Modified Newton Raphson Method

A simple modification to the standard Newton method for approximating the root of a *Multiple* root function is described and analyzed. For the same number of function and derivative evaluations, the modified method converges faster.

$$x_{i+1} = x_i - \frac{f(x_i)}{f'[x_i + a(x_i) + f(x_i)]}$$

where $$a(x_i) = -\frac{1}{2f'(x_i)}$$

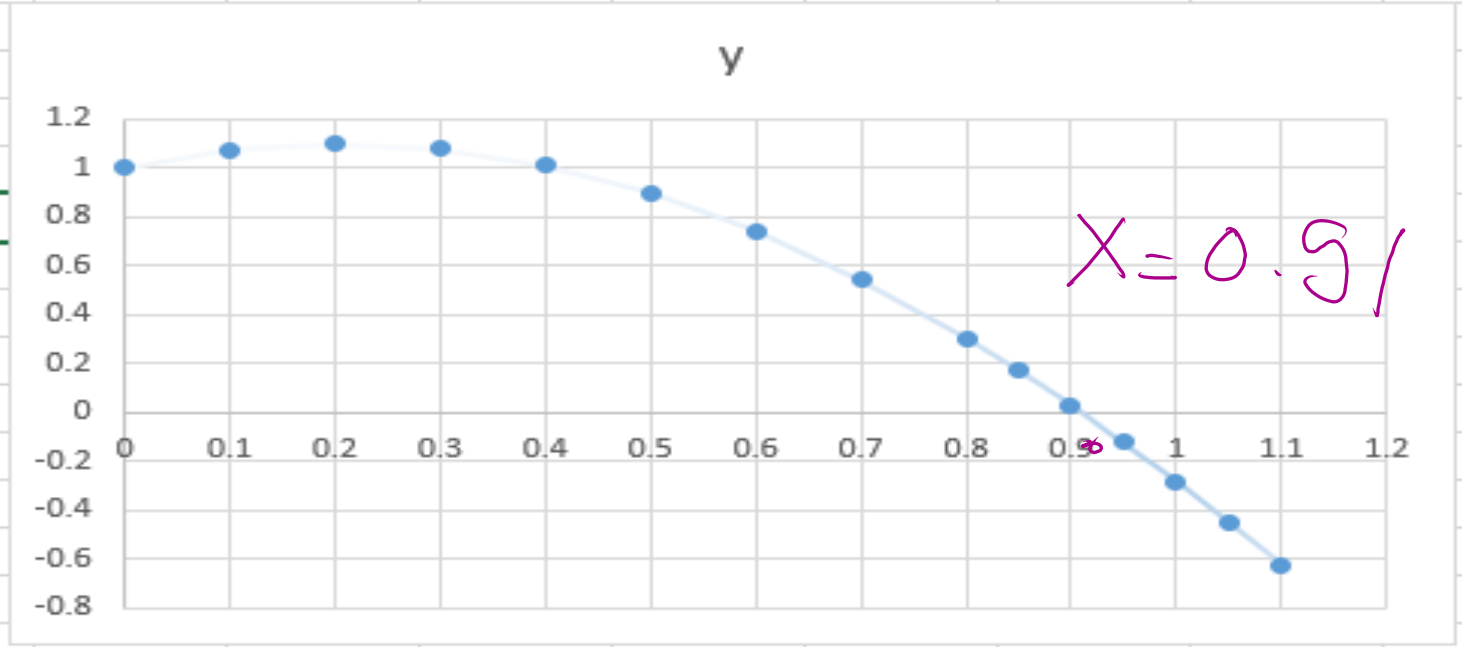
$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

Example # 7 let f(x) = e^x -3x^2 to an accuracy of 3 digits , root is between 0.50 &1.

Solution : from analytical Solution

F(x) = e^x -3x^2

x	y
0	1
0.1	1.075171
0.2	1.101403
0.3	1.079859
0.4	1.011825
0.5	0.898721
0.6	0.742119
0.7	0.543753
0.8	0.305541
0.85	0.172147
0.9	0.029603
0.95	-0.12179
1	-0.28172
1.05	-0.44985
1.1	-0.62583



Example # 7 let f(x) = e^x -3x^2 to an accuracy of 3 digits , root is between 0.50 &1 .

Solution

f(x) = e^x -3x^2

f'(x)= e^x -6 x.

f''(x)= e^x -6

$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

Use $x_0 = 0.50$

$f(0.5) = e^{0.5} - 3(0.50)^2 = 0.8987$

$f'(0.5)$ $e^{0.50} - 6(0.50) = -1.3513$

$f''(0.5) = e^{0.5} - 6 = -4.351$

$X_1 = 0.50 - \frac{(0.8987)(-1.3513)}{(-1.3513)^2 - (0.8987)(-4.351)} = 0.7117$

Example # 7 let f(x) = $e^x - 3x^2$ to an accuracy of 3 digits , root is between 0.50 &1 .

Solution

f(x) = $e^x - 3x^2$

f'(x)= $e^x - 6x$.

f''(x)= $e^x - 6$

$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

Use $x_0 = 0.50$

Use $x_1 = 0.712$ $f(0.712) = e^{0.712} - 3(0.712)^2$

$= 0.517$

$f'(0.712) = e^{0.712} - 6(0.712) = -2.334$ → Rounded

$f'(0.712)^2 = (-2.334)^2 = 5.445$

$f''(0.712) = e^{0.712} - 6 = -3.962$

$x_2 = 0.876$

Example # 7 let f(x) = e^x -3x^2 to an accuracy of 3 digits , root is between 0.50 &1.

$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

Table for detailed illustration

f'(x) f''(x) n d n/d xi

Modified Newton -raphson method

	xi	f(x)	f'(x)	f''(x)	Numerator	Denomir	n/d	Xi
x0	0.5	0.899	-1.351	-4.351	-1.215	5.737	-0.212	0.712
x1	0.712	0.517	-2.234	-3.962	-1.155	7.039	-0.164	0.876
x2	0.876	0.099	-2.855	-3.599	-0.283	8.506	-0.033	0.909
x3	0.909	0.003	-2.972	-3.518	-0.009	8.844	-0.001	0.910
x4	0.91	0	-2.976	-3.516	0	8.855	0	0.910
x5	0.91	0	-2.976	-3.516	0	8.855	0	0.910
x6	0.91	0	-2.976	-3.516	0	8.855	0	0.910
x7	0.91	2.3E-05	-2.976	-3.516	-6.705E-05	8.8547	-8E-06	0.910

Prepared by Eng.Maged Kamel.

$x_0 = 0.50 \rightarrow x_4 = 0.910$ Rounded

Example #7

Modified Newton -raphson method

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