Objective of lecture.

Understand the following points ,Root finding.

- Fixed point iteration.
- linear approximation.
- Newton Method .
- Modified Newton Raphson Method .

Accompanied by Solved problems .

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Fixed point iteration , or one point iteration .

Normally, we write a function as Y=f(x).

But if we want to get an expression for x, what can we do?

We can put x on the left-hand side and re-adjust our equation to get an expression.

f(x)=0 to be rearranged to

x=g(x) give a value to x let us say a we call that x_1=a

Rewrite equation as $X_{i+1}=g(Xi)$

If I put i=1 we can write X_2=g(X1) then we put x1=a

The value of x2 can thus now be evaluated

Next step, put the x2 value just obtained, substitute, and get x3 accordingly.

Repeat the process until the result of x obtained in successive steps is the same.

Example # 1

Solve Numerically the following equation X^3+5x=20 , give answer to three decimal places , start with X_0=2.

Solution

Rearrange the equation to X^3=20-5x then $X=\sqrt[3]{20-5x}$ =g(x)

PLUG in by x_0 value → get $x_1 = g(x_0)$

X_0=2.00 $x_1=\sqrt[3]{20-5(2)}=\sqrt[3]{10}$ =2.154

X_1= 2.154 $x_2=\sqrt[3]{20-5(2.154)}=\sqrt[3]{20-10.77}$ =2.098

X_2= 2.098 $x_3=\sqrt[3]{20-5*(2.098)}=\sqrt[3]{20-10.49}$ =2.119

X_3= 2.119 $x_4=\sqrt[3]{20-5(2.119)}=\sqrt[3]{20-10.595}$

=2.111

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Fixed point iteration , or one point iteration

X_4= 2.111 then x5= = $\sqrt[3]{20-5*(2.111)}$ =
$\sqrt[3]{20-10.555}$ =2.114
X_5= 2.114 then x_6= = $\sqrt[3]{20-5(2.114)}$ =
$\sqrt[3]{20-10.57}$ =2.113

X_6= 2.113 then x_7 = $\sqrt[3]{20-5(2.113)}$ =
$\sqrt[3]{20-10.565}$ =2.113

X_7= 2.113 then x_8 = $\sqrt[3]{20-5(2.114)}$ =
$\sqrt[3]{20-10.565}$ =2.113

The solution is said to be convergent to 2.113 closer to 2.113

Check the equation=(2.113)^3+5*(2.113)

= ***Close to 20.00***

x0	2	10	2.154
x1	2.154	9.23	2.098
x2	2.098	9.51	2.119
x3	2.119	9.405	2.111
x4	2.111	9.445	2.114
x5	2.114	9.43	2.113
x6	2.113	9.435	2.113
x7	2.113	9.435	2.113

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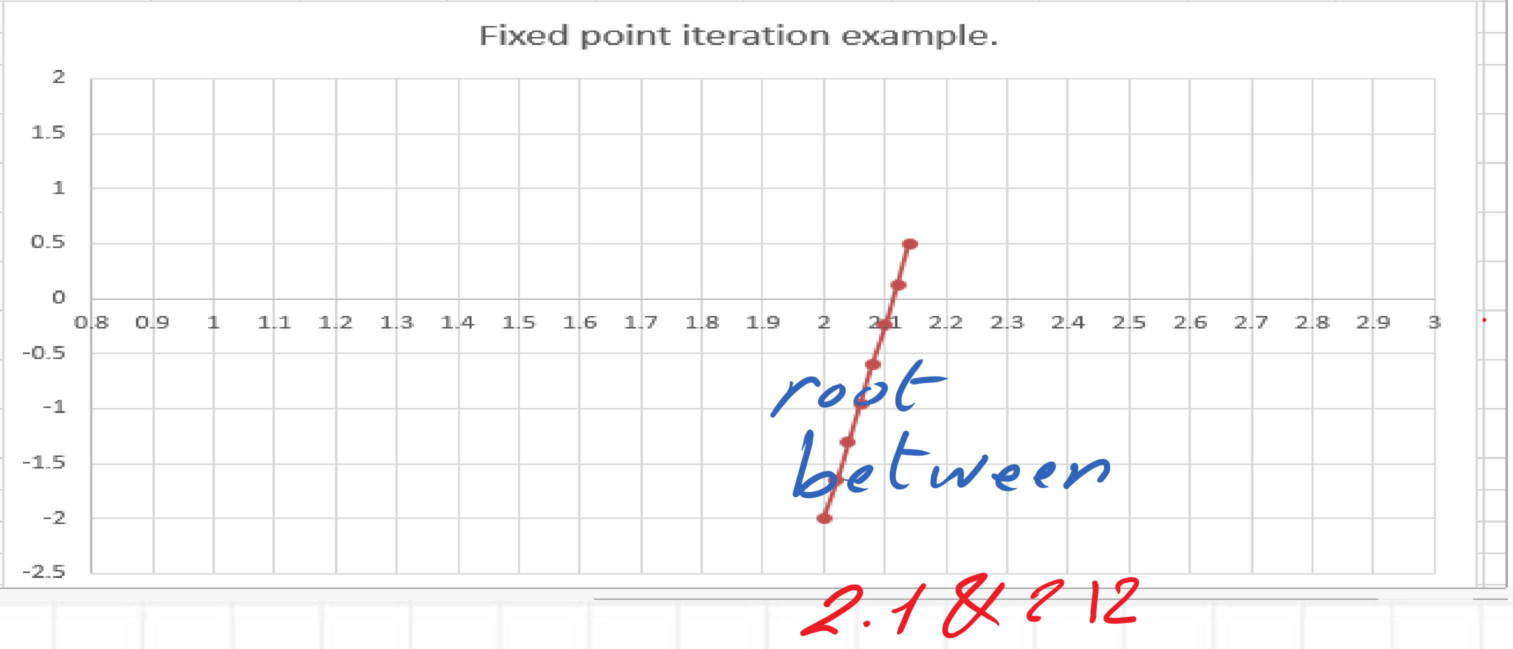
$X^3+5x=20$

X_0	2	-2
X_1	2.02	-1.65759
X_2	2.04	-1.31034
X_3	2.06	-0.95818
X_4	2.08	-0.60109
X_5	2.1	-0.239
X_6	2.12	0.128128
X_7	2.14	0.500344



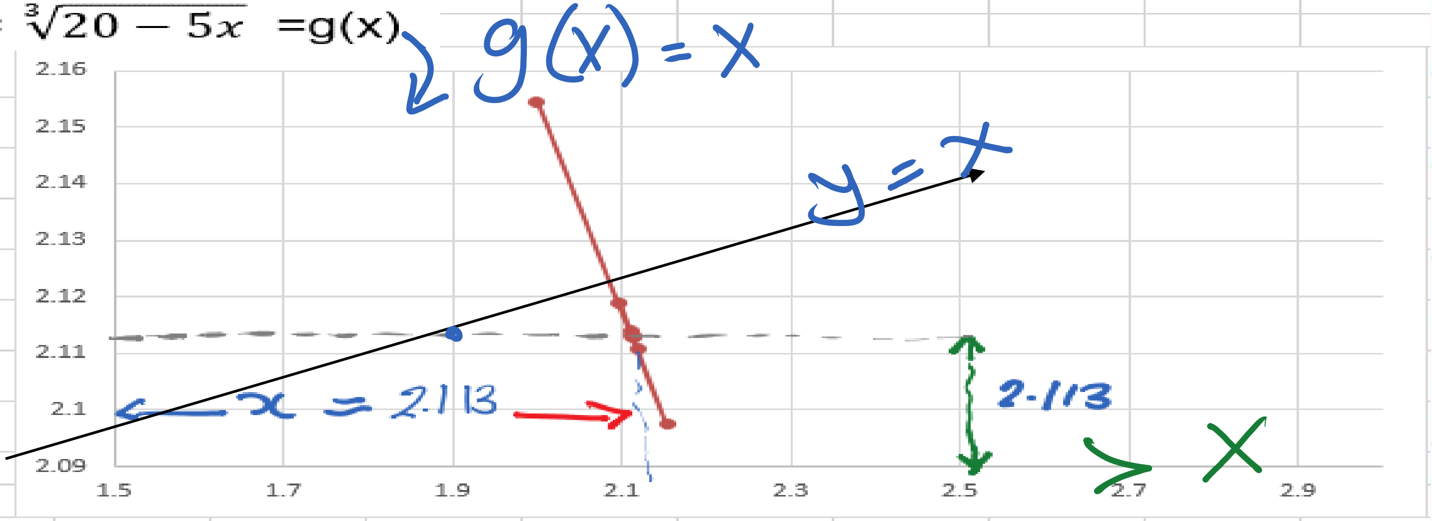
Fixed point iteration example. g(x)

X_0	2	2.154435 x_0
X_1	2.154435	2.09749
X_2	2.09749	2.118844
X_3	2.118844	2.110887
X_4	2.110887	2.113859
X_5	2.113859	2.11275
X_6	2.11275	2.113164
X_7	2.113164	2.113009

g(x)

$\to g(x_6)$

$X = \sqrt[3]{20-5x} = g(x)$



Example # 2

Find the root of $x^4 - x - 10 = 0$

$x_0 = 4$ Consider $x_0 = 4$

Solution Put $x^4 = 10 + x$ re-arrang terms

$x = (10 + x)^{1/4}$

Let $(x) = g(x)$ start with $x_0 = 4$

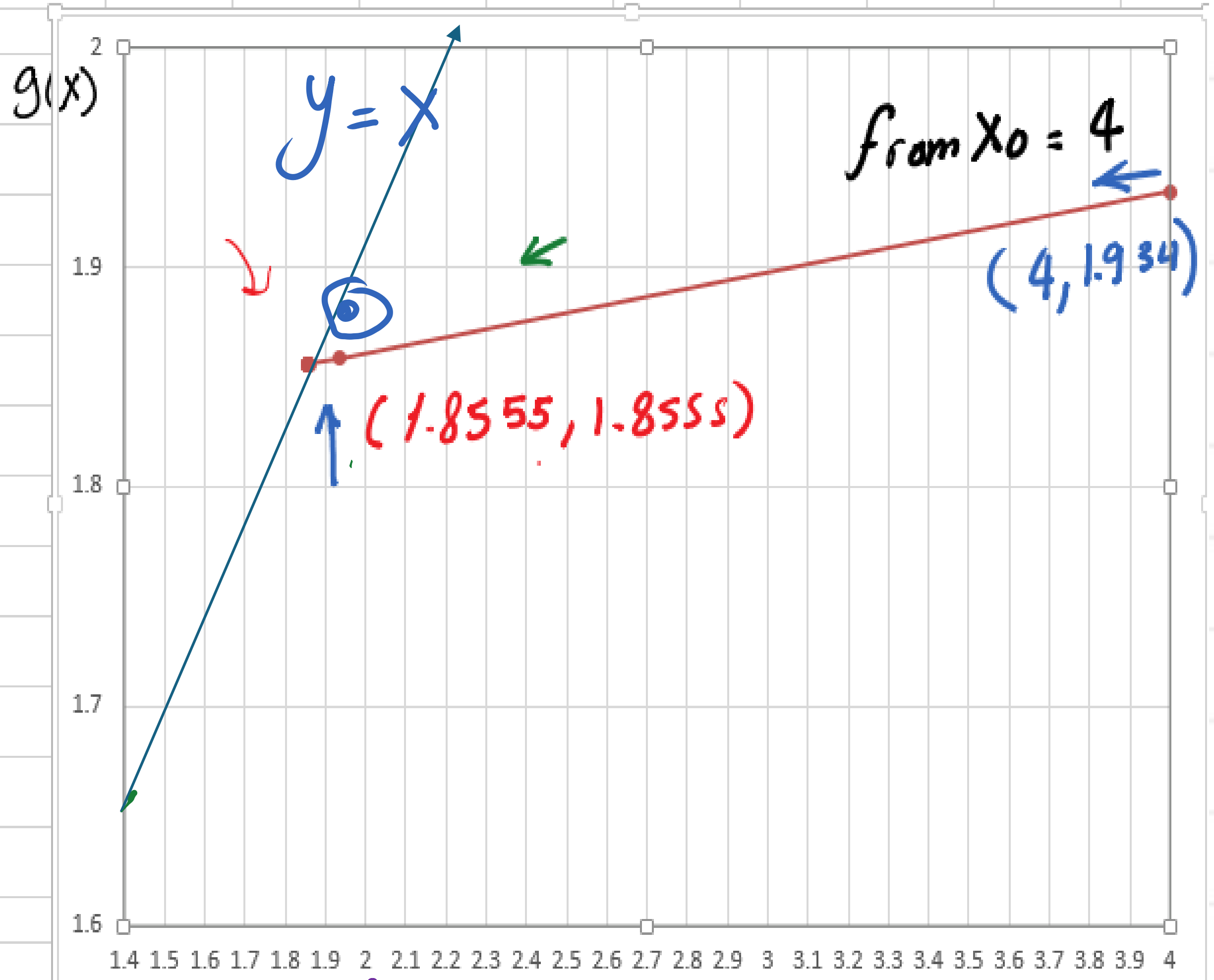
$x_0 = 4.00$ $g(x_0 = 4) = (10 + 4)^{1/4} \rightarrow x_1 = 1.9343$

$x_1 = 1.9343$ $g(x_1 = 1.9343) = (10 + 1.9343)^{1/4} \Rightarrow x_2 = 1.8583$

$\rightarrow$ pluG in $g(x_2 = 1.8583)$

	x		g(x)
X_0	4		1.934336
X_1	1.934336		1.858658
X_2	1.858658		1.855705
X_3	1.855705		1.855589
X_4	1.855589		1.855585
X_5	1.855585		1.855585
X_6	1.855585		1.855585
X_7	1.855585		1.855585
X_8	1.855585		1.855585
X_9	1.855585		1.855585
X_{10}	1.855585		1.855585

$\leftarrow g(x_0)$



intersection Point is
(1.8555, 1.8555)

Find the root of x^4-x-10 = 0

root point is between

1.75 & 2.00

From Fixed Point iteration

For $x = 1.8555$

$(1.8555)^4 - (1.8555) - 10 = -2.075(10^{-3})$

close to zero

X_0	4	242
X_1	3.75	184
X_2	3.5	136.56
X_3	3.25	98.316
X_4	3	68
X_5	2.75	44.441
X_6	2.5	26.563
X_7	2.25	13.379
X_8	2	4
X_9	1.75	-2.371
X_{10}	1.5	-6.438
X_{11}	1.25	-8.809
X_{12}	1	-10
X_{13}	0.75	-10.43
X_{14}	0.5	-10.44

original functio x^4-x-10=0

Test function at $x_0 = 4 \rightarrow 242$

$x_i = 1 \rightarrow -10$

root is between 4 & 1