EXAMPLE 2.11 College Savings Plan: Find *A*, Given *F*, *N*, and *i*

You want to set up a college savings plan for your daughter. She is currently 10 years old and will go to college at age 18. You assume that when she starts college, she will need at least $100,000 in the bank. How much do you need to save each year in order to have the necessary funds if the current rate of interest is 7%? Assume that end-of-year deposits are made.

DISSECTING THE PROBLEM

Given: Cash flow diagram in Figure 2.18; $i = 7\%$ per year, and $N = 8$ years.
Find: A.

METHODOLOGY

Method

SOLUTION

ıking-fund factors, we obtain

$$A = \$100{,}000(A/F, 7\%, 8)$$
$$= \$9{,}746.78.$$

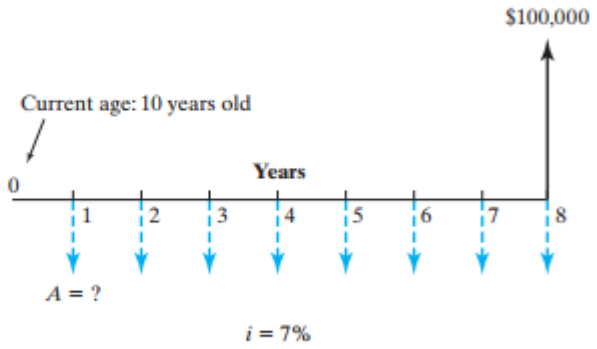

Figure 2.18 Cash flow diagram.

$i = 7\%$ $F = \$100{,}000$
$N = 8$ years

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$F = \$100{,}000$

$i = 7\%$

$n = 8$

Find A ?

$$-A(1+i)^{-1} - A(1+i)^{-2} - A(1+i)^{-3} - A(1+i)^{-4} - A(1+i)^{-5} - A(1+i)^{-6} - A(1+i)^{-7} + 100{,}000(1+i)^{-8} = 0$$

Readjust $\Rightarrow$ -8

$$100{,}000(1.07) = A\left[(1.07)^{-1} + 1.07^{-2} + 1.07^{-3} + 1.07^{-4} + 1.07^{-5} + 1.07^{-6} + 1.07^{-7} + 1.07^{-8}\right]$$

Pivot Point $t=0$

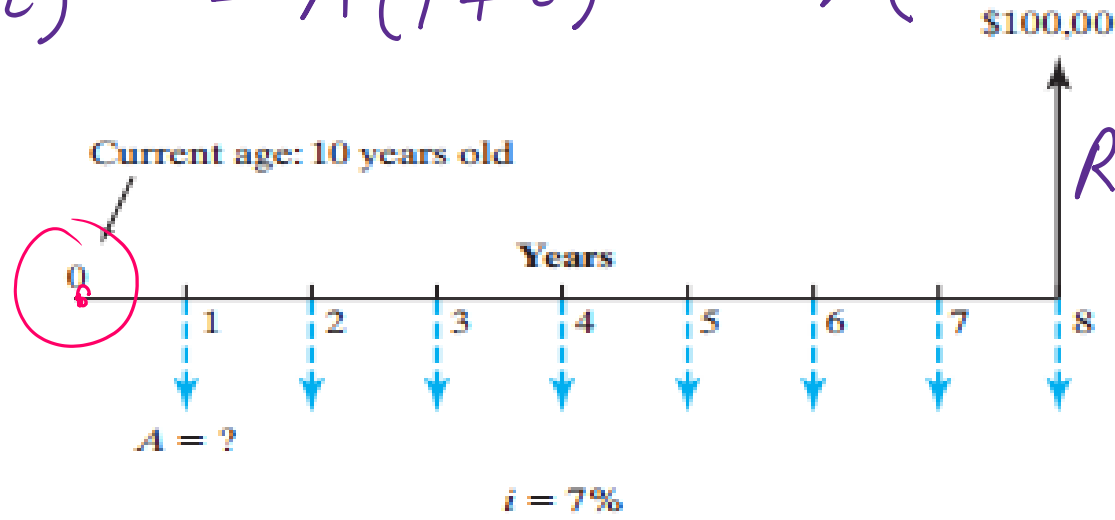


Figure 2.18 Cash flow diagram.

$$A(5.97129) = \frac{100{,}000}{(1.7181)} \Rightarrow A = \frac{100{,}000}{10.2598}$$

$\therefore$ $\$9746.78$

Chan. S. Park : Fundamentals of engineering Economics.

$F = \$100,000$

$i = 7\%$

$n = 8$

Find A ?

Engineering Economics

Factor Name	Converts	Symbol	Formula
Single Payment Compound Amount	to F given P	$(F/P, i\%, n)$	$(1+i)^n$
Single Payment Present Worth	to P given F	$(P/F, i\%, n)$	$(1+i)^{-n}$
Uniform Series Sinking Fund	to A given F	$(A/F, i\%, n)$	$\frac{i}{(1+i)^n - 1}$

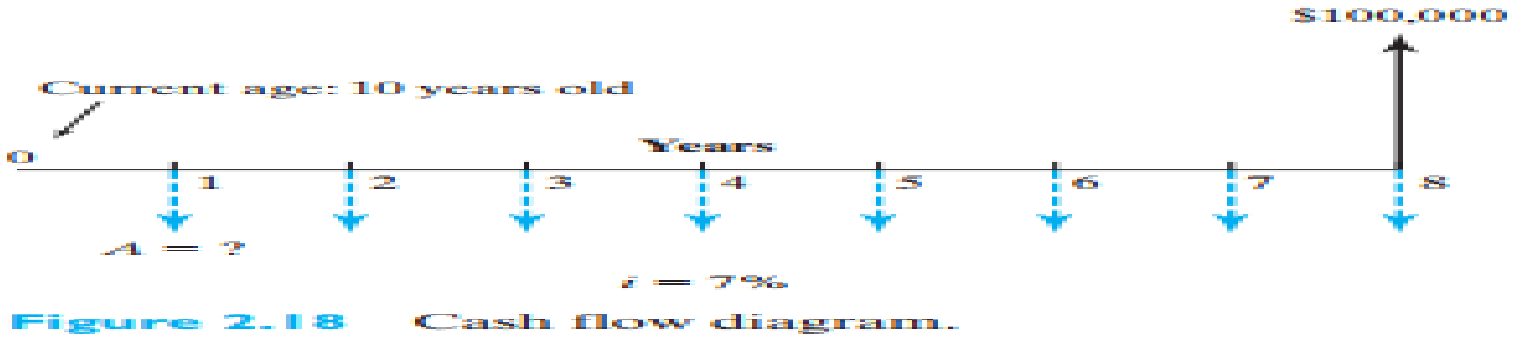


Figure 2.18 Cash flow diagram.

$$\frac{A}{F} = \left[\frac{i}{(1+i)^n - 1}\right] = \frac{0.07}{(1.07)^8 - 1} = 0.09746$$

$$A = F\left(\frac{A}{F}, 7\%, 8\right) = 100,000(0.09746) =$$

$$A = \$9746.78$$

→ From table $i = 7\%$

7% **Compound Interest Factors** 7%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
n	Compound Amount Factor Find *F* Given *P* *F/P*	Present Worth Factor Find *P* Given *F* *P/F*	Sinking Fund Factor Find *A* Given *F* *A/F*	Capital Recovery Factor Find *A* Given *P* *A/P*	Compound Amount Factor Find *F* Given *A* *F/A*	Present Worth Factor Find *P* Given *A* *P/A*	Gradient Uniform Series Find *A* Given *G* *A/G*	Gradient Present Worth Find *P* Given *G* *P/G*	n
1	1.070	.9346	1.0000	1.0700	1.000	0.935	0	0	1
2	1.145	.8734	.4831	.5531	2.070	1.808	0.483	0.873	2
3	1.225	.8163	.3111	.3811	3.215	2.624	0.955	2.506	3
4	1.311	.7629	.2252	.2952	4.440	3.387	1.416	4.795	4
5	1.403	.7130	.1739	.2439	5.751	4.100	1.865	7.647	5
6	1.501	.6663	.1398	.2098	7.153	4.767	2.303	10.978	6
7	1.606	.6227	.1156	.1856	8.654	5.389	2.730	14.715	7
→ 8	1.718	.5820	(.0975)	.1675	10.260	5.971	3.147	18.789	8
9	1.838	.5439	.0835	.1535	11.978	6.515	3.552	23.140	9
10	1.967	.5083	.0724	.1424	13.816	7.024	3.946	27.716	10

given: $i = 7\%$, given F, given n.

need $\frac{A}{F}$?? ⟹ 0.0975 ≈ 0.09746

Formula

Method 2: Excel's PMT Function

Or, using the Excel's built-in financial function, we get the same result:

$$= \text{PMT}(7\%, 8, 0, 100000)$$
$$= -\$9{,}746.78.$$

Using PMT built in Excel

Excel PMT function - syntax and basic uses

The PMT function has the following arguments:

```
PMT(rate, nper, pv, [fv], [type])
```

i ⇒ C19

Excel built in PMT

C19

C21

rate of interest	7%
n-periods	8
F-future value	100000
A value	($9,746.78)

C20

- **Nper** (required) - the number of payments for the loan, i.e. the total number of periods over which the loan should be paid.

  For example, if you make **annual** payments on a 5-year loan, supply 5 for nper. If you make **monthly** payments on the same loan, then multiply the number of years by 12, and use 5*12 or 60 for nper.

- **Pv** (required) - the present value, i.e. the total amount that all future payments are worth now. In case of a loan, it's simply the original amount borrowed.
- **Fv** (optional) - the future value, or the cash balance you wish to have after the last payment is made. If omitted, the future value of the loan is assumed to be zero (0).
- **Type** (optional) - specifies when the payments are due:
  - 0 or omitted - payments are due at the end of each period.
  - 1 - payments are due at the beginning of each period.

$9,746.78

result

```
=PMT(C19,C20,0,+C21)
```

Prepared by Eng.Maged Kamel.

EXAMPLE 2.12 Paying Off an Educational Loan: Find *A*, Given *P*, *i*, and *N*

You borrowed $21,061.82 to finance the educational expenses for your senior year of college. The loan will be paid off over five years. The loan carries an interest rate of 6% per year and is to be repaid in equal annual installments over the next five years. Assume that the money was borrowed at the beginning of your senior year and that the first installment will be due a year later. Compute the amount of the annual installments (Figure 2.20).

$n = 5$ years

$P = \$21061.82$

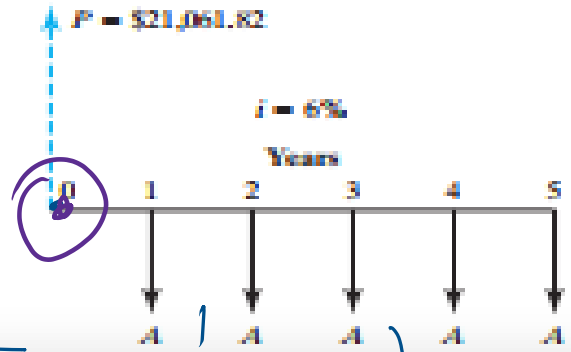
$i = 6\%$

As moment – pivot point time 0

DISSECTING THE PROBLEM

Given: $P = \$21,061.82$, $i = 6\%$ per year, and $N = 5$ years.

Find: A.



$$P(1+i)^0 - A(1+i)^{-1} - A(1+i)^{-2} - A(1+i)^{-3} - A(1+i)^{-4} - A(1+i)^{-5} = 0$$

$$P = A\left(\frac{1}{1.06} + \frac{1}{(1.06)^2} + \frac{1}{(1.06)^3} + \frac{1}{(1.06)^4} + \frac{1}{(1.06)^5}\right)$$

$$A(2.1236) = 21061.82 \Rightarrow A = \$5000$$

EXAMPLE 2.12 Paying Off an Educational Loan: Find A, Given P, i, and N

You borrowed $21,061.82 to finance the educational expenses for your senior year of college. The loan will be paid off over five years. The loan carries an interest rate of 6% per year and is to be repaid in equal annual installments over the next five years. Assume that the money was borrowed at the beginning of your senior year and that the first installment will be due a year later. Compute the amount of the annual installments (Figure 2.20).

$n = 5$ years

$P = \$21,061.82$

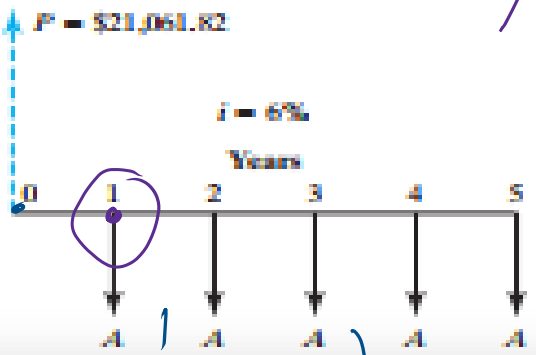
$i = 6\%$

alternative

As moment - pivot point time $\Rightarrow t = 1$

DISSECTING THE PROBLEM

Given: $P = \$21,061.82$, $i = 6\%$ per year, and $N = 5$ years.
Find: A.



$P(1+i)^0 - A(1+i)^{-1} - A(1+i)^{-2} - A(1+i)^{-3} - A(1+i)^{-4} - A(1+i)^{-5} = 0$

$P(1+i)^1 - A(1+i)^0 - A(1+i)^{-1} - A(1+i)^{-2} - A(1+i)^{-3} - A(1+i)^{-4} = 0$

$P = A\left(\frac{1}{1.06} + \frac{1}{(1.06)^2} + \frac{1}{(1.06)^3} + \frac{1}{(1.06)^4} + \frac{1}{(1.06)^5}\right)$

$P(1.06) = A\left[\left(1 + \frac{1}{1.06}\right) + \frac{1}{(1.06)^2} + \frac{1}{(1.06)^3}\right]$

$A(2.1236) = 21061.82 \Rightarrow A = \$5000$

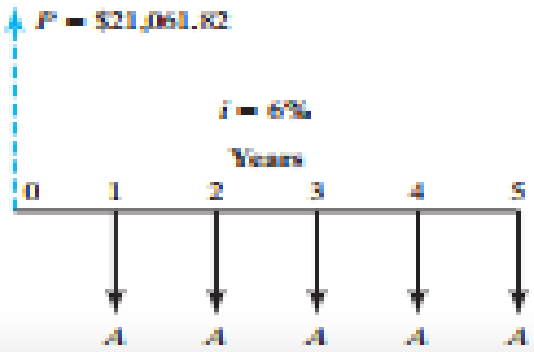
$A = \frac{21061.82}{4.465} = \$5000$

EXAMPLE 2.12 Paying Off an Educational Loan: Find *A*, Given *P*, *i*, and *N*

You borrowed $21,061.82 to finance the educational expenses for your senior year of college. The loan will be paid off over five years. The loan carries an interest rate of 6% per year and is to be repaid in equal annual installments over the next five years. Assume that the money was borrowed at the beginning of your senior year and that the first installment will be due a year later. Compute the amount of the annual installments (Figure 2.20).

DISSECTING THE PROBLEM

Given: $P = \$21{,}061.82$, $i = 6\%$ per year, and $N = 5$ years.
Find: A.



$$A = P\left(\frac{A}{P}, i\%, n\right)$$

$$A = 21{,}061.82\left(\frac{A}{P}, 6\%, 5\right)$$

$$\frac{A}{P} = (0.06)(1.06)^5 / [(1.06)^5 - 1] = \frac{0.0803}{0.3382} = 0.2374$$

$$\Rightarrow A = 21{,}061(0.2374) = \$5000$$

A/P	$\frac{i(1+i)^n}{(1+i)^n - 1}$