Solve the following system of linear equations, using Gauss-Jordan elimination:

33.
$$\begin{cases} x + 2y - z = -2 \\ x + z = 0 \\ 2x - y - z = -3 \end{cases}$$

1st Example

Solved Problems

Stewart – College Algebra

@ Write the matrix in the augmented Form

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 1 & 0 & 1 & 0 \\ 2 & -1 & -1 & -3 \end{array}\right] \rightarrow R_2 \rightarrow (-1R_1 + R_2) \quad \rightarrow R_3 \rightarrow (-2R_1 + R_3)$$

to be Zeros

Solve the following system of linear equations, using Gauss-Jordan elimination:

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$$\begin{bmatrix} 1 & 2 & -1 & -2 \\ 1 & 0 & 1 & 0 \\ 2 & -1 & -1 & -3 \end{bmatrix} \rightarrow R_2 \rightarrow (-1R_1 + R_2) \quad \rightarrow R_3 \rightarrow (-2R_1 + R_3)$$

Zeros

$$\begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & -2 & 2 & 2 \\ 0 & -5 & 1 & 1 \end{bmatrix}$$

make +1

$$(R_1, \ -R_2/2, \ R_3 \rightarrow) \Rightarrow \begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & -5 & 1 & 1 \end{bmatrix} \rightarrow R_1, \ \rightarrow R_2, \ \rightarrow R_3 \rightarrow \left(\frac{+5}{1} R_2 + R_3\right)$$

Zero

Solve the following system of linear equations, using Gauss-Jordan elimination:

33. $$\begin{cases} x + 2y - z = -2 \\ x + z = 0 \\ 2x - y - z = -3 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & -5 & 1 & 1 \end{bmatrix} \begin{matrix} \to R_1 \\ \to R_2 \\ \to R_3 \to (+5R_2 + R_3) \end{matrix} \Rightarrow$$

(R1, -R2/2, R3→)

zero

$$\begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -4 & -4 \end{bmatrix} \begin{matrix} \to R_1 \\ \to R_2 \\ \to -R_3/4 \end{matrix}$$

make it = 1

x y z

$$\begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

33. $$\begin{cases} x + 2y - z = -2 \\ x + z = 0 \\ 2x - y - z = -3 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

x y z — zero — pivot

$R_1 \to (\pm 1 R_3 + R_1)$

$R_2 \to (+ R_3 + R_2)$

$R_3 \to R_3$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

zero — pivot

$R_1 \to (-2R_1 + R_2$

$R_2 \to$

$R_3 \to$

⇓ upper matrix

R. Echelon form

→ Proceed to R.R.E. form

33.
$$\begin{cases} x + 2y - z = -2 \\ x + z = 0 \\ 2x - y - z = -3 \end{cases}$$

Zero

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right] \rightarrow R_1 \rightarrow (-2R_2 + R_1)$$
$$\rightarrow R_2$$
$$\rightarrow R_3$$

$$\Rightarrow \left[\begin{array}{ccc|c} x & y & z & \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Pivot

R R E. F

Gauss_Jordan_Formula

$x = -1$

$y = 0$ & $z = 1$

Check our solution

$x = -1$
$y = 0$
$z = 1$

Back substitute

33. $\begin{cases} x + 2y - z = -2 \\ x + z = 0 \\ 2x - y - z = -3 \end{cases}$

Back to (I)

$(-1) + 2(0) - (1) = -2$
R.H.S

Substitute in the 2nd Equation: $(-1) + 0 + (1) = 0$
R.H.S

* Substitute in the 3rd Equation: $(2)(-1) - (0) - (1)$
$= -2 - 1 = -3$
R.H.S

Stewart- College Algebra 2nd Example

Solve the following system of linear equations, using Gauss-Jordan elimination:

38.
$$\begin{cases} 10x + 10y - 20z = 60 \\ 15x + 20y + 30z = -25 \\ -5x + 30y - 10z = 45 \end{cases}$$

Solution

Write matrix in the augmented Form

make First leading =1

$$\begin{bmatrix} 10 & 10 & -20 & 60 \\ 15 & 20 & 30 & -25 \\ -5 & 30 & -10 & 45 \end{bmatrix} \rightarrow R_1 \left[R_1/10\right] \rightarrow \begin{bmatrix} 1 & 1 & -2 & 6 \\ 15 & 20 & 30 & -25 \\ -5 & 30 & -10 & 45 \end{bmatrix}$$

$\rightarrow R_2$

$\rightarrow R_3$

Zeros

$R_1 \rightarrow R_1$

$R_2 \rightarrow [-15R_1 + R_2]$

$R_3 \rightarrow [5R_1 + R_3]$

$$\begin{bmatrix} 1 & 1 & -2 & 6 \\ 0 & 5 & 60 & -115 \\ 0 & 35 & -20 & 75 \end{bmatrix}$$

1

38.

$$\begin{cases} 10x + 10y - 20z = 60 \\ 15x + 20y + 30z = -25 \\ -5x + 30y - 10z = 45 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & -2 & 6 \\ 0 & 5 & 60 & -115 \\ 0 & 35 & -20 & 75 \end{bmatrix} \begin{matrix} \rightarrow R_1 \\ \rightarrow R_2/5 \\ \rightarrow R_3 \end{matrix}$$

R_3 / 5

2nd leading make it = 0

$$\begin{bmatrix} 1 & 1 & -2 & 6 \\ 0 & 1 & 12 & -23 \\ 0 & 7 & -4 & 15 \end{bmatrix} \begin{matrix} \rightarrow R_1 \\ \rightarrow R_2 \\ \rightarrow -7R_2 + R_3 \end{matrix}$$

Zero

$$\begin{bmatrix} 1 & 1 & -2 & 6 \\ 0 & 1 & 12 & -23 \\ 0 & 0 & -88 & 176 \end{bmatrix} \rightarrow \frac{R_3}{-88}$$

$$\begin{bmatrix} 1 & 1 & -2 & 6 \\ 0 & 1 & 12 & -23 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

38.

$$\begin{cases} 10x + 10y - 20z = 60 \\ 15x + 20y + 30z = -25 \\ -5x + 30y - 10z = 45 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & -2 & 6 \\ 0 & 1 & 12 & -23 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

0 → zero

1 → pivot

$R_1 \to R_1 + (2R_3 + R_1)$

$R_2 \to (-12R_3 + R_2)$

$R_3 \to R_3$

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

x y z

R.E. Form → RREF

Upper matrix

Gauss-Jordan

38.

$$\begin{cases} 10x + 10y - 20z = 60 \\ 15x + 20y + 30z = -25 \\ -5x + 30y - 10z = 45 \end{cases}$$

Zero

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \rightarrow R_1 \Rightarrow \begin{bmatrix} -R_2 + R_1 \\ R_2 \\ R_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$x \quad y \quad z$

$$x = 1$$

$$y = 1$$

$$z = -2$$

Back Substitute ⇒ by $x = 1$, $y = 1$ & $z = -2$

38.
$$\begin{cases} 10x + 10y - 20z = 60 \\ 15x + 20y + 30z = -25 \\ -5x + 30y - 10z = 45 \end{cases}$$

$10(1) + 10(1) - 20(-2) = +60$

$15(1) + 20(1) + 30(-2) = -25 \rightarrow$

$-5(1) + 30(1) - 10(-2) = 45$ ok