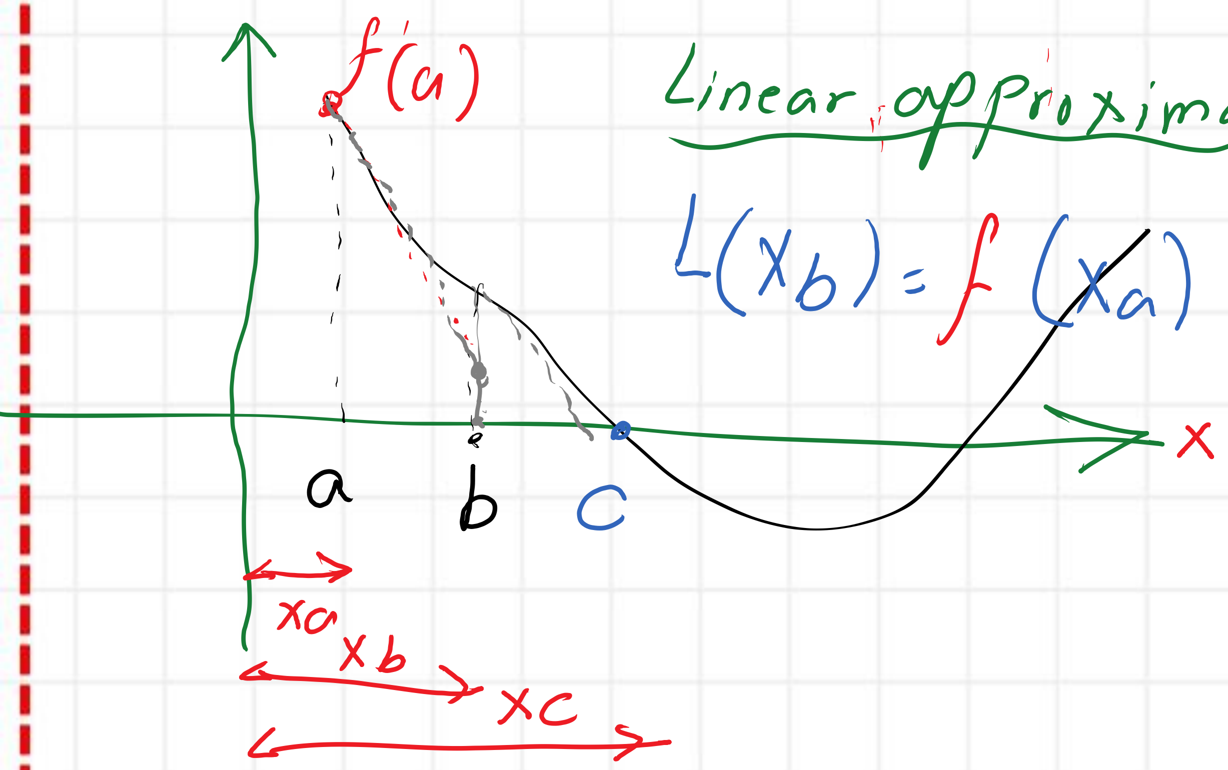
Introdution to Newton-Raphson method



Linear approximation $f(x_b) \approx L(x_b)$

$$L(x_b) = f(x_a) + f'(a)(x_b - x_a)$$

We can get an expression for x_b

$$x_b - x_a = \frac{1}{f'(a)}\left[(f(x_b) - f(x_a))\right.$$

$$x_b = x_a + \frac{1}{f'(a)}(f(x_b - f(x_a))$$

$$x_{new} = x_{intial} + \frac{1}{f'(a)}[f(x_{new}) - f(x_{intial})]$$

When $f(x_{new}) = 0$ at x_c

$$x_{new} = x_{intial} + 0 - \frac{1}{f'(a)} f(x_{new})$$

→ Thus creating series of approximation will Lead to x_c

Prepared by Eng.Maged Kamel.

Newton- Raphson expression for Root finding

Getting use of the previous linear function expression

Step -1 choose x1 and get f(x1) value and f'(x1)-Then for y =0 which is y value for point x2 on the line - 0=f(x1)+f'(x1)(x2-x1) then X2=X1+f(x1)/f'(x1) point 2 obtained if close to real root point then f(x2) is close to zero.

Else repeat by putting x2 value in lieu of x1 in the previous expression and get X3 continue the process .

Prepared by Eng.Maged Kamel.

Newton- Raphson expression for Root finding

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

X_0 : initial point x value
X1: The first iteration x value after substitution.
f(x0) : y value at x=x0 & f'(x0) : First derivative value at x0

Example #4 :Use Newton method to find the roots of $\sqrt[2]{29}$

Solution : We will write the equation as modify to

$x = \sqrt[2]{29} \rightarrow x^2 = 29 \Rightarrow x^2 - 29 = 0$

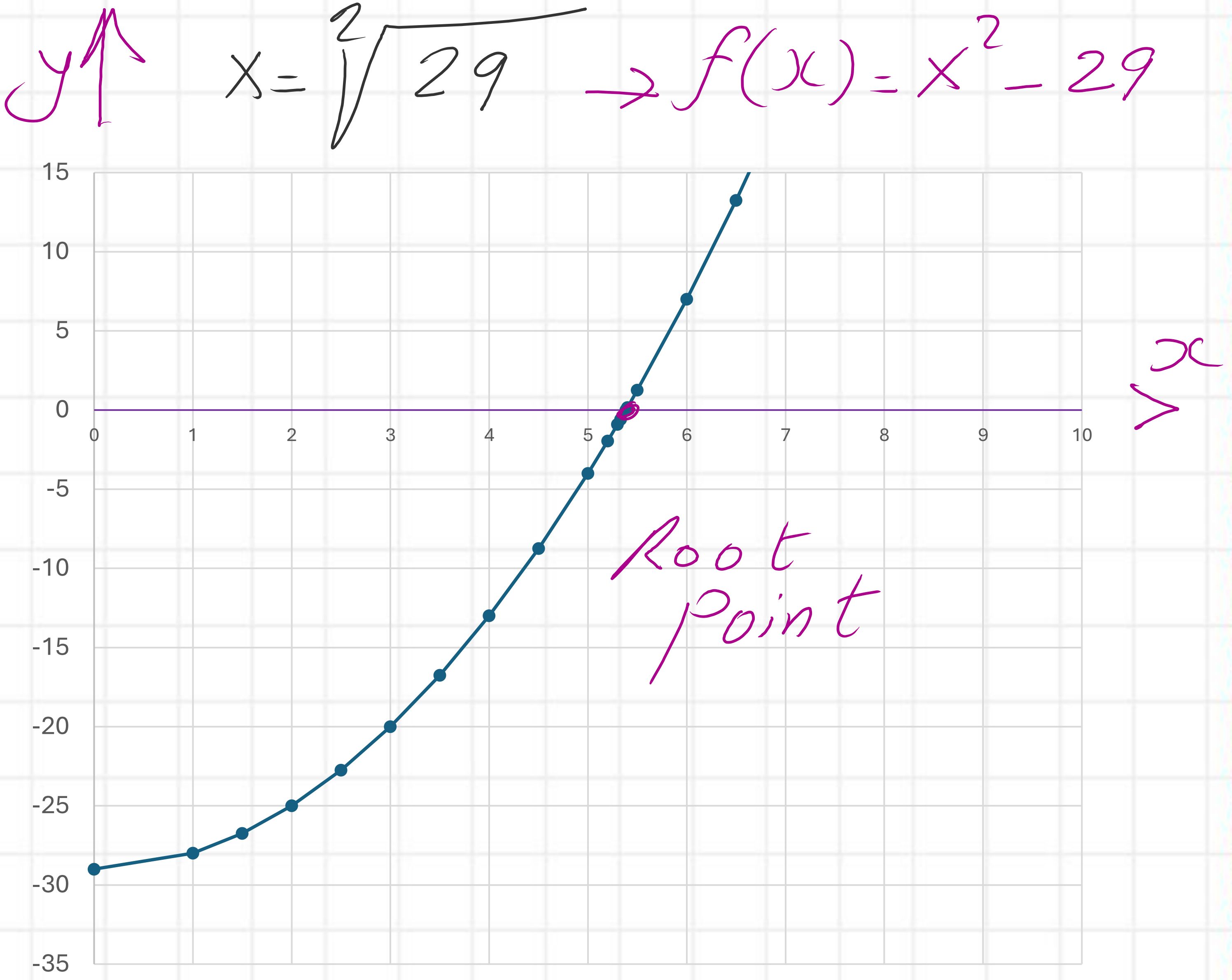
$f(x) = x^2 - 29 \Rightarrow f'(x) = 2x$

Based on analytic Solution $6 > x > 5$ x: root point

Prepared by Eng.Maged Kamel.

x	y
0	-29
1	-28
1.5	-26.75
2	-25
2.5	-22.75
3	-20
3.5	-16.75
4	-13
4.5	-8.75
5	-4
5.2	-1.96
5.3	-0.91
5.33	-0.591
5.3852	-5E-05
5.391	0.0629
5.4	0.16
5.5	1.25
6	7
6.5	13.25
7	20
7.5	27.25
8	35
8.5	43.25
9	52

$x = \sqrt[2]{29} \rightarrow f(x) = x^2 - 29$



Example 4

Example #4 :Use Newton method to find the roots of $\sqrt[2]{29}$

X_1 value

$$X_i = X_0 - \frac{f(x_0)}{f'(x_0)}$$

if we select $x_0 = 5$

$f(x) = x^2 - 29$

$f(5) = 5^2 - 29 = -4$

$f'(x) = 2x \rightarrow f'(x_0 = 5) = 10$

$X_1 = 5 - (-4)/(10) = 5 + 0.40 = 5.40$

For 2nd iteration Use $X_1 = 5.40$

To find x_2

$$X_2 = X_1 - \frac{f(x_1)}{f'(x_1)}$$

$f(5.40) = (5.4)^2 - 29 = 0.16$

$f'(5.40) = 2(5.40) = 10.80$

Prepared by Eng.Maged Kamel.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$x_1 = 5.40$

$f(5.4) = 0.16$

$f'(5.4) = 10.80$

$$x_2 = 5.40 - \frac{(0.16)}{(10.80)} = 5.3851$$

For x_3 $\rightarrow$ Use $x_2 = 5.3852$

$$x_3 = x_2 - f(x_2)/f'(x_2)$$

$$f(x_2 = 5.3851) = (5.3852)^2 - 29 = 3.7904(10^{-4})$$

$$f'(x_2 = 5.3851) = 2(5.3852) = 10.7704$$

$$x_3 = 5.3852 - \frac{(3.7904)(10^{-4})}{10.7704} = 5.3852$$

Prepared by Eng.Maged Kamel.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

$X_0=5$

f(x)=X^2-29

Newton -Raphson method

f'(x)= 2x

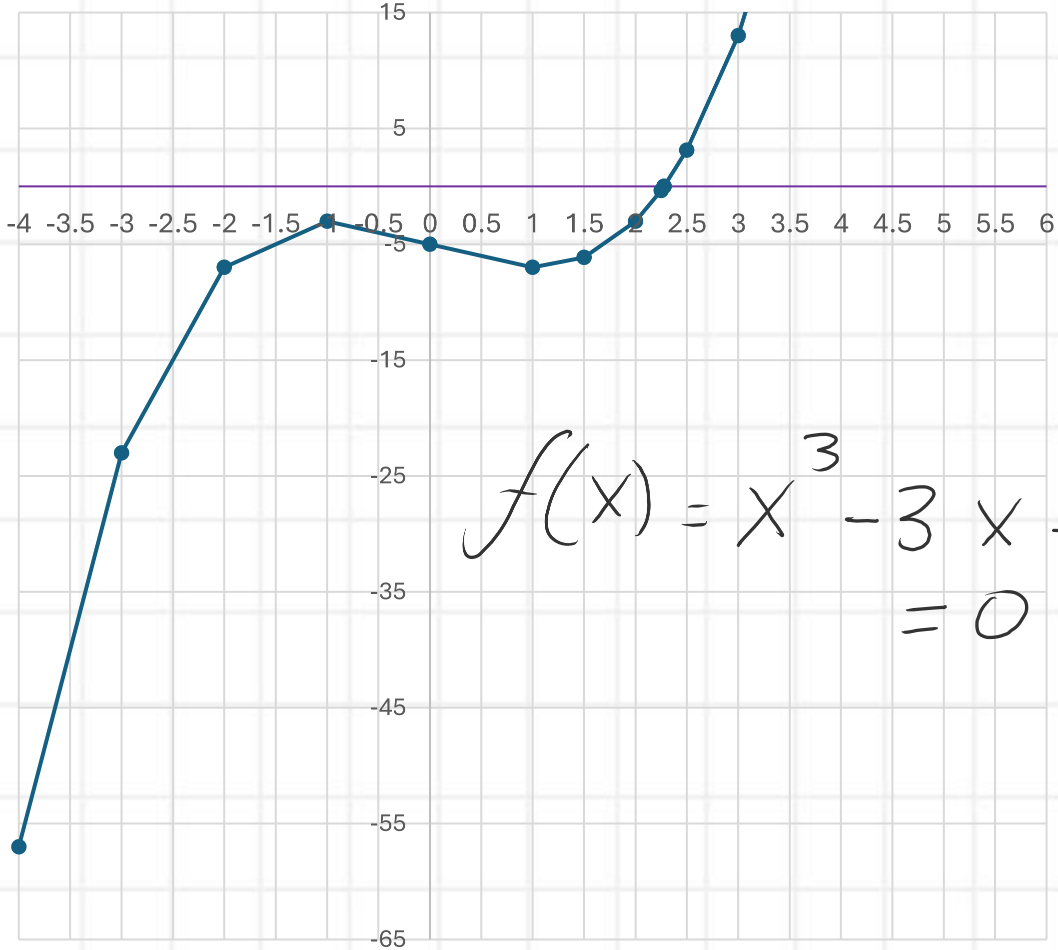
	x	f(x)	f'(x)	Numerator	Denomin	N/D	Xi
x0	5	-4	10	-4	10	-0.4	5.4
x1	5.4	0.16	10.8	0.16	10.8	0.0148	5.3852
x2	5.3852	0.00038	10.7704	0.00038	10.77	0.00000	5.3852
x3	5.3852	0.0004	10.7704	0.0004	10.77	0.00000	5.3852
x4	5.3852	0.0004	10.7704	0.0004	10.77	0.00000	5.3852

Example #5: Use Newton method for root extraction to find the roots of X^3-3x-5=0 starting with x0=3.00

Solution

By drawing using excel root between 2 &2.50

x	y
0	-5
1	-7
1.5	-6.125
2	-3
2.5	3.125
3	13
-0.5	-3.625
-1	-3
-2	-7
-3	-23
-4	-57



$f(x) = x^3 - 3x - 5$
$= 0$

Prepared by Eng.Maged Kamel.

Example #5: Use Newton method for root extraction to find the roots of X^3-3x-5=0 starting with x0=3.00

Solution

$$f(x) = X^3 - 3X - 5$$

$$X_0 = 3.00$$

$$f'(x) = 3X^2 - 3$$

$$f(x_0) = (3)^3 - 3(3) - 5 = 27 - 9 - 5 = 13$$

$x_0 = 3$

$$f'(x_0) = 3(3)^2 - 3 = 27 - 3 = 24$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{(13)}{24} = 2.45833$$

Example #5: Use Newton method for root extraction to find the roots of X^3-3x-5=0 starting with x0=3.00

$X_0=3$ $\quad f(x)=X^3-3x-5$

Newton -Raphson method $\quad f'(x)= 3x^2-3$

Newton -raphson

	xi	f(x)	f'(x)	Numerator	Denomir	N/D	Xi
x0	3	13	24	13	24	0.54166667	2.4583
x1	2.458333333	2.4817	15.1302083	2.481698	15.13	0.16402276	2.2943
x2	2.294310576	0.194	12.7915831	0.194	12.792	0.01516624	2.2791
x3	2.279144331	0.00158	12.5834966	0.00158	12.583	0.00012554	2.279
x4	2.279018795	1.1E-07	12.58178	1.08E-07	12.582	8.5641E-09	2.279

X_1

X_1

$x_1 = 2.4583$ $\quad f(X_1) = 2.4817$

for x_2 :

$f'(x_1) = 15.1302$

$x_2 = 2.4583 - \dfrac{(2.4817)}{15.1302}$

$= 2.2943$

Example #6-Use Newton method for root extraction to find the roots of

$\boldsymbol{x}.\boldsymbol{e}^{\boldsymbol{x}}$-2=0

Solution

Here no starting point was given , it is good to make guess for the root range

By substituting:

for x=0 f(0)=0* $.e^0$ -2= -2

for x=1 f(1) =1. $.e^1$ -2= 2.718281-2 =0.718281.

Our root between 0,1

intial point

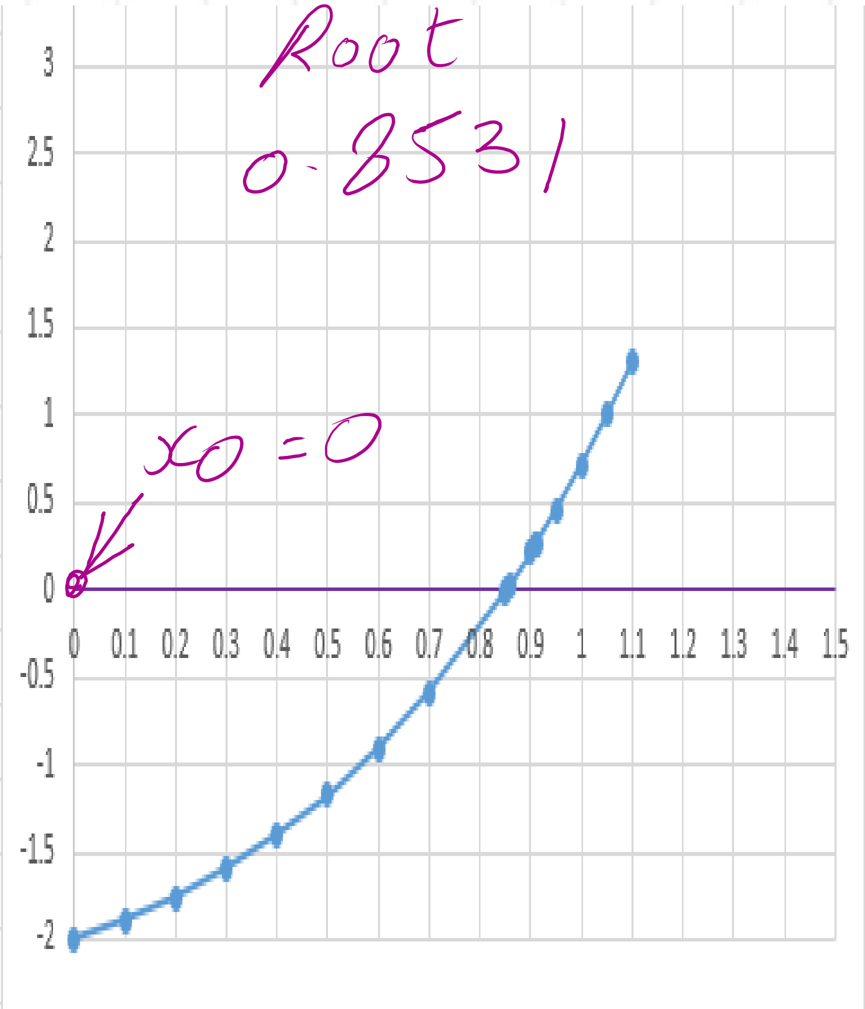
$X_0 = 0$

$f(x_0) = (0)(e^0) - 2 = -2$

$f'(x_0) = x e^x + e^x \Rightarrow$

$f'(0) = 0 + 1 = 1$

$x_1 = 0 - \frac{(-2)}{1} = +2$



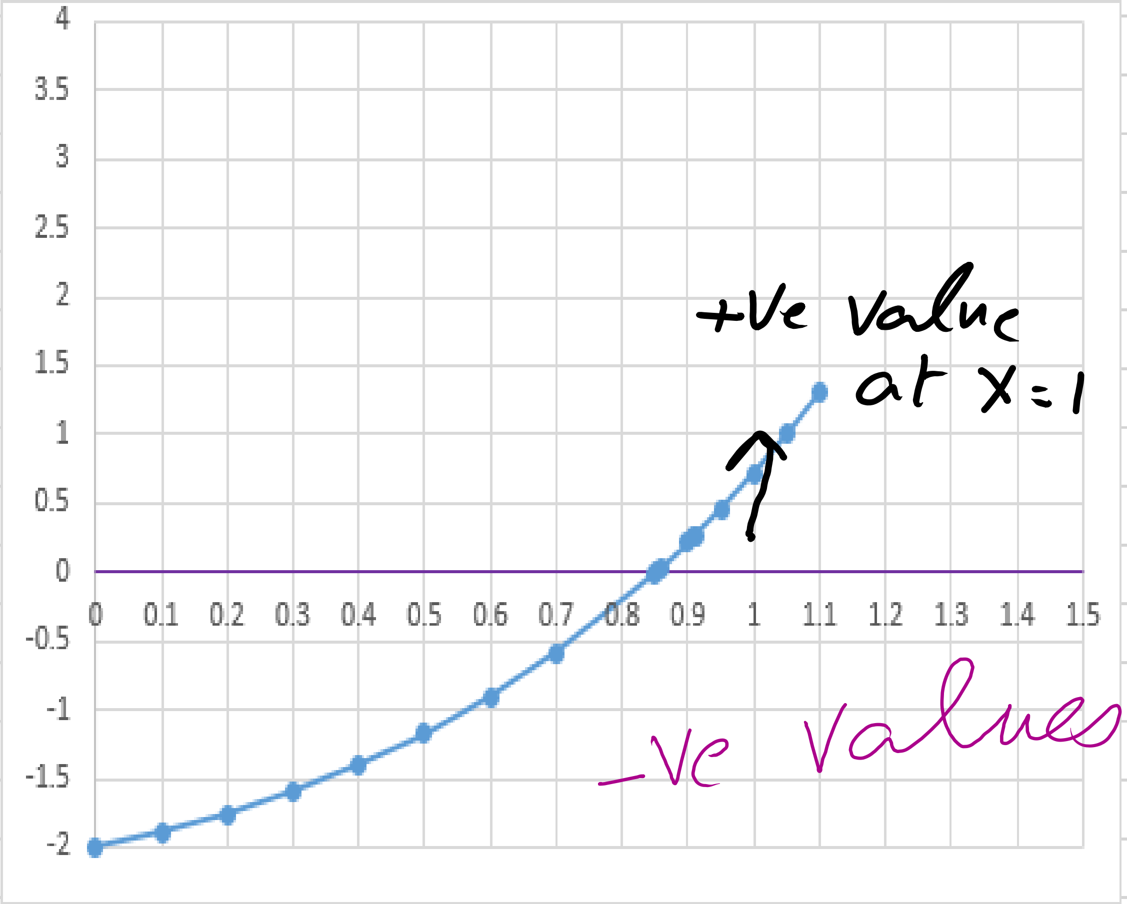
Prepared by Eng.Maged Kamel.

Find the function value for different x- values.

	xi	f(x)
x0	0	-2
x1	0.25	-1.679
x2	0.5	-1.1756
x3	0.75	-0.4122
x4	1	0.7183

F(x) =x e^x -2

root=0.8531



Prepared by Eng.Maged Kamel.

$x_1 = 2.00$ to get x_2 $\quad x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$f(2) = e^2(2) - 2 = 12.7781$

$f'(2) = e^2(1+2) = 22.167$

$f(x) = e^x . x - 2$

$f'(x) = e^x(1+x)$

$x_2 = 2.0 - \frac{12.7781}{22.167} = 1.4236$

Proceed with using an Excel sheet

Prepared by Eng.Maged Kamel.

Use $x_0 = 0$

Using the formula for Newton -raphson

	xi	f(x)	f'(x)	Numerator	Denomina	N/D	Xi
x0	0	-2	1	-2	1	-2	2
x1	2	12.7781	22.1672	12.7781	22.1672	0.576	1.4236
x2	1.4236	3.9108	10.0629	3.9108	10.0629	0.389	1.035
x3	1.035	0.9136	5.7287	0.9136	5.7287	0.159	0.8755
x4	0.8755	0.1013	4.5013	0.1013	4.5013	0.023	0.853
x5	0.853	0.0017	4.3484	0.0017	4.3484	4E-04	0.8526
x6	0.8526	-2E-05	4.3457	-2.39E-05	4.3457	-0	0.8526

$x_6 = 0.8526$

Example #6-Use Newton method for root extraction to find the roots of

$\boldsymbol{x}.\boldsymbol{e}^{\boldsymbol{x}}$-2=0 Solution

if $x_0 = 1.00$

$f(x_0) = (1)(e^1) - 2 = 0.7183$

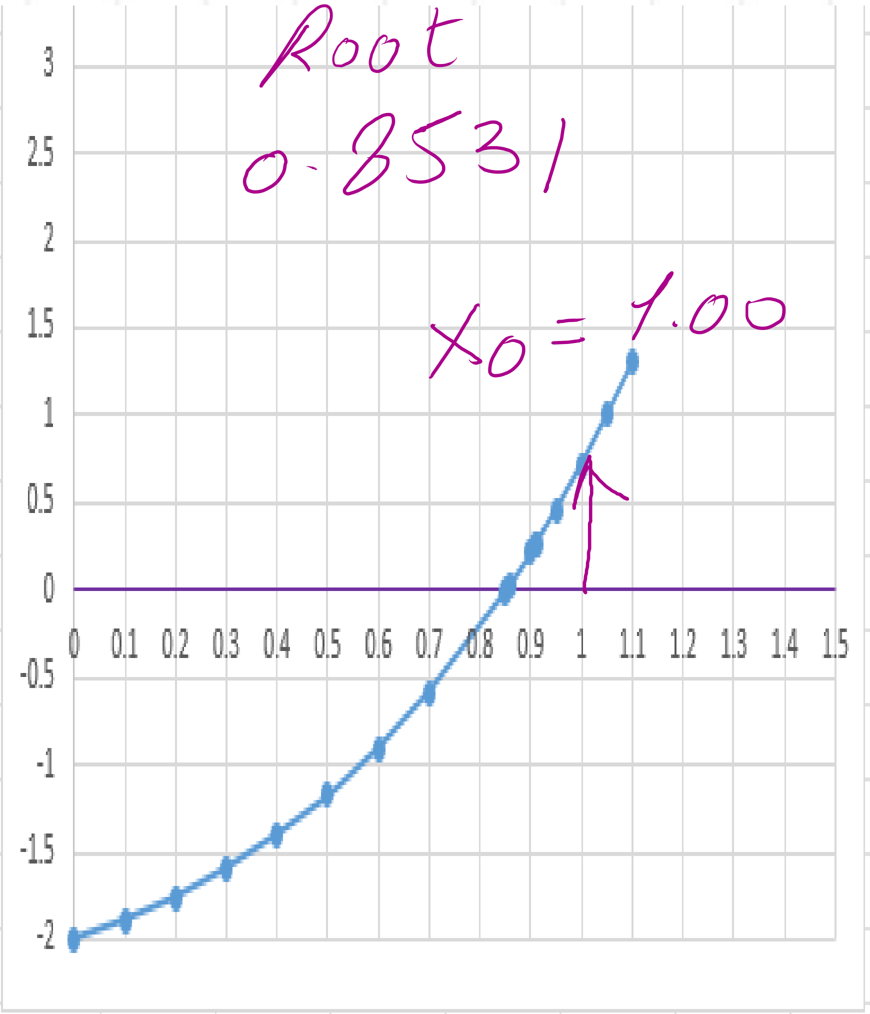
$f'(x_0) = (1)e^1 + e^1 =$

$x_0 = 1$ $= e^1(2) = 5.437$

$x_1 = 1.00 - \dfrac{(0.7183)}{5.437}$

$x_1 = 0.8679$

→ Proceed with Excel table



Prepared by Eng.Maged Kamel.

$x_0 = 1 \longrightarrow x_3 = 0.8526$

	Newton -raphson			X_0=1			
	xi	f(x)	f'(x)	Numerator	Denomina	N/D	Xi
x0	1	0.7183	5.4366	0.7183	5.4366	0.132	0.8679
x1	0.8679	0.0673	4.4492	0.0673	4.4492	0.015	0.8528
x2	0.8528	0.0008	4.3471	0.0008	4.3471	2E-04	0.8526
x3	0.8526	-2E-05	4.3457	-2.39E-05	4.3457	-0	0.8526
x4	0.8526	-2E-05	4.3457	-2.39E-05	4.3457	-0	0.852606