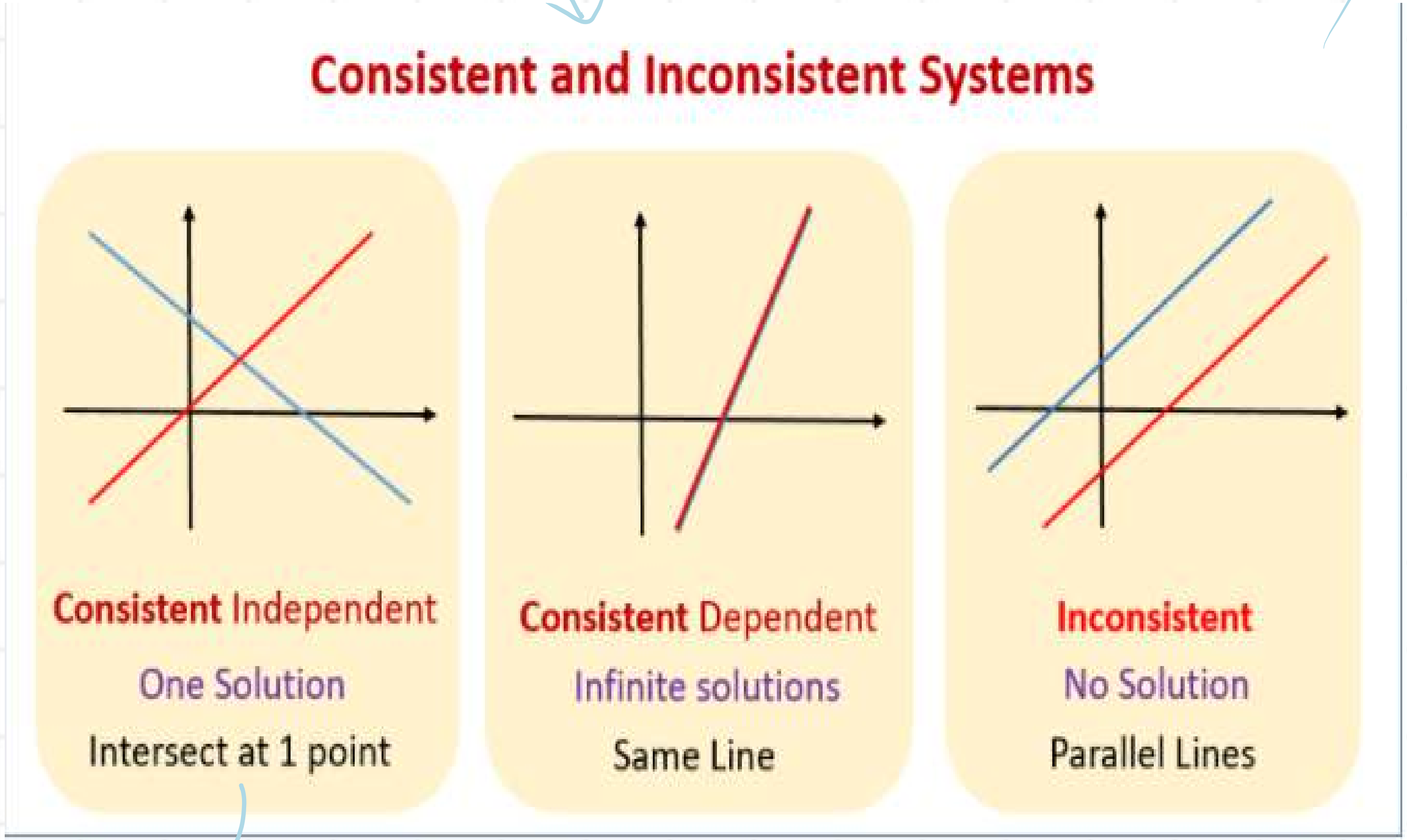
https://study.com/skill/learn

Consistent Dependent: A system of linear equations is consistent dependent if it has an infinite number of solutions. When this is the case, the graphs of the lines in the system are the same, meaning the equations in the system represent the same line.



Inconsistent: A system of linear equations is inconsistent if it has no solutions. When this is the case, the graphs of the lines in the system do not intersect, meaning they are parallel.

Consistent Independent: A system of linear equations is consistent independent when it has exactly one solution. When this is the case, the graphs of the lines in the system cross at exactly one point.

https://www.khanacademy.org/math/algebra-home/alg-matrices/alg-elementary-matrix-row-operations/a/matrix-row-operations

Matrix row operations

The following table summarizes the three elementary **matrix row operations**.

Matrix row operation	Example
Switch any two rows	$\begin{bmatrix} 2 & 5 & 3 \\ 3 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 4 & 6 \\ 2 & 5 & 3 \end{bmatrix}$ (Interchange row 1 and row 2.)
Multiply a row by a nonzero constant	$\begin{bmatrix} 2 & 5 & 3 \\ 3 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 3 \cdot 2 & 3 \cdot 5 & 3 \cdot 3 \\ 3 & 4 & 6 \end{bmatrix}$ (Row 1 becomes 3 times itself.)
Add one row to another	$\begin{bmatrix} 2 & 5 & 3 \\ 3 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 5 & 3 \\ 3+2 & 4+5 & 6+3 \end{bmatrix}$ (Row 2 becomes the sum of rows 2 and 1.)

OBJECTIVE 3 Use row operations to solve a system with two equations.

Row operations can be used to rewrite a matrix until it is the matrix of a system whose solution is easy to find. The goal is a matrix in the form

Diagonal =1

$$\left[\begin{array}{cc|c} 1 & a & b \\ 0 & 1 & c \end{array}\right] \quad \text{or} \quad \left[\begin{array}{ccc|c} 1 & a & b & c \\ 0 & 1 & d & e \\ 0 & 0 & 1 & f \end{array}\right]$$

upper matrix with 1 as diagonal

for systems with two and three equations, respectively. Notice that there are 1's down the diagonal from upper left to lower right and 0's below the 1's. A matrix written this way is in **row echelon form.**

row echelon For in these Examples are upper matrices

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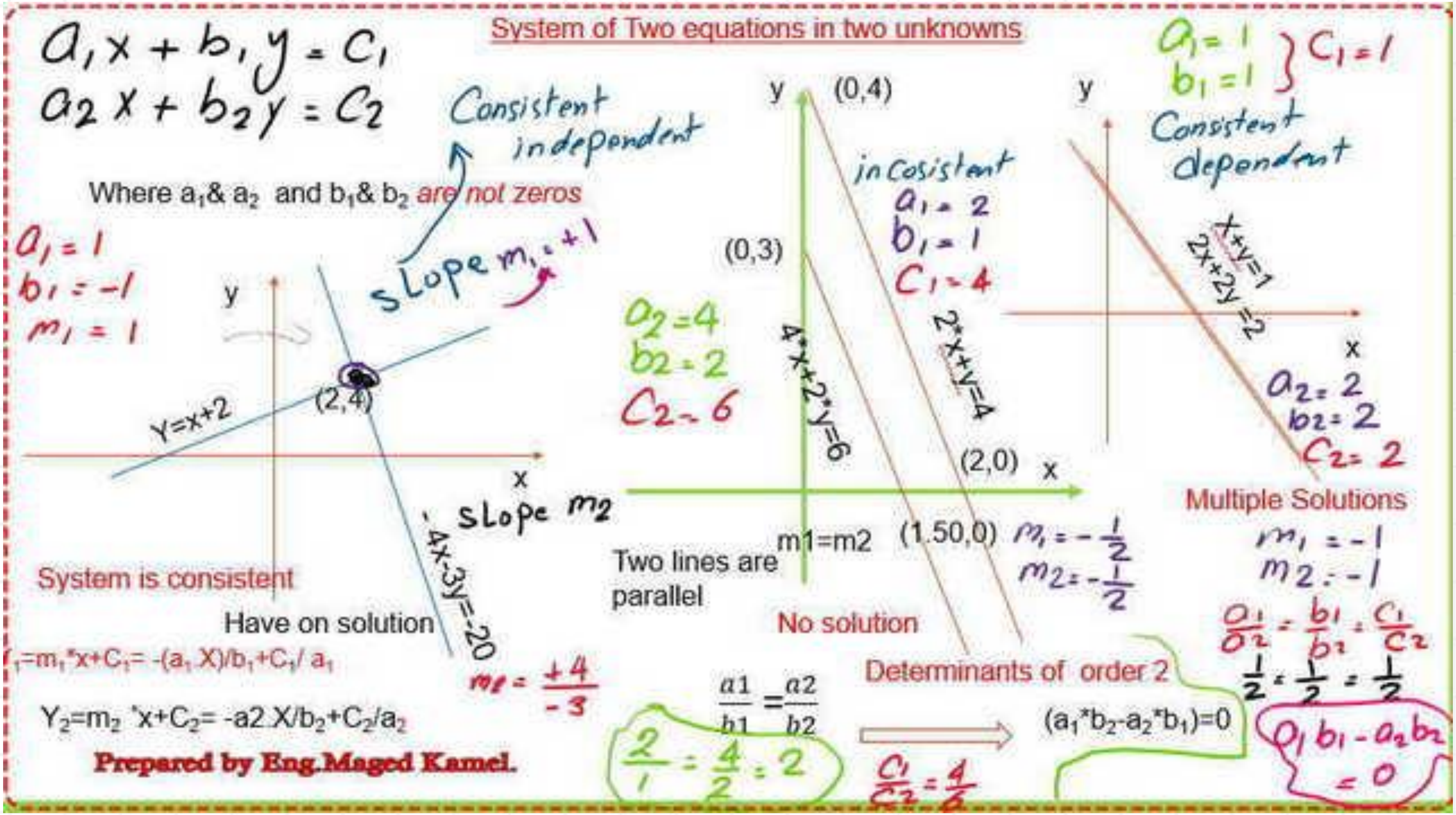
Gaussian Elimination Method

The Gaussian elimination method is an algorithm[1] that uses elementary row operations to solve a system of linear equations. The goal of this method is to rewrite an augmented matrix in row echelon form.


System of Two equations in two unknowns
$a_1x+b_1y=c_1$
$a_2x+b_2y=c_2$
Where a_1& a_2 and b_1& b_2 are not zeros
Consistent independent
slope $m_1=+1$
$a_1=1$
$b_1=-1$
$m_1=1$
y
x
Y=x+2
(2,4)
slope m_2
-4x-3y=-20
$m_2=\frac{+4}{-3}$
System is consistent
Have on solution
$Y_1=m_1*x+C_1=-(a_1X)/b_1+C_1/a_1$
$Y_2=m_2*x+C_2=-a2X/b_2+C_2/a_2$
Prepared by Eng.Maged Kamel.
$a_2=4$
$b_2=2$
$C_2=6$
(0,4)
(0,3)
inconsistent
$a_1=2$
$b_1=1$
$C_1=4$
4*x+2*y=6
2*x+y=4
(2,0)
(1.50,0)
m1=m2
Two lines are parallel
$m_1=-\frac{1}{2}$
$m_2=-\frac{1}{2}$
No solution
$\frac{a1}{b1}=\frac{a2}{b2}$
$\frac{2}{1}=\frac{4}{2}=2$
$\frac{C_1}{C_2}=\frac{4}{6}$
Deteminants of order 2
$(a_1*b_2-a_2*b_1)=0$
$a_1=1$
$b_1=1$
$C_1=1$
Consistent dependent
X+y=1
2x+2y=2
$a_2=2$
$b_2=2$
$C_2=2$
Multiple Solutions
$m_1=-1$
$m_2=-1$
$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$
$\frac{1}{2}=\frac{1}{2}=\frac{1}{2}$
$a_1b_1-a_2b_2=0$

Example solve for

$$-x + y = 2 \quad \text{I}$$
$$-4x - 3y = -20 \quad \text{II}$$

Case of Consistent independent

Write
$$\begin{array}{cc} x & y \end{array}$$
$$\left[\begin{array}{cc|c} -1 & 1 & 2 \\ -4 & -3 & -20 \end{array}\right]$$

1. Step # 1 Write in the Augmented Form

2. Make sure that $a_{11} = 1$ → first row, first column

$$\left[\begin{array}{cc|c} -1 & 1 & 2 \\ -4 & -3 & -20 \end{array}\right] \begin{array}{l} -R_1 \\ \longrightarrow R_2 \end{array} \left[\begin{array}{cc|c} 1 & -1 & -2 \\ -4 & -3 & -20 \end{array}\right]$$

a_{11} is a pivot

3. For first Column / 2nd row is to be 0.

Unchanged.

$$\left[\begin{array}{cc|c} 1 & -1 & -2 \\ -4 & -3 & -20 \end{array}\right] \xrightarrow{-(-4)R_1 + R_2} \left[\begin{array}{cc|c} 1 & -1 & -2 \\ 0 & -7 & -28 \end{array}\right]$$

4. Use Row operation, Multiply R_1 by 4 and to R_2 $\longrightarrow R_2$

Example solve for

$$-x + y = 2 \quad \text{I}$$
$$-4x - 3y = -20 \quad \text{II}$$

Case of two intersecting Lines at a point

Write
$$\begin{array}{cc|c} x & y & \\ \end{array}$$
$$\left[\begin{array}{cc|c} -1 & 1 & 2 \\ -4 & -3 & -20 \end{array}\right]$$

5
$$\left[\begin{array}{cc|c} 1 & -1 & -2 \\ 0 & -7 & -28 \end{array}\right] \begin{array}{l} \rightarrow R_1 \\ \rightarrow -\dfrac{R_2}{7} \rightarrow R_2 \end{array} \quad \begin{array}{cc|c} x & y & \\ \end{array} \left[\begin{array}{cc|c} 1 & -1 & -2 \\ 0 & 7 & 28 \end{array}\right]$$

to be $=1$

We have a new system of Equations

6 $\quad x - y = -2$

$0x + 7y = 28 \rightarrow y_{value} = \dfrac{28}{7} = 4$

Back subsititute

7 $\quad x - (4) = -2 \rightarrow x = 4 - 2 = 2$

Example solve for

$$x + y = 1$$
$$2x + 2y = 2$$

I Multi Solution

II Consistant dependent

write
$$\begin{array}{cc} x & y \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & 2 & 2 \end{array}\right]$$

1. step # 1 write in the augmented form
2. Make sure that $a_{11} = 1$ → first row, first column

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & 2 & 2 \end{array}\right]$$ OK

3. For first column/2nd row is to be 0.

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & 2 & 2 \end{array}\right] \begin{array}{l} \to R_1 \\ \to \underbrace{-\frac{2}{1}R_1 + R_2}_{R_2} \end{array} \Rightarrow \begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right] \end{array}$$

$$\boxed{\begin{array}{l} x + y = 1 \\ 0 + 0 = 0 \end{array}}$$

$0 = 0$

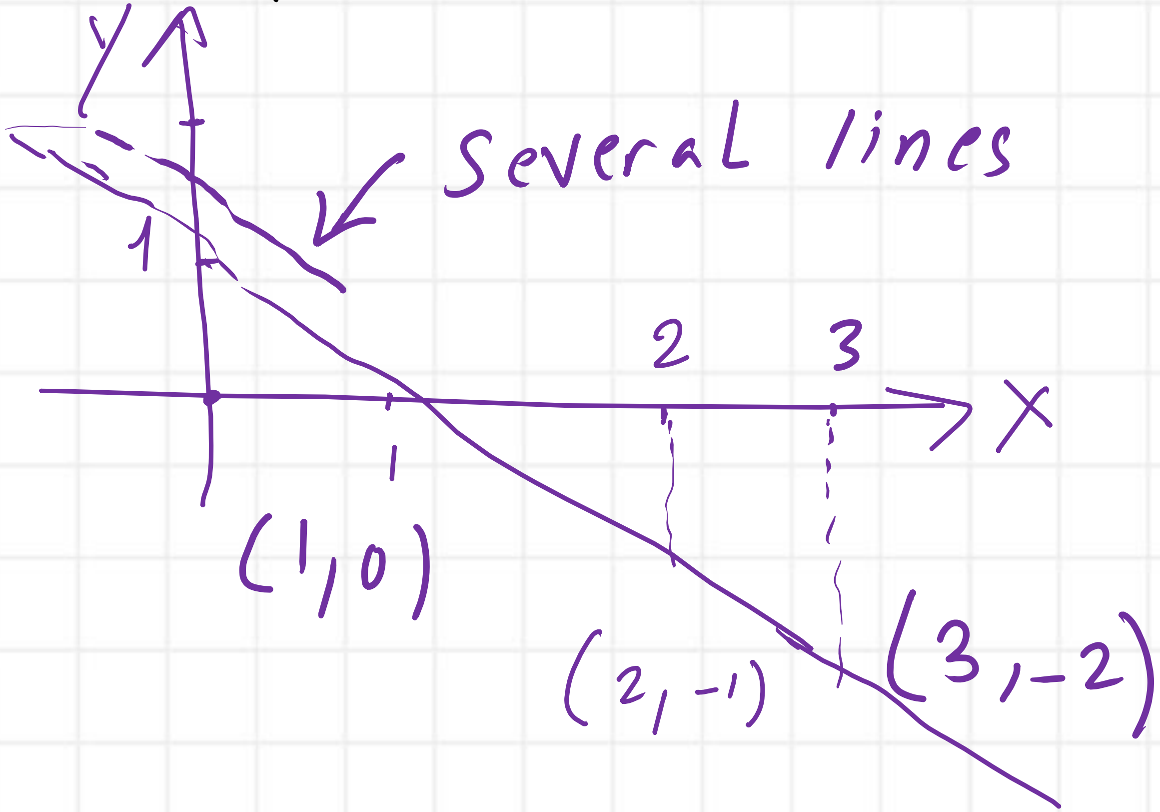
For different y values → many x

LS RS Hint $\frac{R_2}{R_1} = 2$ $\frac{\text{II}}{\text{I}} = 2$

$$\left.\begin{aligned} \text{I} \quad & x + y = 1 \\ \text{II} \quad & 2x + 2y = 2 \end{aligned}\right\} \Longrightarrow \begin{aligned} x + y &= 1 \\ 0 &= 0 \end{aligned}$$

$x = 0$, $y = 1$; $x = 1$, $y = 0$; $x = 2$, $y = -1$

$x = 3$, $y = -2$



Example Solve For

$$2x + y = 4 \quad \text{I}$$
$$4x + 2y = 6 \quad \text{II}$$

Write
$$\begin{array}{cc} x & y \end{array}$$
$$\left[\begin{array}{cc|c} 2 & 1 & 4 \\ 4 & 2 & 6 \end{array}\right]$$

In Consistent → Two parallel Lines

① Step # 1 Write in the Augmented Form

② Make sure that $a_{11} = 1$ → first row, first Column

$$\left[\begin{array}{cc|c} 2 & 1 & 4 \\ 4 & 2 & 6 \end{array}\right] \rightarrow R_1/2 \Rightarrow \left[\begin{array}{cc|c} 1 & \frac{1}{2} & 2 \\ 4 & 2 & 6 \end{array}\right] \begin{array}{l} \rightarrow R_1 \\ \rightarrow \dfrac{-4}{1} R_1 + R_2 \\ \qquad \overline{\quad R_2 \quad} \end{array}$$

③ make $a_{21} = 0$

④ $$\left[\begin{array}{cc|c} 1 & \frac{1}{2} & 0 \\ 0 & 0 & -2 \end{array}\right]$$

$0 \neq -2$

Contradication Or inConsistent Solution

$$2x + y = 4 \quad \text{I}$$

$$4x + 2y = 6 \quad \text{II}$$

$x = 0 \; \} \; y = 0$

$y = 4 \; \} \; x = 2$

Graph

I $\quad y = -2x + 4$

$y = mx + c$

$m = -2, \; c = 4$

$x = 1$

$y = 1$

$x = 3/2$

$y = 0$

II $\quad 2y = -4x + 6$

$y = -2x + 3$

$m = -2$

$c = 3$

Two Parallel Lines

$m_1 = m_2$