Powers of MATrix

$A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \Rightarrow$ Find $A^2 \Rightarrow$

$$\begin{array}{ccc} A & \cdot & A \\ \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} & \cdot & \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \end{array}$$

It is easy to get values of various elements

$$(AB)_{11} \Rightarrow \left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = 1 + 2(0) = 1$$

First row A

First A Column

$$(AB)_{12} \Rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} = (1)(2) + (2)(-1) = 2 - 2 = 0$$

$$(AB)_{21} \Rightarrow \begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = (0)(1) - (1)(0) = 0$$

Finally $(AB)_{22} = \begin{bmatrix}0\\-1\end{bmatrix} \cdot \begin{bmatrix}2\\-1\end{bmatrix} = 0-(1)(-1) = +1$

2nd row A   2nd Column A

$A^2 = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Recall $A = \begin{bmatrix}1 & 2\\ 0 & -1\end{bmatrix} \Rightarrow$ A^2 not $\neq \begin{bmatrix}1^2 & 2^2\\ 0^2 & -1^2\end{bmatrix}$

$A^n = \begin{bmatrix}2^{n-1} & 2^{n-1}\\ 2^{n-1} & 2^{n-1}\end{bmatrix} \Rightarrow n=2 \quad \begin{bmatrix}2^1 & 2\\ 2 & 2\end{bmatrix}$

For Identity matrix

2nd item

$A \cdot B \# B \cdot A$

(3×3) A $\qquad$ $B\ (3 \times 4)$ $\qquad$ $[3 \times 4]$

AB Dim $\Rightarrow$ $(3 \times 3)(3 \times 4)$

Common

$$A = \begin{array}{c} \begin{bmatrix} 3 & 5 & -1 \\ 4 & 0 & 2 \\ -6 & -3 & 2 \end{bmatrix} \end{array} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

while

$$B \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \\ \end{array}$$
$$B = \begin{bmatrix} 2 & -2 & 3 & 1 \\ 5 & 0 & 7 & 8 \\ 9 & -4 & 1 & 1 \end{bmatrix}$$

C_1

$$A \cdot B_{11} \Rightarrow \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix} = 3(2) + 5(5) + (1)(-9) =$$

$$= 6 + 25 + (-9) = 22$$

C_2

$$AB12 \Rightarrow \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 0 \\ -4 \end{bmatrix} = -6 + 0 + 4 = -2$$

$$A = \begin{array}{c c} \begin{bmatrix} 3 & 5 & -1 \\ 4 & 0 & 2 \\ -6 & -3 & 2 \end{bmatrix} & \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \end{array} \quad \times \quad B = \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 2 & -2 & 3 & 1 \\ 5 & 0 & 7 & 8 \\ 9 & -4 & 1 & 1 \end{bmatrix} \end{array}$$

First row 3rd column B

$$(AB)_{13} = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 7 \\ 1 \end{bmatrix} = (3)(3) + 5(7) - 1(1)$$

$$= 9 + 35 - 1 = 43$$

$$(AB)_{14} = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix} = 3(1) + 5(8) - 1(1)$$

$$= 3 + 40 - 1 = 42$$

$$(AB) \rightarrow \begin{bmatrix} 22 & -2 & 43 & 42 \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$$

AB_{11} AB_{12} AB_{13} AB_{14}

$$A = \begin{bmatrix} 3 & 5 & -1 \\ 4 & 0 & 2 \\ -6 & -3 & 2 \end{bmatrix} \quad \times \quad B = \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \\ \end{array} \begin{bmatrix} 2 & -2 & 3 & 1 \\ 5 & 0 & 7 & 8 \\ 9 & -4 & 1 & 1 \end{bmatrix}$$

(AB) → First row

$$\begin{bmatrix} 22 & -2 & 43 & 42 \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$$

$$(AB)_{21} \Rightarrow \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix} = (4)(2) + 0(5) + 2(9) = 8 + 0 + 18 = 26$$

$$(AB)_{22} \Rightarrow \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 0 \\ -4 \end{bmatrix} = (4)(-2) + 0 + (2)(-4) = -16$$

$$(AB)_{23} \Rightarrow \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 7 \\ 1 \end{bmatrix} \Rightarrow 4(3) + 0 + 2(1) = 12 + 2 = 14$$

$$A = \begin{bmatrix} 3 & 5 & -1 \\ 4 & 0 & 2 \\ -6 & -3 & 2 \end{bmatrix} \times \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ B = \begin{bmatrix} 2 & -2 & 3 & 1 \\ 5 & 0 & 7 & 8 \\ 9 & -4 & 1 & 1 \end{bmatrix} \end{array}$$

$$(AB)_{24} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix} = 4 + 0 + 2 = 6$$

$$AB \Rightarrow \begin{bmatrix} 22 & -2 & 43 & 42 \\ 26 & -16 & 14 & 6 \\ * & * & * & * \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 5 & -1 \\ 4 & 0 & 2 \\ -6 & -3 & 2 \end{bmatrix} \times B = \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 2 & -2 & 3 & 1 \\ 5 & 0 & 7 & 8 \\ 9 & -4 & 1 & 1 \end{bmatrix} \end{array}$$

$$(AB)_{31} \Rightarrow \begin{bmatrix} -6 \\ -3 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix} = -6(2) + (-3)(5) + 2(9) = -12 - 15 + 18$$

$$= -9$$

$$(AB)_{32} \Rightarrow \begin{bmatrix} -6 \\ -3 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 0 \\ -4 \end{bmatrix} = -6(-2) + 0 + 2(-4) = 12 - 8 = 4$$

$$(AB)_{33} \Rightarrow \begin{bmatrix} -6 \\ -3 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 7 \\ 1 \end{bmatrix} \Rightarrow -6(3) - 3(7) + 2(1)$$

$$-18 - 21 + 2 = -37$$

$$(AB)_{34} = \begin{bmatrix} -6 \\ -3 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix} = -6(1) - 3(8) + (2)(1)$$

$$= -6 - 24 + 2 = -28$$

$A \cdot B \Rightarrow \begin{bmatrix} 22 & -2 & 43 & 42 \\ 26 & -16 & 14 & 6 \\ -9 & 4 & -37 & -28 \end{bmatrix}$

$$\begin{array}{cc} B & A \\ (3 \times 4) & (3 \times 3) \end{array}$$

$B \cdot A$

Does not match

We Cannot make multplication

$$\begin{pmatrix} 3 & 5 & -1 \\ 4 & 0 & 2 \\ -6 & -3 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & -2 & 3 & 1 \\ 5 & 0 & 7 & 8 \\ 9 & -4 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 22 & -2 & 43 & 42 \\ 26 & -16 & 14 & 6 \\ -9 & 4 & -37 & -28 \end{pmatrix}$$

Commutative Property

The commutative property states that the numbers on which we operate can be moved or swapped from their position without making any difference to the answer. The property holds for Addition and Multiplication, but not for subtraction and division.

$(a+b)=(b+a)$

$(ab)=(ba)$

What is Associative Property?

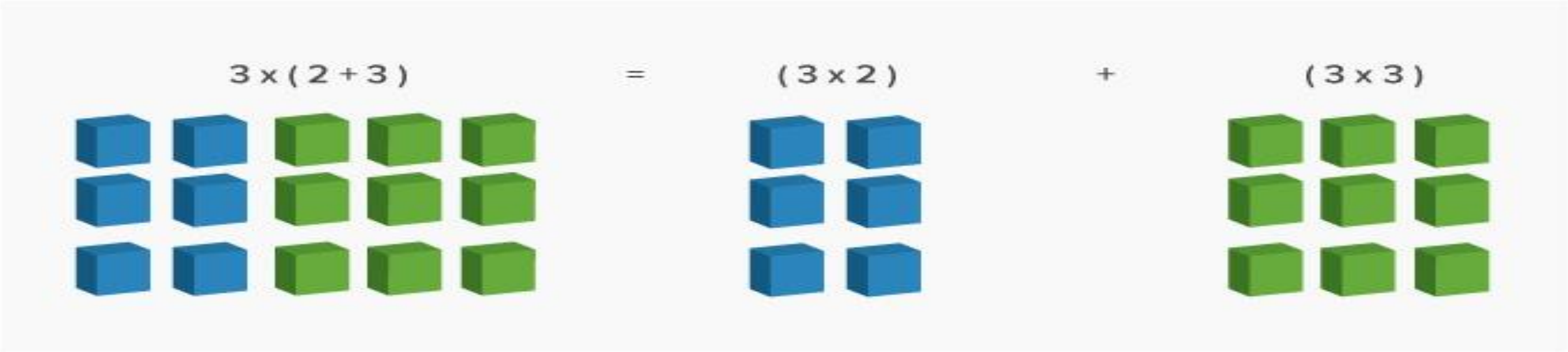
This property states that when three or more numbers are added (or multiplied), the sum (or the product) is the same regardless of the grouping of the addends (or the multiplicands).

$A+B+C$

$=A+(B+C)$

What is Distributive Property?

To "distribute" means to divide something or give a share or part of something.
According to the distributive property, multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together.



Check which that can be

Commutative and Associative Properties

For any real numbers a, b, and c,

$$\left.\begin{aligned} a + b &= b + a \\ \text{and} \quad ab &= ba. \end{aligned}\right\} \quad \text{Commutative properties}$$

For real numbers

Reverse the order of two terms or factors.

Also,

$$\left.\begin{aligned} a + (b + c) &= (a + b) + c \\ \text{and} \quad a(bc) &= (ab)c. \end{aligned}\right\} \quad \text{Associative properties}$$

Shift parentheses among three terms or factors; order stays the same.

1. Matrix addition is commutative

If A and B are two matrices of the same order, then $A + B = B + A$.

Proof. Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$

Then,

$$\begin{aligned} A + B &= [a_{ij}]_{m \times n} + [b_{ij}]_{m \times n} \\ &= [a_{ij} + b_{ij}]_{m \times n} \\ &= [b_{ij} + a_{ij}]_{m \times n} \\ &= [b_{ij}]_{m \times n} + [a_{ij}]_{m \times n} \\ &= B + A \end{aligned}$$

$\because$ Addition of scalars is commutative.

Theorem (1.12). Properties of Matrix Addition.

Let $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ be matrices of the same size $m \times n$ (m rows by n columns). Then

(a) $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

(b) $(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$ (Associative law for addition)

(c) $\underbrace{\mathbf{A} + \mathbf{A} + \mathbf{A} + \cdots + \mathbf{A}}_{k \text{ copies}} = k\mathbf{A}$

(d) There is an $m \times n$ matrix called the zero matrix, denoted $\mathbf{O}$, which has the property

$$\mathbf{A} + \mathbf{O} = \mathbf{A}$$

⇒ additive identity

(e) There is a matrix denoted $-\mathbf{A}$ such that

$$\mathbf{A} + (-\mathbf{A}) = \mathbf{A} - \mathbf{A} = \mathbf{O}$$

This matrix $-\mathbf{A}$ is called the **additive inverse** of $\mathbf{A}$.

Linear Algebra

Step by Step

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Example 1.22

Let $\mathbf{A} = \begin{pmatrix} 1 & -1 & 3 & 7 \\ 2 & 9 & 5 & -6 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 7 & 6 & -3 & 2 \\ 1 & 4 & 5 & 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} -2 & -7 & 8 & 6 \\ 3 & -9 & 2 & 1 \end{pmatrix}$. Determine

(a) $\mathbf{A} + \mathbf{B}$ (b) $\mathbf{B} + \mathbf{A}$ (c) $(\mathbf{A} + \mathbf{B}) + \mathbf{C}$ (d) $\mathbf{A} + (\mathbf{B} + \mathbf{C})$

(e) $\mathbf{A} + \mathbf{O}$ (f) $\mathbf{A} + \mathbf{A} + \mathbf{A} + \mathbf{A} + \mathbf{A}$ (g) $\mathbf{C} + (-\mathbf{C})$

$$A = \begin{pmatrix} 1 & -1 & 3 & 7 \\ 2 & 9 & 5 & -6 \end{pmatrix} \ \& \ B \begin{pmatrix} 7 & 6 & -3 & 2 \\ 1 & 4 & 5 & 3 \end{pmatrix}$$

A+B

Part (a)

$$= \begin{pmatrix} (1+7) & (-1+6) & (3+-3) & (7+2) \\ (2+1) & (9+4) & (5+5) & (-6+3) \end{pmatrix}$$

$$\begin{pmatrix} 8 & 5 & 0 & 9 \\ 3 & 13 & 10 & -3 \end{pmatrix}$$

Part (B.)

$A+B = B+A$

Example 1.22

Let $\mathbf{A} = \begin{pmatrix} 1 & -1 & 3 & 7 \\ 2 & 9 & 5 & -6 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 7 & 6 & -3 & 2 \\ 1 & 4 & 5 & 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} -2 & -7 & 8 & 6 \\ 3 & -9 & 2 & 1 \end{pmatrix}$. Determine

(a) $\mathbf{A} + \mathbf{B}$ **(b)** $\mathbf{B} + \mathbf{A}$ **(c)** $(\mathbf{A} + \mathbf{B}) + \mathbf{C}$ **(d)** $\mathbf{A} + (\mathbf{B} + \mathbf{C})$

(e) $\mathbf{A} + \mathbf{O}$ **(f)** $\mathbf{A} + \mathbf{A} + \mathbf{A} + \mathbf{A} + \mathbf{A}$ **(g)** $\mathbf{C} + (-\mathbf{C})$

$$A+B = \begin{pmatrix} 8 & 5 & 0 & 9 \\ 3 & 13 & 10 & -3 \end{pmatrix} \qquad C = \begin{pmatrix} -2 & -7 & 8 & 6 \\ 3 & -9 & 2 & 1 \end{pmatrix}$$

From Part b

Part c)

$$(A+B)+C = \begin{pmatrix} 8-2 & 5-7 & 0+8 & 9+6 \\ 3+3 & 13+(-9) & 10+2 & -5+1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 2 & 8 & 15 \\ 6 & 4 & 12 & -4 \end{pmatrix}$$

Associative

Example 1.22

Let $\mathbf{A} = \begin{pmatrix} 1 & -1 & 3 & 7 \\ 2 & 9 & 5 & -6 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 7 & 6 & -3 & 2 \\ 1 & 4 & 5 & 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} -2 & -7 & 8 & 6 \\ 3 & -9 & 2 & 1 \end{pmatrix}$. Determine

(a) $\mathbf{A} + \mathbf{B}$ (b) $\mathbf{B} + \mathbf{A}$ (c) $(\mathbf{A} + \mathbf{B}) + \mathbf{C}$ (d) $\mathbf{A} + (\mathbf{B} + \mathbf{C})$

(e) $\mathbf{A} + \mathbf{O}$ (f) $\mathbf{A} + \mathbf{A} + \mathbf{A} + \mathbf{A} + \mathbf{A}$ (g) $\mathbf{C} + (-\mathbf{C})$

Part e

$A = \begin{bmatrix} 1 & -1 & 3 & 7 \\ 2 & 9 & 5 & -6 \end{bmatrix}$

$A + 0 = A$

$A + 0 = \begin{bmatrix} 1 & -1 & 3 & 7 \\ 2 & 9 & 5 & -6 \end{bmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 3 & 7 \\ 2 & 9 & 5 & -6 \end{pmatrix}$

Part f $A + A + A + A + A = KA$ where $K = 5$

$KA = 5\begin{pmatrix} 1 & -1 & 3 & 7 \\ 2 & 9 & 5 & -6 \end{pmatrix} = \begin{pmatrix} 5 & -5 & 15 & 35 \\ 10 & 45 & 25 & -30 \end{pmatrix}$

Part g $C + (-C) = 0$

$\begin{pmatrix} -2-2 & -7+7 & 8-8 & 6-6 \\ 3-3 & -9+9 & 2-2 & 1-1 \end{pmatrix}$

$0 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$