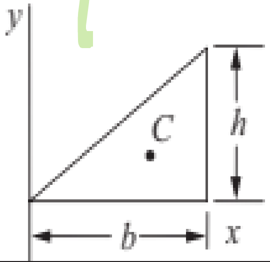
Case - 2

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = bh/2$ $x_c = 2b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/4$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/2$	$I_{x_c y_c} = Abh/36 = b^2h^2/72$ $I_{xy} = Abh/4 = b^2h^2/8$

	Product of Inertia
	$I_{x_c y_c} = Abh/36 = b^2h^2/72$ $I_{xy} = Abh/4 = b^2h^2/8$

Ixy at cg

Exterior axis

First method — Product of inertia

Use → a VL strip

Case - 2 : where is the opposite side is at the right

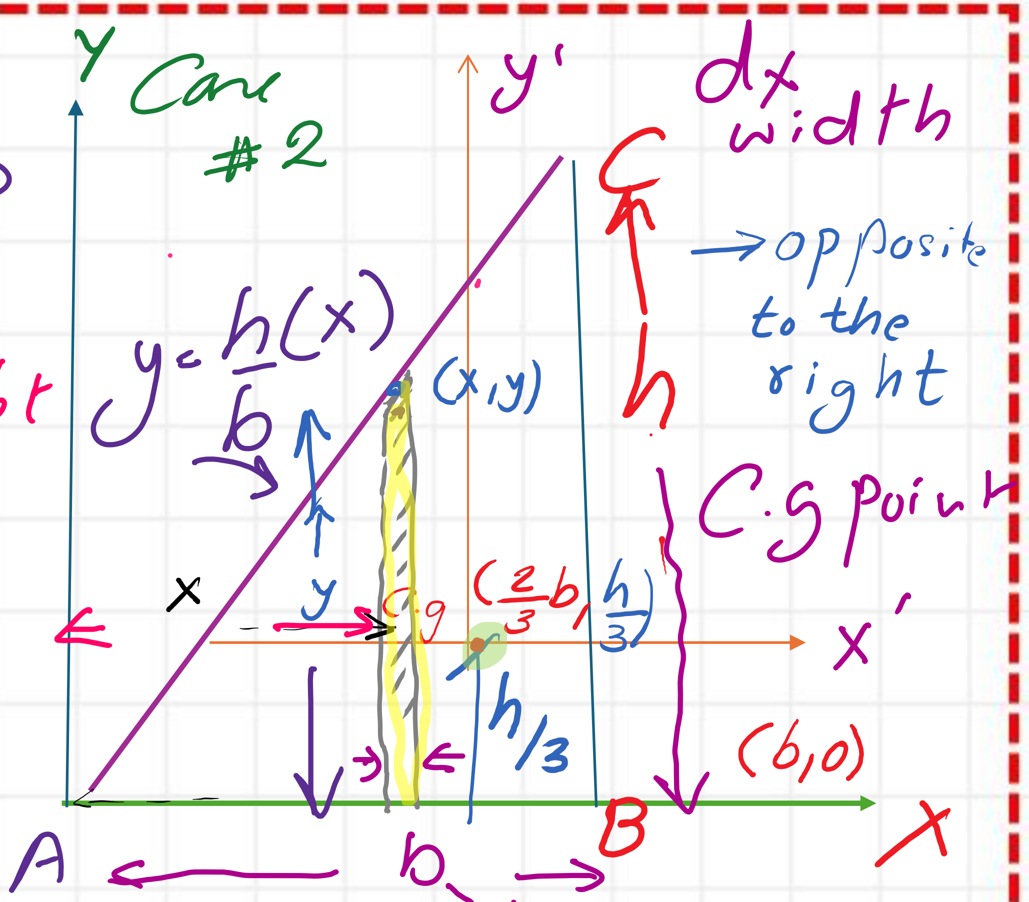
Equation for the inclined line is $y = \frac{h}{b} x$

Strip width $= dx$, $ht = y$

$$dI_{xy} = dA\,(x)\left(\overset{c.g}{\frac{y}{2}}\right) = \frac{xy}{2}\,dA$$

bwt $y = \frac{h}{b} x$; $dI_{xy} = \frac{x}{2}\, dx \left(\frac{h^2}{b^2}\right) x^2$

$$I_{xy} = \int_0^b \frac{x^3}{2}\left(\frac{h^2}{b^2}\right) dx = \frac{h^2}{2b^2}\, \frac{x^4}{4}\Big|_0^b = \frac{h^2}{2b^2}\left(\frac{b^4}{4}\right) = \frac{h^2 b^2}{8}$$



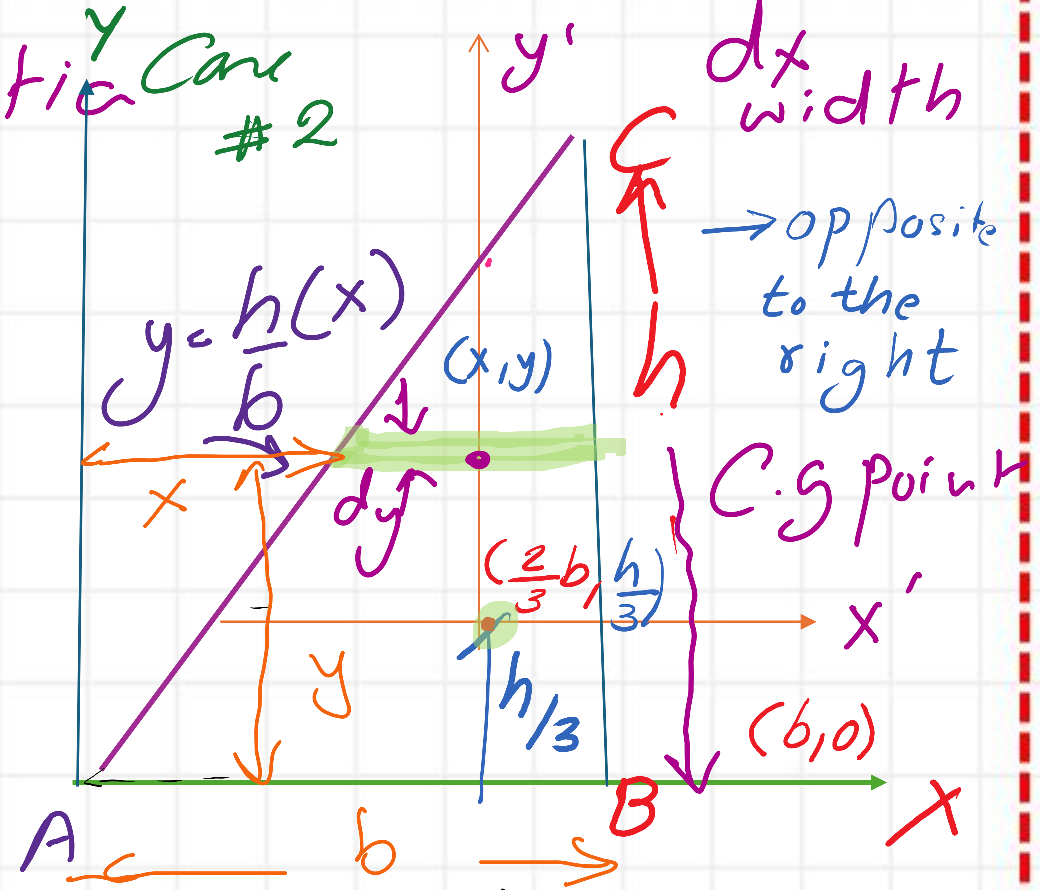
x : hL distance to Y-axis

Prepared by Eng.Maged Kamel.

2nd method: Product of inertia

Use a hL strip

Case 2 — hL strip, width dx, ht y from base



$$dI_{xy} = dA(x)(y)$$

$$dA = (b-x)dy \quad x : \left(x + \frac{(b-x)}{2}\right)$$

$$y : y$$

$$y = \frac{h}{b}(x) \Rightarrow \left(x = \frac{by}{h}\right)$$

$$dI_{xy} = (b-x)dy\left(\frac{b+x}{2}\right)(y).$$

$$x^2 = \frac{b^2y^2}{h^2}$$

$$dI_{xy} = \frac{1}{2}(b^2 - x^2)y\,dy = \frac{1}{2}\left(b^2 - \frac{b^2y^2}{h^2}\right)y\,dy$$

$$\int dI_{xy} = \int \frac{1}{2}b^2y\,dy - \int \frac{b^2}{2h^2}y^3dy$$

$\underbrace{I_{xy}}$ Using a HL strip

$$\int dI_{xy} = \int_0^h \frac{1}{2} b^2 y \, dy - \int_0^h \frac{b^2}{2h^2} y^3 \, dy$$

$$I_{xy} = \frac{b^2}{2} \frac{y^2}{2} \Bigg|_0^h - \frac{b^2}{2h^2} \frac{y^4}{4} \Bigg|_0^h$$

$$I_{xy} = \frac{b^2}{4} h^2 - \frac{b^2}{2h^2} \left(\frac{h^4}{4} \right) = b^2 h^2 \left(\frac{1}{4} - \frac{1}{8} \right)$$

$$I_{xy} = b^2 h^2 \left(\frac{1}{8} \right) \Longrightarrow$$ Same result as by using VL strip

I_{xy} right angle

Case - 2

Product of inertia

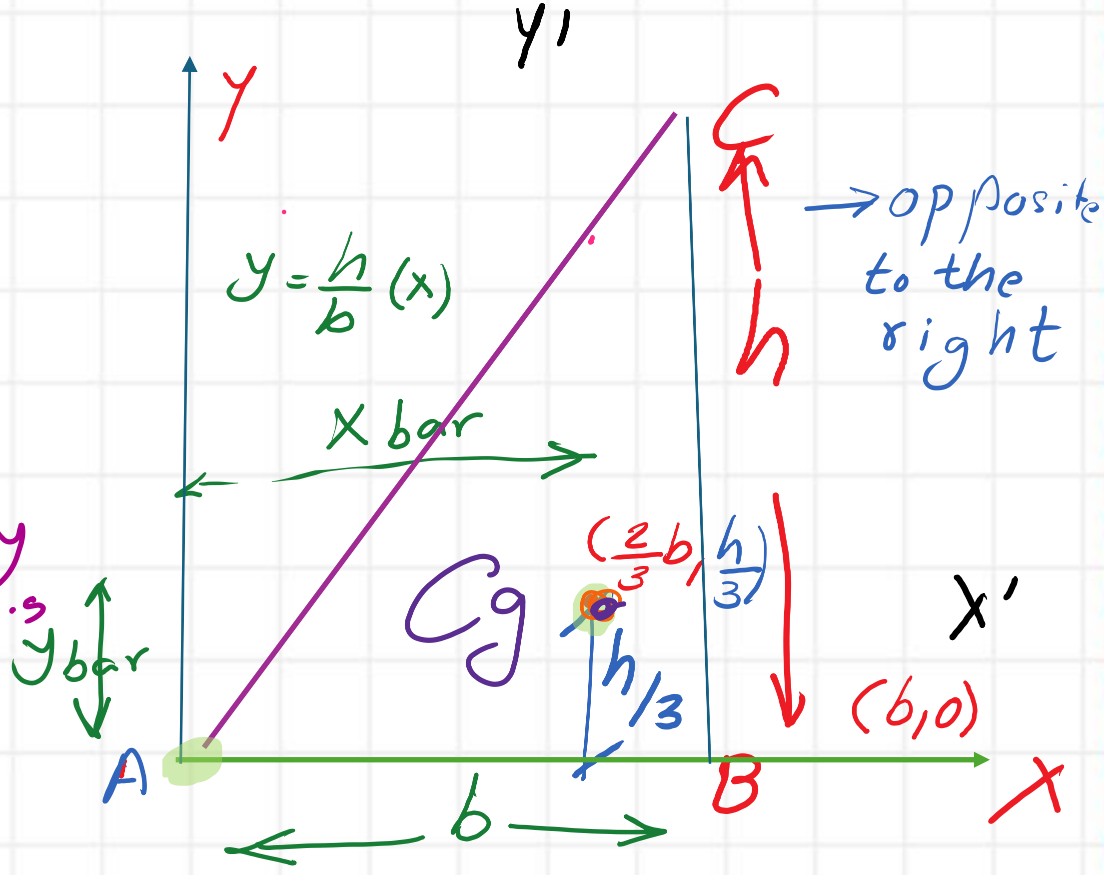
$I_{xy} = \frac{b^2h^2}{8}$ at Point A

$I_{xy_{c.g}}$ use $I_{xy_{cg}} = I_{xy} - A\,x_{c.g}\cdot y_{c.g}$

$x_{cg\,bar} = \frac{2}{3}b, \quad y_{cg} = \frac{h}{3}$

$$I_{xy_{cg}} = \frac{b^2h^2}{8} - \frac{1}{2}(bh)\left(\frac{2}{3}b\right)\left(\frac{h}{3}\right)$$

$$= b^2h^2\left(\frac{1}{8} - \frac{1}{9}\right) = \frac{b^2h^2}{72}(1)$$



Cg Coordinate

$\left(\frac{2}{3}b, \frac{h}{3}\right)$