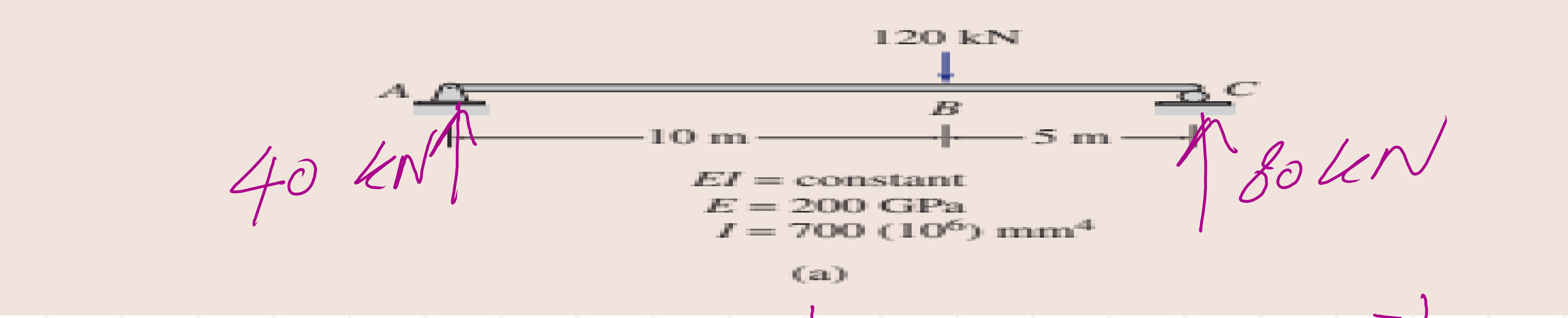
$f''(x_i)$: shear $\varphi(x)$ $\frac{d^3y}{dx^3}$

Root is the point of zero slope

Source : Aslam Kassimali 6.4



We want to use Newton-Raphson method to Determine the x distance to maximum deflection → max deflection occurs at 8.165 m From A



(a)

$$w(x) = 40\langle x-0\rangle^{-1} - 120\langle x-10\rangle^{-1} + 80\langle x-15\rangle$$
ignored

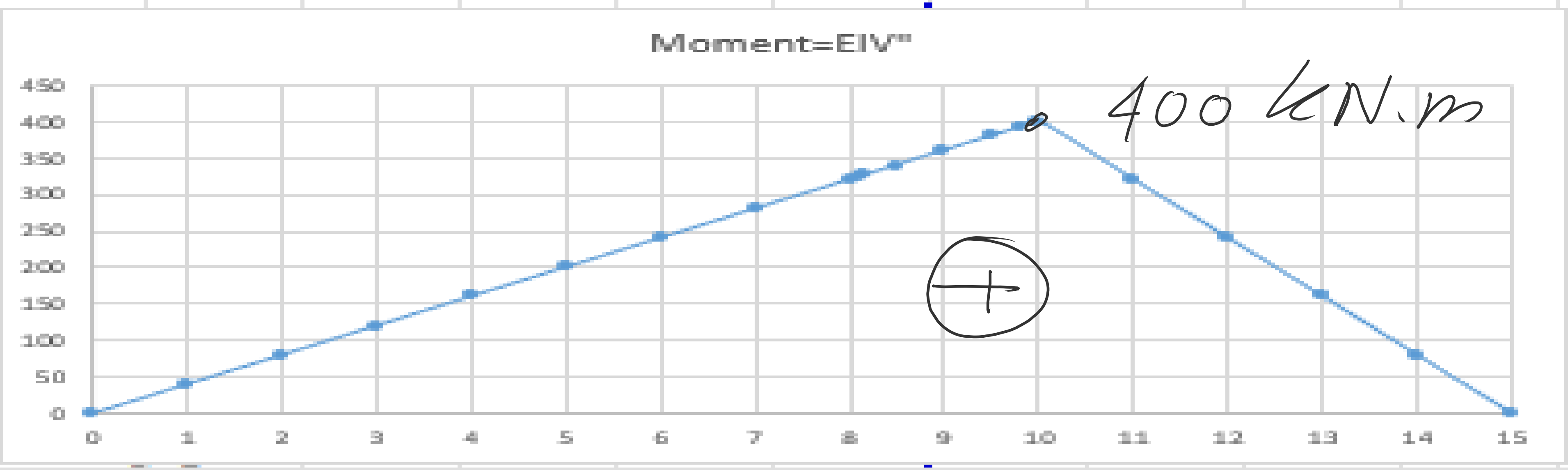
$$V(x) = 40\langle x-0\rangle^{0} - 120\langle x-10\rangle^{0}$$

$$EI\,v'' = M(x) = 40\langle x-0\rangle^{1} - 120\langle x-10\rangle^{1}$$

$$EI\,v' = \frac{40}{2}\langle x-0\rangle^{2} - \frac{120}{2}\langle x-10\rangle^{2} - 1333.333$$

$$EI\,v = \frac{20}{3}\langle x-0\rangle^{3} - \frac{60}{3}\langle x-10\rangle^{3} - 1333.33x$$

For $x < 10$ mm ignore



$L = 15m$

$\longrightarrow 200 \frac{kN}{m^2}(10^6)$

$EI \Rightarrow \overset{E}{200\,GPA} = 200(10^9)\frac{N}{m^2}$

$I = 700(10^6)\,mm^4 = \frac{700(10^6)\,mm^4}{(10^3)^4\,\frac{mm^4}{m^4}}(m^4) = 7(10^{-4})\,m^4$

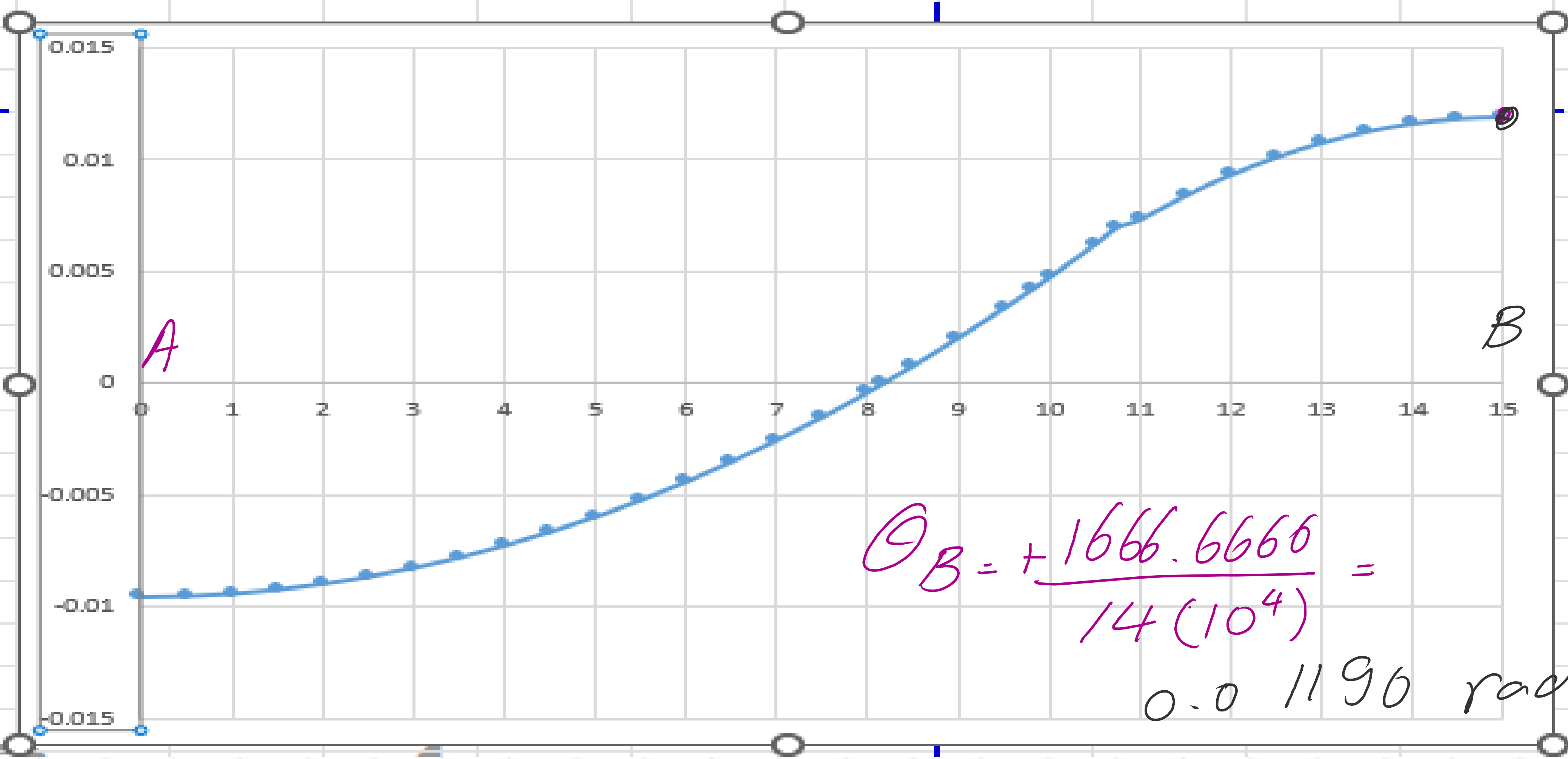
$EI = 200(10^6)\frac{kN}{m^2}(7)(10^{-4})\,m^4$

$= 14(10^{+4})\;kN.m^2$

$= 140000$

Slope = 0 at $x = 8.16497\,m$

$EI\,\theta_A = 1333.3333$ kN.m² $\rightarrow \theta_A = \dfrac{-1333.3333}{140000} = -0.00952$ rad

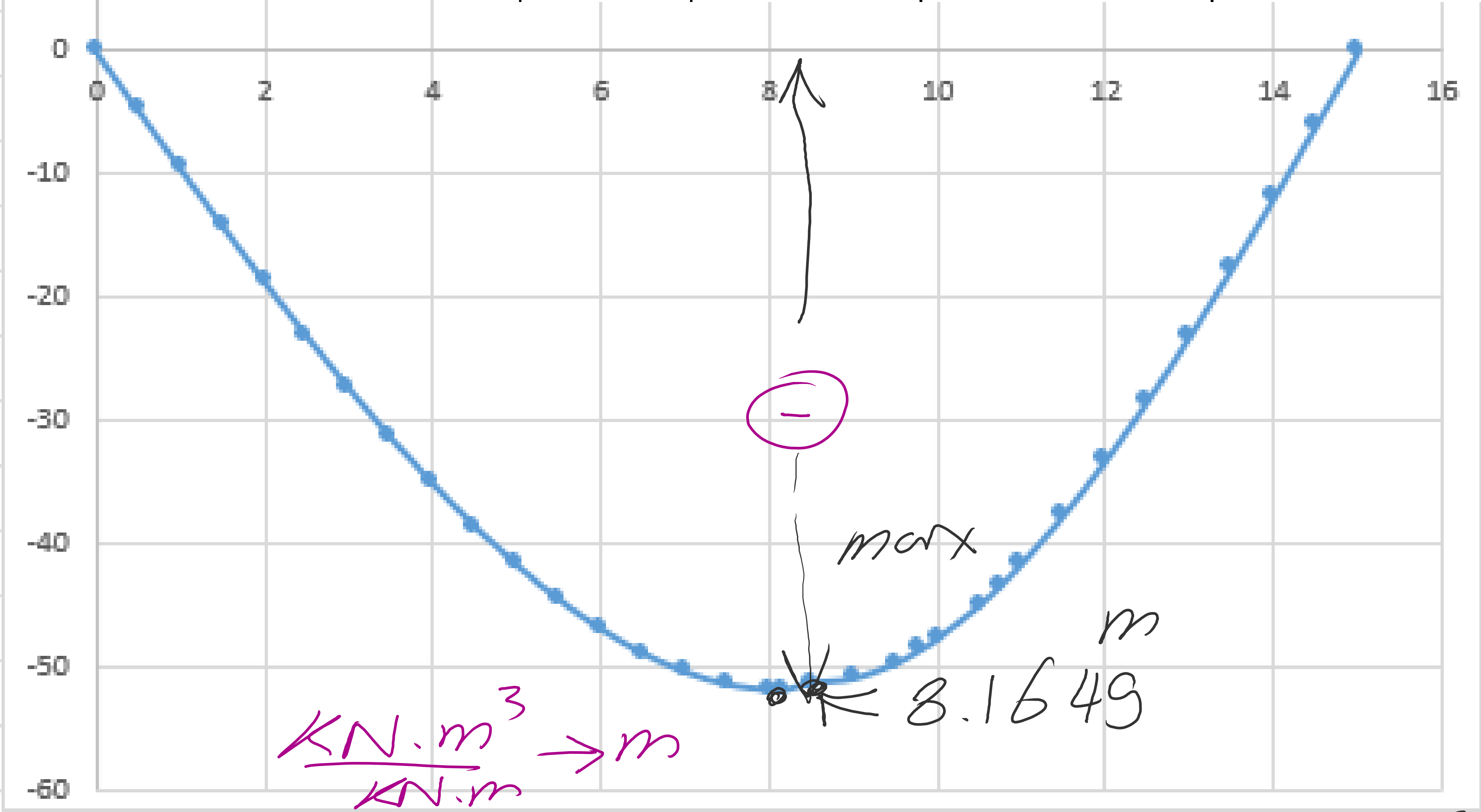


$\theta_B = +\dfrac{1666.6666}{14(10^4)} =$

0.01190 rad

$EI\nu_{max} = -7257.747$

x	$EI\nu''$	$EI\nu'$	$EI\nu$
8.164965	326.5986	-0.000230976	-7257.747114



$\nu_{max} = -7257.747 / 140000 = -0.0518 \, m \Rightarrow 51.84 \, mm$

From MACaulay 's discontinuity function

$EI\vartheta' = 20\langle x-0\rangle^2 - 1333.33$ $\quad x > 0 \quad < 10\,m$

$EI\vartheta' = 20x^2 - 1333.33 \rightarrow$ numerator

$EI\vartheta'' = 40x \rightarrow$ De-numerator

Select $x_0 = 4\,m \longrightarrow EI\vartheta' = 20(4)^2 - 1333.33$

$EI\vartheta'' = 40(4)$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 4 - \frac{(-1013.33)}{160} = 10.3333\,m$$

$> 10\,mm$

From MACaula discontinuity function

$X > 10\ m$ $\quad X_1 = 10.3333\ m$

$$\begin{cases} EIv' = 20\langle x-0\rangle^2 - 66\langle x-10\rangle^2 - 1333.33 \\ EIv'' = 40\langle x-0\rangle - 120\langle x-10\rangle = \end{cases}$$

Find $EIv' = 20(10.3333)^2 - 60(0.3333)^2 - 1333.33$

while $EIv'' = 40(10.3333) - 120(0.3333)$

$$X_2 = X_1 - \frac{EIv'}{EIv''} = 10.333 - \frac{(782.2138)}{(373.336)} = 8.2381\ m$$

$< 10\ m$

$$X_3 = X_2 - \frac{EIv'}{EIv''} \quad \text{at } x < 10\ m$$

From MACaulay 's discontinuity function

$$EI\nu' = 20\langle x-0\rangle^2 - 1333.33 \qquad x > 0,\ x < 10\,m$$

$$EI\nu' = 20x^2 - 1333.33 \rightarrow \text{numerator}$$

$$\rightarrow \text{De-numerator}$$

$$EI\nu'' = 40x$$

$$x_2 = 8.2381\,m \longrightarrow EI\nu' = 20(8.2381)^2 - 1333.33$$

$$EI\nu'' = 40(8.2381)^2$$

$$x_3 = x_2 - \frac{EI\nu'}{EI\nu''} \quad \Big\} \quad x_3 = 8.2381 - \frac{23.9958}{329.524} = 8.165\,m$$

$\downarrow$

more iterations

Using Newton-Raphson method

X	f(X)	f'(X)	f(x0)/f'(x0)	x new
4	-1013.3300	160	-6.3333	10.3333
10.3333	782.2138	373.336	2.0952	8.2381
8.2381	23.9958	329.524	0.0728	8.1653
8.1653	0.1125	326.612	0.0003	8.1650
8.165	0.0145	326.6	0	8.1650

f(x)=20x^2-1333.33-slope

f'(x)=40*(x-0)

$\xi < 10$ m

f(x)=20x^2-60(x-10)^2-1333.33- slope for bi

f'(x)=40(x-0)-120(x-10)

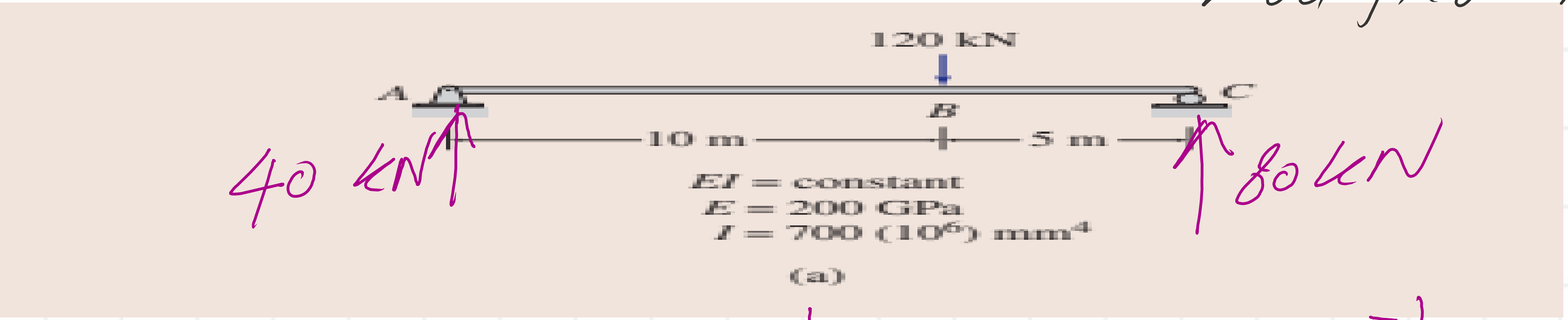
$x > 10$ m

f''(x)=Q(x)=40

$x_0 = 4$

$x_i = 8.1650$ m

Modified Newton Raphson



$$w(x) = 40\langle x-0\rangle^{-1} - 120\langle x-10\rangle^{-1} + 80\langle x-15\rangle$$

ignored

$$V(x) = 40\langle x-0\rangle^{0} - 120\langle x-10\rangle^{0}$$

$$EI\,v'' = M(x) = 40\langle x-0\rangle^{1} - 120\langle x-10\rangle^{1}$$

$$EI\,v' = \frac{40}{2}\langle x-0\rangle^{2} - \frac{120}{2}\langle x-10\rangle^{2} - 1333.333$$

ignore Green term for $x < 10$ m

$$f'''(x) = V(x) \Rightarrow EI\,v''$$

Start $x_0 = 4$

$x < 10$ m

Calculate

$f''(x) = q(x) = 40$ kN

EIv'''

$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

Slope

EIv'' Curvature

$\frac{(EI)^2}{(EI)^2}$

$\overline{}^2$ Curvature

EIv' Slope

$q(x)$ EIv''' Shear

$f'(x) = EIv'' = 40(x_0) = 160$ = Curvature

$f(x) = EIv' = 20\langle x - 0\rangle^2 - 1333.33 = 320 - 1333.33 = -1013.33$

slope

$f'(x) = (160)^2$

$$x_1 = 4 - \frac{(1013.33)(160)}{(160)^2 - 1013.33(40)}$$

Use

Modified Newton - Raphson Table for Xi values

	X	f(x)	f'(X)	f''(X)	f(x)*f'(x)	denominator	xi
X_0	4	-1013.3300	160	40	-162132.8	66133.2	6.4516
X_1	6.4516	-500.8671	258.064	40	-129255.767	86631.712	7.9436
X_2	7.9436	-71.3144	317.744	40	-22659.7227	103813.83	8.1619
X_3	8.1619	-0.9978	326.476	40	-325.757753	106626.49	8.165
X_4	8.165	0.0145	326.6	40	4.7357	106666.98	8.165

$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$