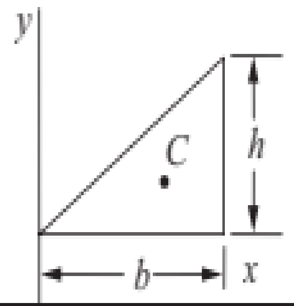
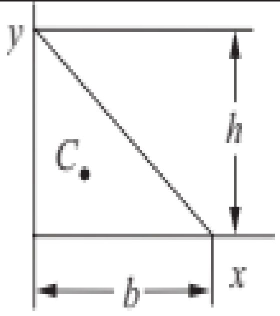
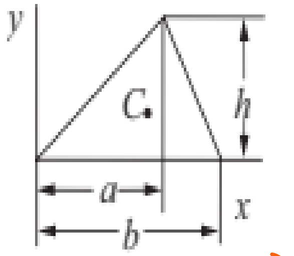
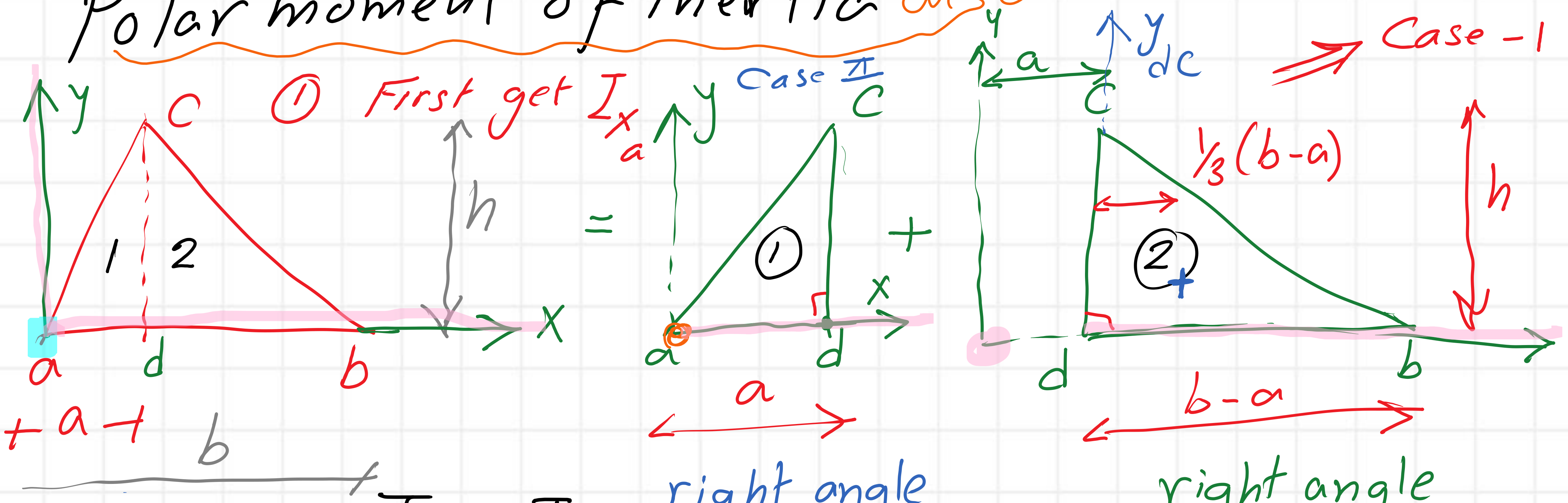
Polar moment of Inertia Triangle

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = bh/2$ $x_c = 2b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/4$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/2$	$I_{x_c y_c} = Abh/36 = b^2h^2/72$ $I_{xy} = Abh/4 = b^2h^2/8$
	$A = bh/2$ $x_c = b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/6$	$I_{x_c y_c} = -Abh/36 = -b^2h^2/72$ $I_{xy} = Abh/12 = b^2h^2/24$
	$A = bh/2$ $x_c = (a+b)/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = [bh(b^2 - ab + a^2)]/36$ $I_x = bh^3/12$ $I_y = [bh(b^2 + ab + a^2)]/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = (b^2 - ab + a^2)/18$ $r_x^2 = h^2/6$ $r_y^2 = (b^2 + ab + a^2)/6$	$I_{x_c y_c} = [Ah(2a-b)]/36$ $= [bh^2(2a-b)]/72$ $I_{xy} = [Ah(2a+b)]/12$ $= [bh^2(2a+b)]/24$

$(I_x + I_y)$ & $I_{x_c} + I_{y_c}$ at C.g

Polar moment of inertia at a



I_x Final at a $= I_{x_1} + I_{x_2}$

$$= \frac{a\,\bar{h}^3}{12} + \frac{(b-a)(h^3)}{12} = \frac{1}{12} h^3 (a+b-a)$$

$$I_x \text{ Final} = \frac{h^3}{12}(b)$$

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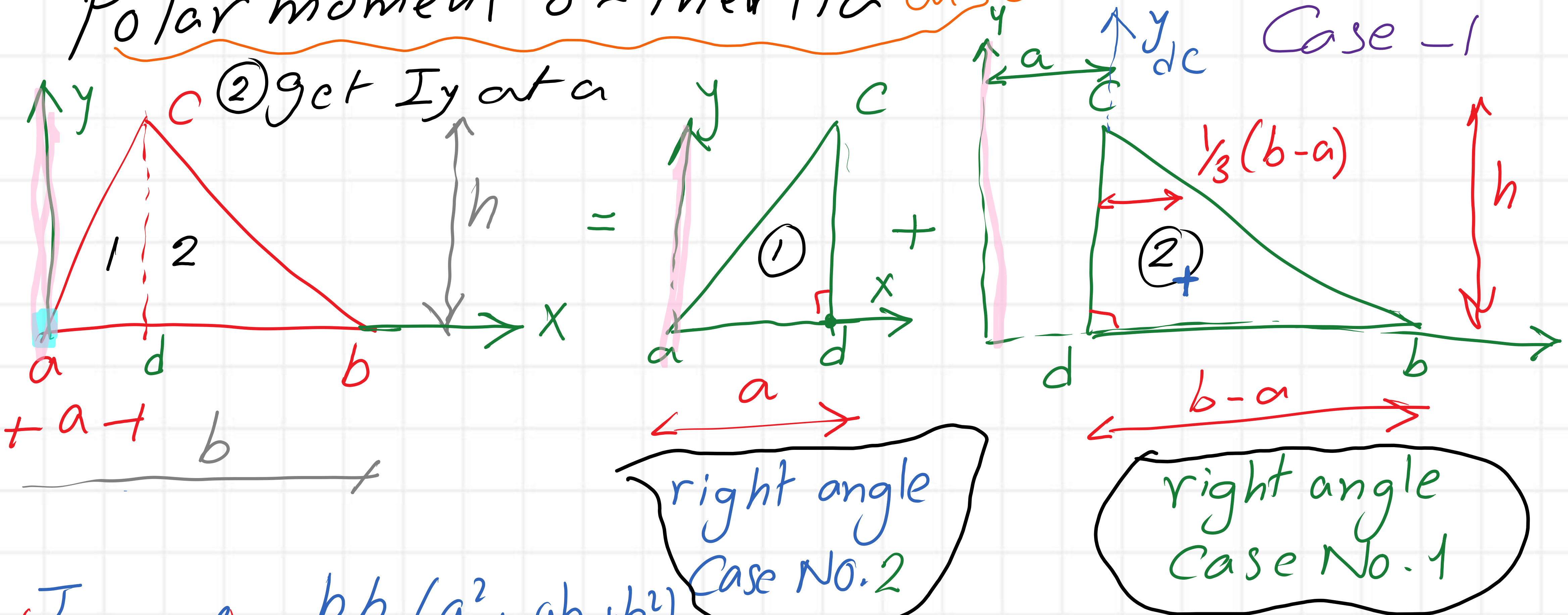
Polar moment o- inertia at a

② get I_y at a



$$\left(I_{y\ Final}\right)_a = \frac{bh}{12}\left(a^2+ab+b^2\right)$$

③ Add $I_x + I_y \rightarrow J_o$ at Point a $\rightarrow$ Proceed

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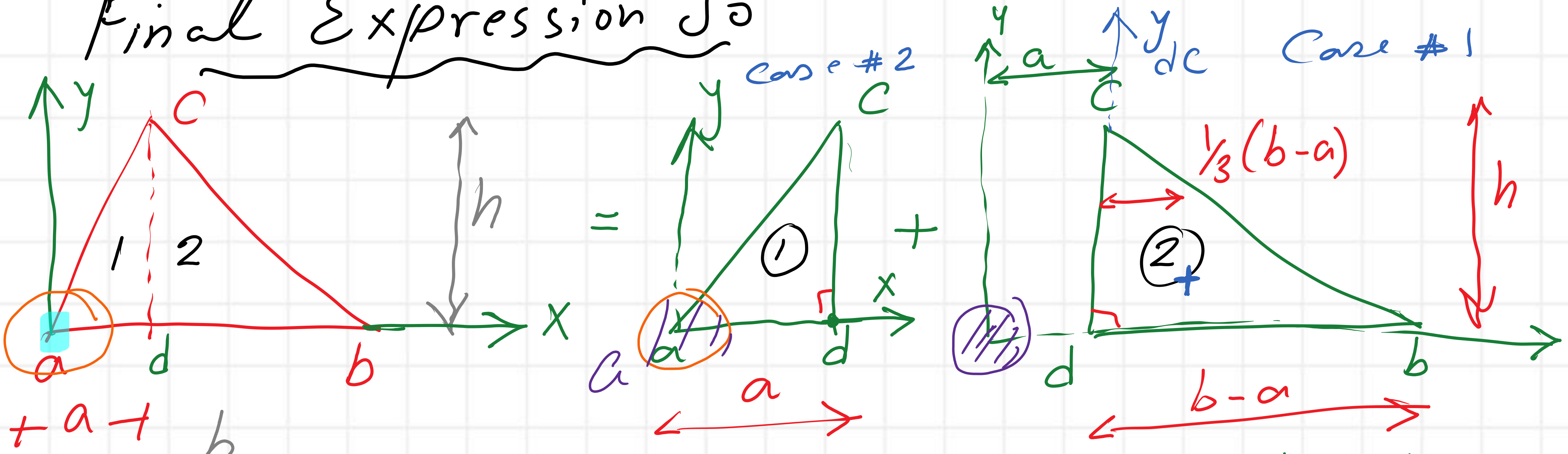
Final Expression J_o



right angle Case No.2

right angle Case No. 1

$$(I_{x\,Final})_a = \frac{h^3 b}{12}$$

$$(I_{y\,Final}) = \frac{bh}{12}(a^2 + ab + b^2)$$

$$J_o = \frac{hb}{12}\left[\underbrace{h^2}_{I_x} + a^2 + ab + b^2\right] = \frac{hb}{12}(h^2 + a^2 + ab + b^2)$$

Final

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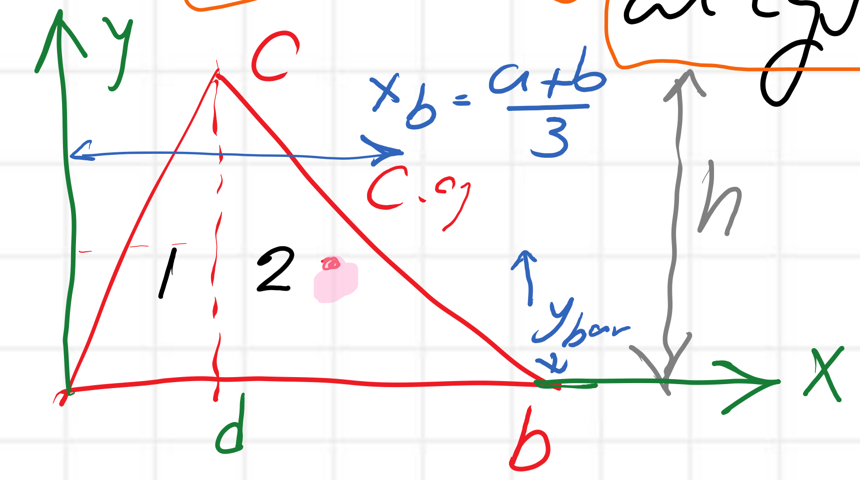
Polar moment of inertia at c.g



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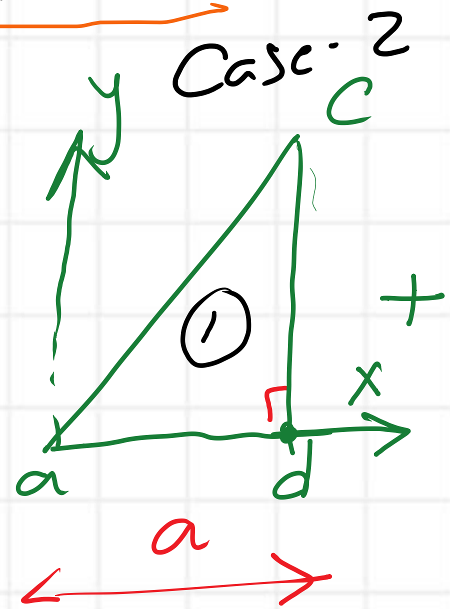

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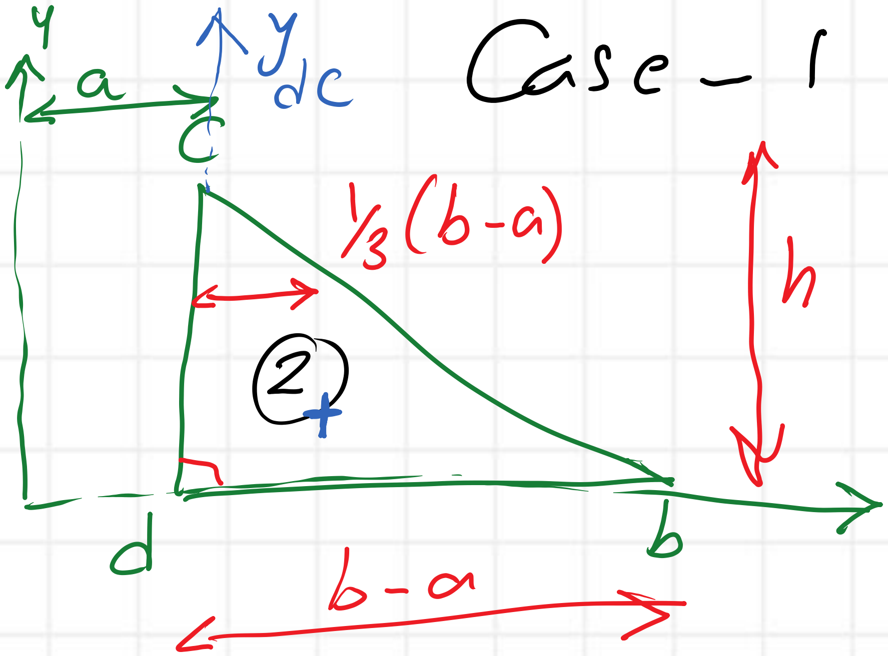

+ a + b

$$(I_{x_{Final}})_a = \frac{h^3 b}{12} - \frac{1}{2} b h \left(\frac{h}{9}\right)^2$$

$$= h^3 b \left[\frac{1}{12} - \frac{1}{18}\right]$$

$$I_x \ c.g = \frac{h^3 b}{36}$$

right angle
Case No.2

right angle
Case No.1

$$J_o = \frac{hb}{12}\left[h^2 + a^2 + ab + b^2\right] = \frac{hb}{12}\left(h^2 + a + ab + b^2\right)$$

For J_g deduct $A(x_b^2 + y_b^2) = \frac{1}{2} hb \left(\left(\frac{a+b}{3}\right)^2 + \left(\frac{h}{3}\right)^2\right]$

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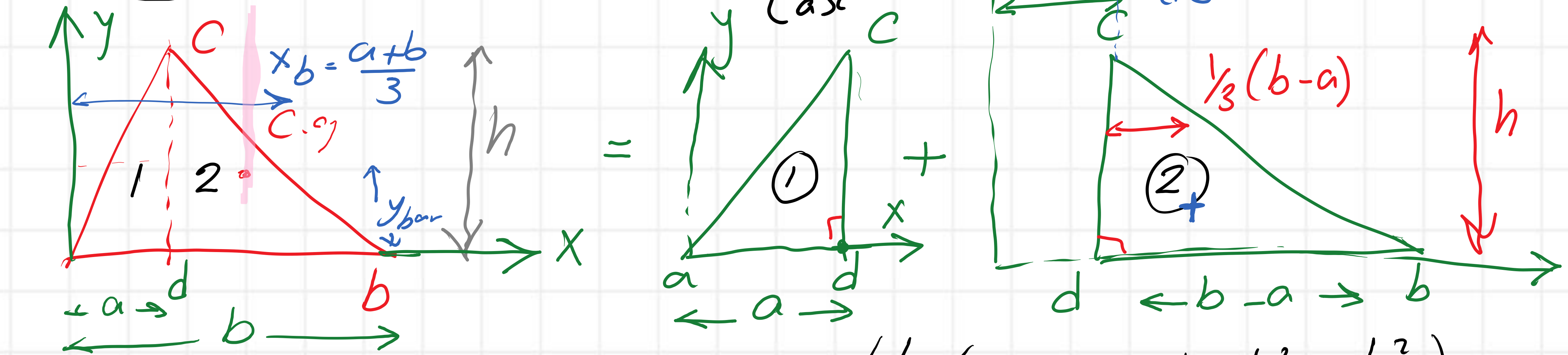
Polar moment of inertia at cg



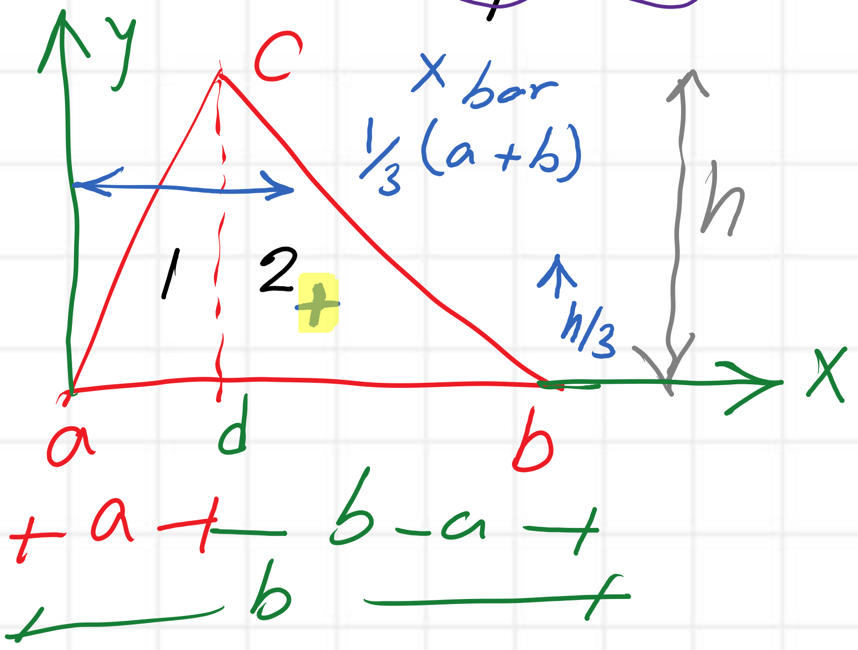
$$J_{g_y} = \frac{hb}{12}\left[h^2+a^2+ab+b^2\right] - \frac{hb}{2}\,\frac{(a^2+2ab+b^2+h^2)}{9}$$

$$\frac{hb}{36}\left[3h^2+3a^2+3ab+3b^2-2a^2-4ab-2b^2-2h^2\right)$$

$$J_{g_y} = \frac{hb}{36}\left[h^2+a^2-ab+b^2\right] \Rightarrow$$

Final Expression J_g

Other way of Estimation



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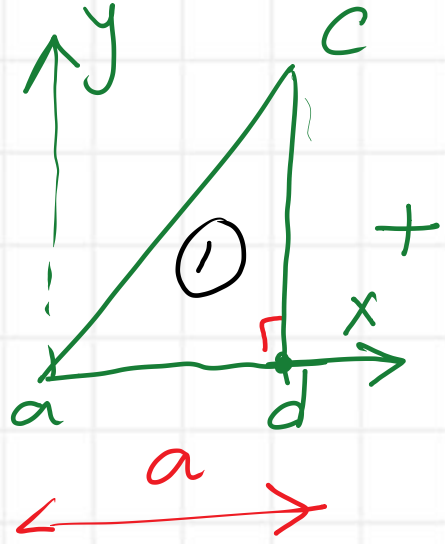

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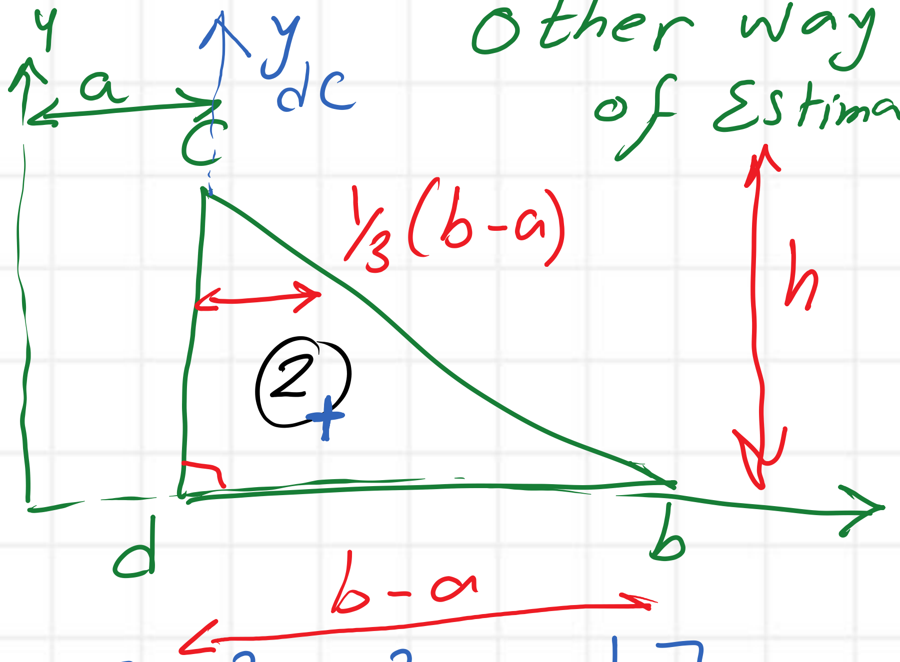

$$I_{xg} = \frac{bh^3}{36}$$

$$I_{ycg} = \frac{bh}{36}\left[b^2 + a^2 - ab\right]$$

→ or by adding

$$J_g = I_{xg} + I_{yg} = \frac{bh^3}{36} + \frac{bh}{36}\left(b^2 - ab + a^2\right)$$

$$J_g = \frac{bh}{36}\Big(\underbrace{h^2}_{I_{xg}} + \underbrace{b^2 - ab + a^2}_{I_y\ c.g}\Big)$$