EXAMPLE 5.10
Strength of Column with Slender Elements

Goal: Determine the available strength of a compression member with a slender web.

Given: Use a W16×26 as a column with $L_{cy} = 5.0$ ft.

Solution:

$A992$ steel $\Rightarrow F_y = 50$ ksi;

① check column whether $\Rightarrow$ Elastic $\rightarrow$ inelastic
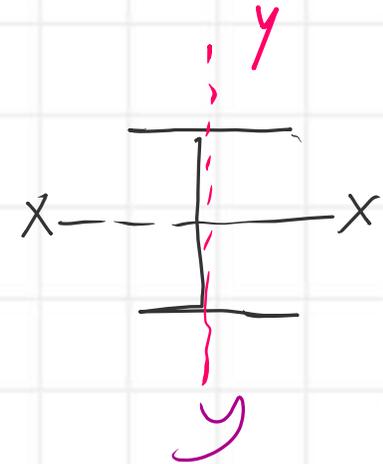
$$\frac{KL}{r} \Rightarrow \text{or } \frac{L_e}{r} = 4.71\sqrt{E/F_y} = 4.71\sqrt{\frac{29000}{50}} = 113.43$$

From Table 1-1 For W16×26

$L_{cy} = 5.00$ FT $r_y = 1.12$ inch^3

$$\left(\frac{KL}{r}\right)_y \Rightarrow \frac{L_{cy}}{r_y} = \frac{5(12)}{1.12} = 53.57 \Rightarrow \text{inelastic}$$
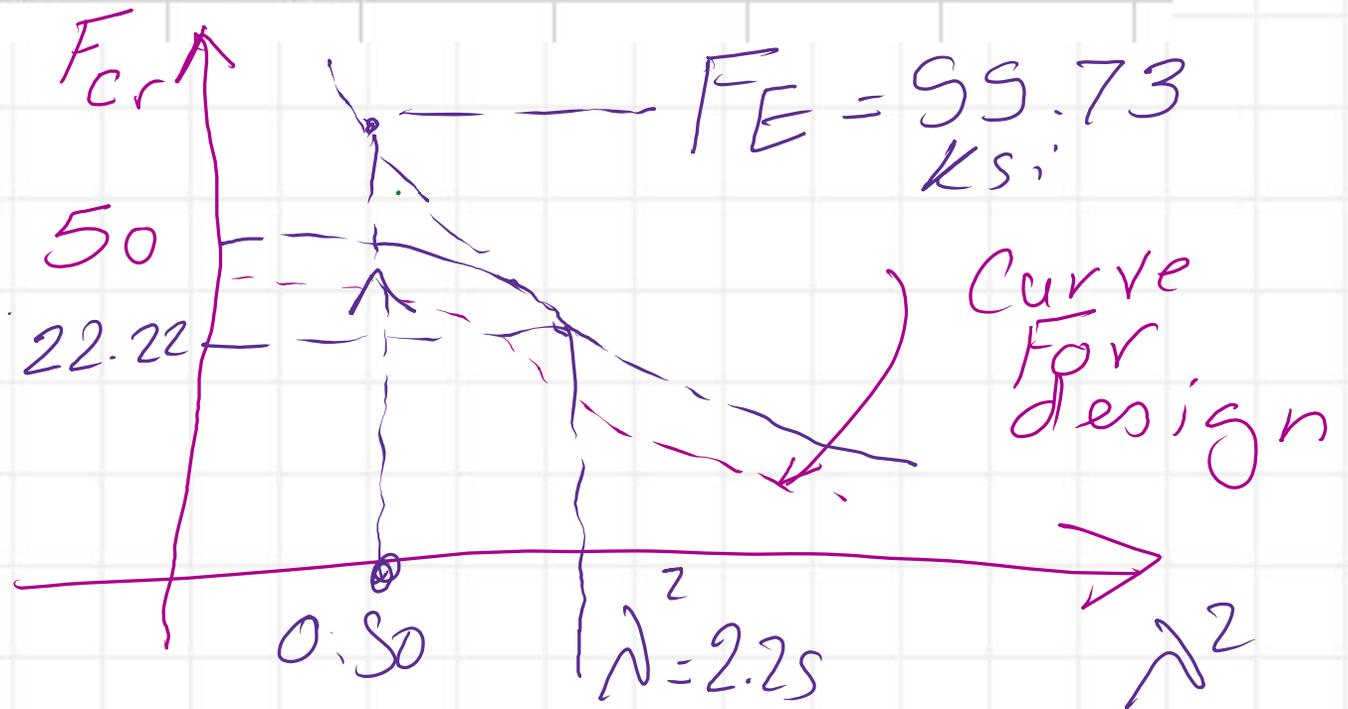
Prepared by Eng.Maged Kamel.

< 113.43

(a) For uniformly compressed slender elements, with $\dfrac{b}{t} \geq 1.49\sqrt{\dfrac{E}{f}}$, except flanges of square and rectangular sections of uniform thickness:

$$b_e = 1.92t\sqrt{\dfrac{E}{f}}\left[1 - \dfrac{0.34}{(b/t)}\sqrt{\dfrac{E}{f}}\right] \leq b \qquad \text{(E7-17)}$$

$$F_E = \dfrac{\pi^2 E}{\left(\dfrac{KL}{r}\right)_y^2} = \dfrac{\pi^2(29000)}{(53.57)^2}$$

$$= 99.73 \; \text{ksi}$$

$$\lambda_c^2 = \dfrac{F_y}{F_E}$$

$$= 50/99.73$$

$$= 0.501$$

F_{cr}

$F_E = 99.73$ ksi

50

22.22

0.50

$\lambda^2 = 2.25$

Curve for design

λ^2

Prepared by Eng.Maged Kamel.

(a) For uniformly compressed slender elements, with $\dfrac{b}{t} \geq 1.49\sqrt{\dfrac{E}{f}}$, except flanges of square and rectangular sections of uniform thickness:
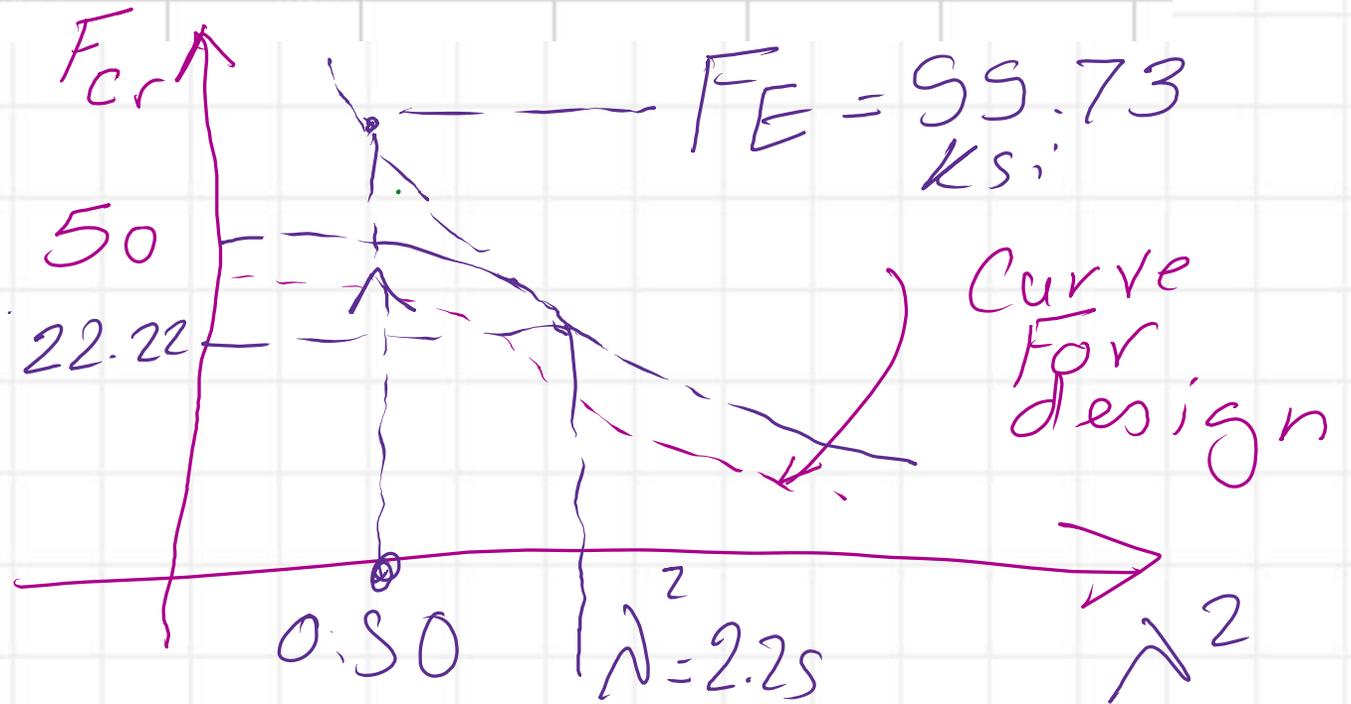
$$b_e = 1.92t\sqrt{\dfrac{E}{f}}\left[1 - \dfrac{0.34}{(b/t)}\sqrt{\dfrac{E}{f}}\right] \leq b \qquad \text{(E7-17)}$$

$Q = 1$

$Ag =$

$\lambda_c^2 < 2.25$

$F_{cr} = 0.658^{\lambda_c^2} \, F_y \, Q$

$= 0.658^{(0.50\,t)} \, (50) \, 1$

$F_{cr} = 40.54 \text{ ksi}$

$\phi_c P_n = 280 \text{ kips}$

$P_n / \Omega = 187 \text{ kips}$

F_{cr}

50

22.22

$F_E = 99.73 \text{ ksi}$

Curve For design

0.50

$\lambda^2 = 2.25$

λ^2

In the previous Post → Post 16

$Q = 1$ ⟿ $\lambda^2 = 0.501$ ⟿

But we will use Q_{new} ↘

$A_e = 6.83 \text{ inch}^2$

$A_g = 7.68 \text{ inch}^2$ → $Q = \dfrac{A_e}{A_g} = \dfrac{6.83}{7.68} = 0.889$

New $F_{cr} = 0.658^{\,Q\lambda_c^2} \, (F_y \, Q)$

$F_{cr} = 0.658^{\,(0.889)(0.50)} \quad (50)(0.889)$

$= 36.92 \text{ ksi}$

2.25

λ^2
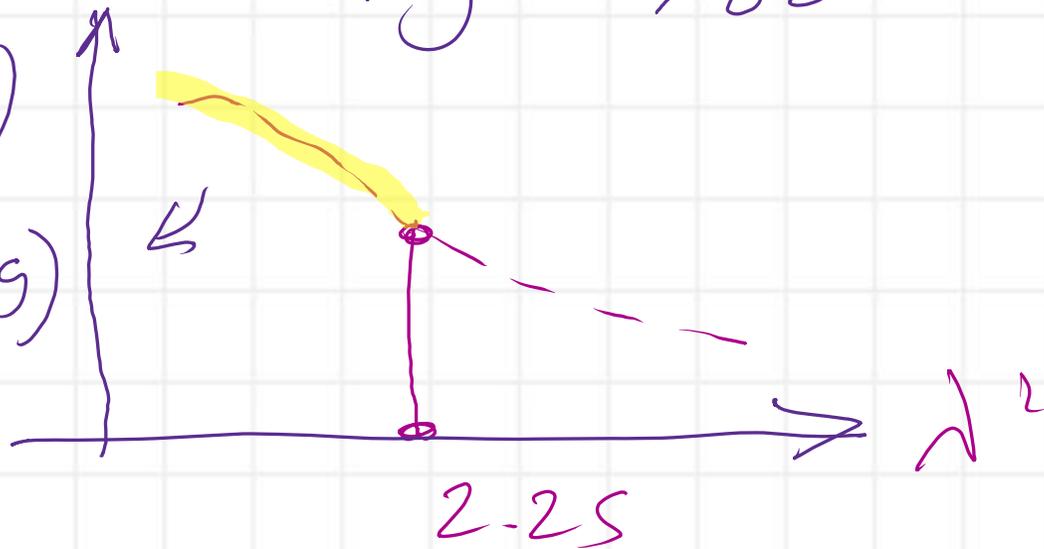
For $Q = 0.889$

$F_{cr} = 36.92$ Ksi

$A_g = 7.68$ inch^2

$A_{eff} = 6.83$ inch^2
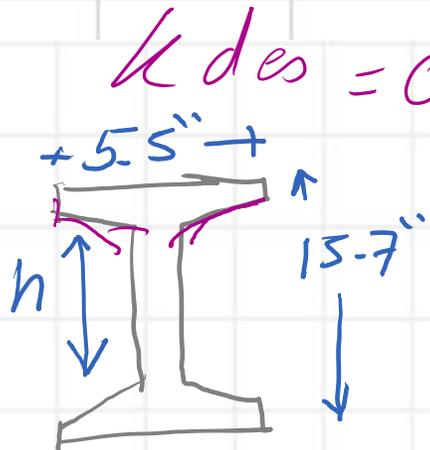
$P_n = A_g f_{cr} = 7.68 (36.92) = 283.55$
≈ 284 kips

$\phi P_n = 0.9 (284) = 256$ Kips

$\dfrac{P_n}{\Omega} = \dfrac{284}{1.67} = 170$ Kips

(a) For uniformly compressed slender elements, with $\dfrac{b}{t} \geq 1.49\sqrt{\dfrac{E}{f}}$, except flanges of square and rectangular sections of uniform thickness:

$$b_e = 1.92t\sqrt{\dfrac{E}{f}}\left[1 - \dfrac{0.34}{(b/t)}\sqrt{\dfrac{E}{f}}\right] \leq b \qquad (E7\text{-}17)$$

Get new he

$k\,des = 0.747''$

+5.5"+

h 15.7"

$E = 29000$ ks:

$t = tweb$

$f = f_{cr} = 36.92$ ks;

$\dfrac{b}{t} = \dfrac{hw}{tw}$

$\dfrac{b}{t} = \dfrac{14.21}{0.25} = 56.80$

Find $h_w = d - 2\,kdes$

$= 15.7 - 2(0.747)$

$h_w = 14.21''$

(a) For uniformly compressed slender elements, with $\frac{b}{t} \geq 1.49\sqrt{\frac{E}{f}}$, except flanges of square and rectangular sections of uniform thickness:

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{(b/t)}\sqrt{\frac{E}{f}}\right] \leq b \qquad \text{(E7-17)}$$

$F_{cr} = 36.92 \text{ ksi};$

$h_e = b_e = 1.92(0.25)\sqrt{\frac{29000}{36.92}}\left[1 - \frac{0.34}{56.8}\sqrt{\frac{29000}{36.92}}\right]$

$h_e = b_e = 11.20'' < 14.21''$

$d - 2k \text{ des}$
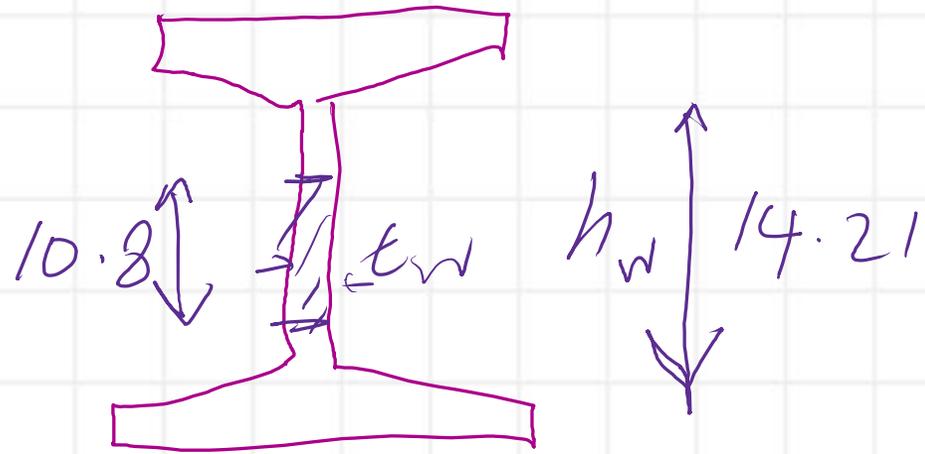
Find A_{eff} Web

Flange is fully active

$\lambda_F < \lambda_r$

Effective web area

$h_e = 11.20''$
$t_w = 0.25'' \Rightarrow A_{ef} = 11.2(0.25)$
$= 2.80$ $inch^2$

$h_w = 14.21''$
$t_w = 0.25''$

$\Downarrow$

$A_w = 3.55$ $inch^2$

reduced $\nearrow$

$A_{sect} = A_g - (h_w)(t_w) + A_{ef} w$

$A_e = 7.68 - 3.55 + 2.8 = 6.93$ $inch^2$

10.8

t_w

h_w 14.21

$$A_e = 6.93 \text{ inch}^2$$
$$A_g = 7.68 \text{ inch}^2$$

$$Q = \frac{A_e}{A_g} = \frac{6.93}{7.68} = 0.902$$
$$\approx 0.90$$

$$\lambda_c^2 = \frac{F_y}{F_E} = \frac{50}{99.73} = 0.5$$

$$F_{cr} = 0.658^{(0.45)} (50)(0.9)$$
$$= 37.27$$
$$ksi$$

$$Q\lambda^2 = 0.9(0.5)$$
$$= 0.45$$

LRFD : $0.9(7.68)(37.27)$
$$= 258 \text{ kips}$$

ASD : $\dfrac{7.68(37.27)}{1.67}$
$$= 171 \text{ kips}$$

DESIGN TABLE DISCUSSION

Table 6-1. W-Shapes in Combined Flexure and Axial Force

Steel W-shapes with $F_y = 50$ ksi (ASTM A992) and subject to combined axial force (tension or compression) and flexure may be checked for compliance with the provisions of Section H1.1 and H1.2 of the AISC *Specification* using values listed in Table 6-1 and the appropriate interaction equations provided in the following sections.

Values p, b_x, b_y, t_y and t_r presented in Table 6-1 are defined as follows.

	LRFD	ASD
Axial Compression	$p = \dfrac{1}{\phi_c P_n}$, (kips)$^{-1}$	$p = \dfrac{\Omega_c}{P_n}$, (kips)$^{-1}$

$$\phi_c P_n = \frac{1000}{P}$$

$$\frac{P_n}{\Omega} = \frac{P/1000}{P}$$

COMPOSITE MEMBERS SUBJECT TO COMBINED FLEXURE AND AXIAL COMPRESSION

For the design of composite members subject to combined flexure and axial compression, see AISC *Specification* Section I5.

DESIGN TABLE DISCUSSION

Table 6-1. W-Shapes in Combined Flexure and Axial Force

Steel W-shapes with $F_y = 50$ ksi (ASTM A992) and subject to combined axial force (tension or compression) and flexure may be checked for compliance with the provisions of Section H1.1 and H1.2 of the AISC *Specification* using values listed in Table 6-1 and the appropriate interaction equations provided in the following sections.

Values p, b_x, b_y, t_y and t_r presented in Table 6-1 are defined as follows.

	LRFD	ASD
Axial Compression	$p = \dfrac{1}{\phi_c P_n} \cdot \text{(kips)}^{-1}$	$p = \dfrac{\Omega_c}{P_n} \cdot \text{(kips)}^{-1}$
Strong Axis Bending	$b_x = \dfrac{8}{9\phi_b M_{nx}} \cdot \text{(kip-ft)}^{-1}$	$b_x = \dfrac{8\Omega_b}{9M_{nx}} \cdot \text{(kip-ft)}^{-1}$
Weak Axis Bending	$b_y = \dfrac{8}{9\phi_b M_{ny}} \cdot \text{(kip-ft)}^{-1}$	$b_y = \dfrac{8\Omega_b}{9M_{ny}} \cdot \text{(kip-ft)}^{-1}$
Tension Yielding	$t_y = \dfrac{1}{\phi_t F_y A_g} \cdot \text{(kips)}^{-1}$	$t_y = \dfrac{\Omega_t}{F_y A_g} \cdot \text{(kips)}^{-1}$
Tension Rupture	$t_r = \dfrac{1}{\phi_t F_u(0.75A_g)} \cdot \text{(kips)}^{-1}$	$t_r = \dfrac{\Omega_t}{F_u(0.75A_g)} \cdot \text{(kips)}^{-1}$

$\phi_c P_n = \dfrac{1000}{3.37} = 297\ kips\ at\ \ell_c = 0$

AISC-360-10

CM #14

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

$F_y = 50$ ksi

W16

W16×26   L = 5'

by Estimate LRFD

$L_{cy} = 5$

258 Kips

Shape		W16×						
		31ᶜ				26ᶜ·ᵛ		
Design		$p \times 10^3$ (kips)⁻¹		$b_x \times 10^3$ (kip–ft)⁻¹		$p \times 10^3$ (kips)⁻¹		$b_x \times 10^3$ (kip–ft)⁻¹	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
	0	4.09	2.72	6.60	4.39	5.06	3.37	8.06	5.36
	6	5.08	3.38	7.28	4.85	6.33	4.21	9.07	6.03
	7	5.52	3.67	7.71	5.13	6.91	4.60	9.66	6.43
	8	6.10	4.06	8.19	5.45	7.68	5.11	10.3	6.87
	9	6.87	4.57	8.74	5.82	8.70	5.79	11.1	7.39
	10	7.89	5.25	9.37	6.23	10.1	6.72	12.0	7.99
	11	9.28	6.17	10.1	6.71	12.0	8.01	13.1	8.69
	12	11.0	7.34	11.1	7.35	14.3	9.53	15.0	10.0
	13	13.0	8.62	12.6	8.39	16.8	11.2	17.2	11.5
	14	15.0	10.0	14.2	9.45	19.5	13.0	19.5	13.0
	15	17.2	11.5	15.8	10.5	22.4	14.9	21.9	14.6
	16	19.6	13.1	17.5	11.6	25.5	16.9	24.3	16.2
	17	22.2	14.7	19.2	12.8	28.7	19.1	26.7	17.8
	18	24.8	16.5	20.9	13.9	32.2	21.4	29.2	19.4
	19	27.7	18.4	22.6	15.0				

(left axis) with respect to least radius of gyration, r_y length, L_b (ft), for X-X axis bending

$\phi_c P_n = \dfrac{1000}{4.21} = 237.5\ kips$ at $L_y = 6'$

$$\frac{P_n}{\Omega} = \frac{1000}{5.06} = 197.60 \text{ kips} \approx 198 \text{ kips} \quad e_y = 0$$

6-62

DESIGN OF MEMBERS SUBJECT TO COMBINED FORCES

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

$F_y = 50$ ksi

W16

Shape		W16×							
		31[c]				26[c,v]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip–ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip–ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
with respect to least radius of gyration, r_y, length, L_b (ft), for X-X axis bending	0	4.09	2.72	6.60	4.39	5.06	3.37	8.06	5.36
	6	5.08	3.38	7.28	4.85	6.33	4.21	9.07	6.03
	7	5.52	3.67	7.71	5.13	6.91	4.60	9.66	6.43
	8	6.10	4.06	8.19	5.45	7.68	5.11	10.3	6.87
	9	6.87	4.57	8.74	5.82	8.70	5.79	11.1	7.39
	10	7.89	5.25	9.37	6.23	10.1	6.72	12.0	7.99
	11	9.28	6.17	10.1	6.71	12.0	8.01	13.1	8.69
	12	11.0	7.34	11.1	7.35	14.3	9.53	15.0	10.0
	13	13.0	8.62	12.6	8.39	16.8	11.2	17.2	11.5
	14	15.0	10.0	14.2	9.45	19.5	13.0	19.5	13.0
	15	17.2	11.5	15.8	10.5	22.4	14.9	21.9	14.6
	16	19.6	13.1	17.5	11.6	25.5	16.9	24.3	16.2
	17	22.2	14.7	19.2	12.8	28.7	19.1	26.7	17.8
	18	24.8	16.5	20.9	13.9	32.2	21.4	29.2	19.4
	19	27.7	18.4	22.6	15.0				

ASD

$L_{cy} = 5$

171 kips

$$\frac{P_n}{\Omega} = \frac{1000}{6.33} = 158 \text{ kips} \quad L_{cy} = 6 \text{ FT}$$