## ECONOMIC EQUIVALENCE VIEWED AS A MOMENT DIAGRAM

The similarity between cash flow and free body diagrams allows an analogy that helps some students better understand economic equivalence.[2] Think of the cash flows as forces that are always perpendicular to the axis. Then the time periods become the distances along the axis.

When we are solving for unknown forces in a free body diagram, we know that a moment equation about any point will be in equilibrium. Typically moments are calculated by using the right-hand rule so that counterclockwise moments are positive, but it is also possible to define moments so that a clockwise rotation is positive. We are assuming that **clockwise rotations are positive**. This allows us to make the normal assumptions that positive forces point up and positive distances from the force to the pivot point are measured from left to right. Thus, negative forces point down, and negative distances are measured

# ENGINEERING ECONOMIC ANALYSIS

ELEVENTH EDITION

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# Four Cases For Forces and distance relation.

from the force on the right to the pivot point on the left. These assumptions are summarized in the following diagram.

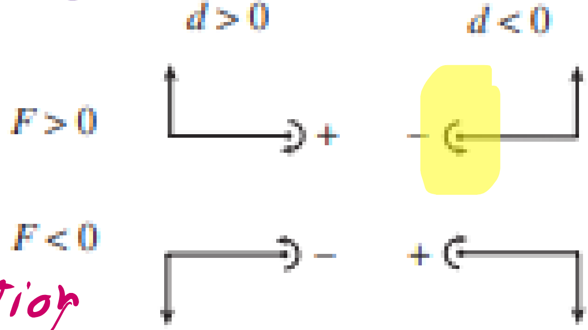
Point to the right d is +ve

① upword Force is +ve

(+)(+) = +

Moment is +ve

Clock wise rotation




Force is +ve
distance is -ve

Moment is -ve

anti-clock rotation

With these assumptions we can write force moment equations at equilibrium for the following diagram.

Force is downwards (-ve)

distance is +ve

Moment will have (−)(+) = (−) negative sign

Force is downwards

F is -ve

distance is to the Left of the Force → (−)

Moment is (−)(−) = +ve

# Cash Flow moment equation

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To write the cash flow moment equation for the cash flow diagram we need:

1. A sign convention for cash flows, such that positive values point up.
2. A way to measure the moment arm for each cash flow. This moment arm must be measured as $(1 + i)^T$, where $T$ is the number of periods measured *from* the cash flow *to* the pivot point or axis of rotation.
   a. Thus the sign of the distance is moved to the exponent.
   b. For cash flows at the pivot point, $T = 0$, and $(1 + i)^0 = 1$.

+ve values are up

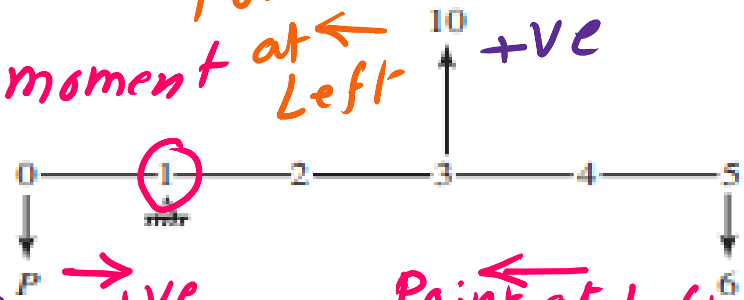
moment arm $= (1+i)^n$

$n = 0$ at the same point

We can redraw the simple example with an unknown present cash flow, $P$.

Taking Cash Flow moment at point ①

Point at Left




+ve

-ve +ve

Point at left

-ve

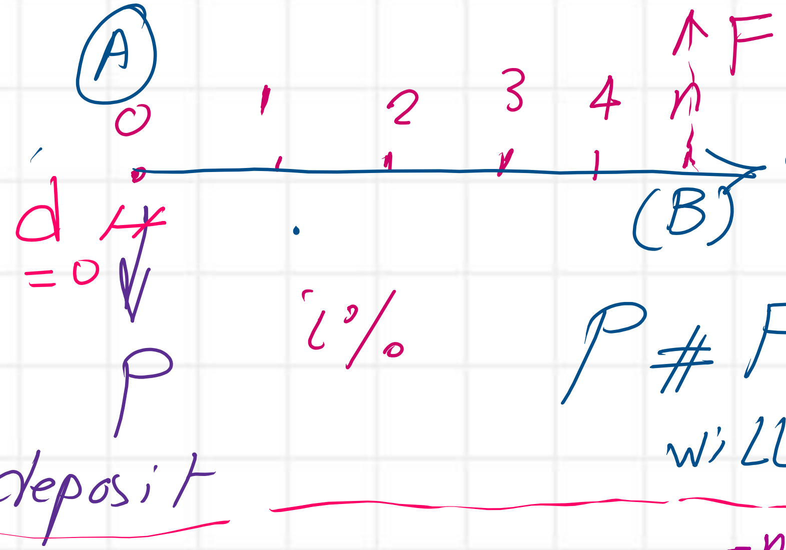
$P(-)$

$\Sigma M = 0$ at point ①

Since $P$ is drawn as a negative cash flow, we put a minus sign in front of it when we write the cash flow moment equation. To rewrite the cash flow moment equation at Year 1, we use the distances from the diagram as the exponents for $(1 + i)^t$:

$$0 = -P \times (1 + i)^1 + 10 \times (1 + i)^{-2} + -6 \times (1 + i)^{-4}$$

We can get the relation between P & Future value
P.V F.V
as Follows:




time Line in years

P is assumed at the right

P # F as Forces, but we will Take moment at Point (B)

$$\sum M_A = 0 \qquad -P(1+i)^0 + F(1+i)^{-n} = 0$$

$$P(1) = \frac{F}{(1+i)^n} \Rightarrow \text{Known Formula}$$

P: is downward Force ⇒ (−P)
F: is upward Force (+F)
A : at distance 0 → From P → A at distance (−n) at Left of F
B : at distance n → From P B at distance (0) From F

**Prepared by Eng.Maged Kamel.**

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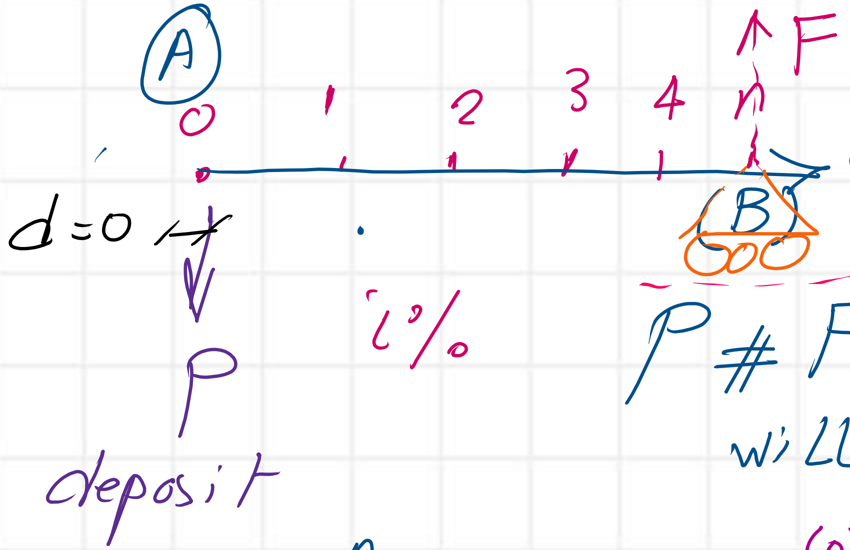
P.V F.V

as Follows:

Cash Flow
- moment




time Line in years

F is assumed to the Left

P # F as Forces, but we will Take moment at Point B

$\sum M_B = 0$

$$-P(1+i)^{n} + F(1+i)^{(0)} = 0$$

$$P(1+i)^n = F(1) \Rightarrow \boxed{F = P(1+i)^n}$$

P: is downward Force ⇒ (−P)

F: is upward Force (+F)

A: at distance 0 → From P → A at distance (-n) at Left of F

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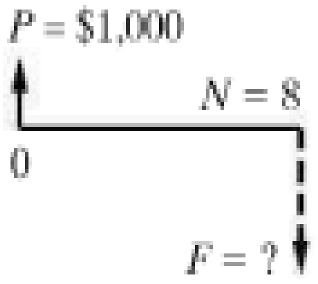
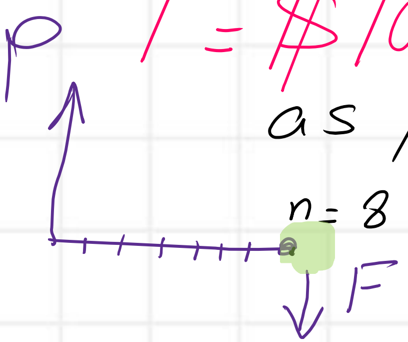
## TABLE 4-2 Discrete Cash-Flow Examples Illustrating Equivalence

Example Problems (All Using an Interest Rate of $i = 10\%$ per Year—See Table C-13 of Appendix C)

| To Find: | Given: | (a) In Borrowing–Lending Terminology: | (b) In Equivalence Terminology: | Cash-Flow Diagram[a] | Solution |
|---|---|---|---|---|---|
| *For single cash flows:* | | | | | |
| $F$ | $P$ | A firm borrows \$1,000 for eight years. How much must it repay in a lump sum at the end of the eighth year? | What is the future equivalent at the end of eight years of \$1,000 at the beginning of those eight years? |   | $F = P(F/P, 10\%, 8)$<br>$= \$1{,}000(2.1436)$<br>$= \$2{,}143.60$ |

$P = \$1000,\ i = 10\%,\ n = 8$. Taking moment at $n = 8$ as pivot point $\Rightarrow P(1+i)^n - F(1+i)^0 = 0$




$100(1+0.10)^8 - F(1.10)^0 = 0$

$F = 100(1.10)^8 / 1 = \$\ 2143.588$

**TABLE 4-2 Discrete Cash-Flow Examples Illustrating Equivalence**

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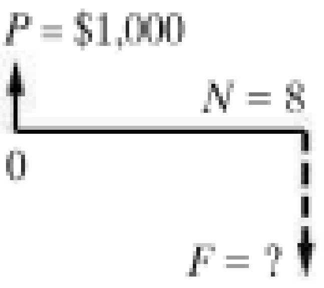
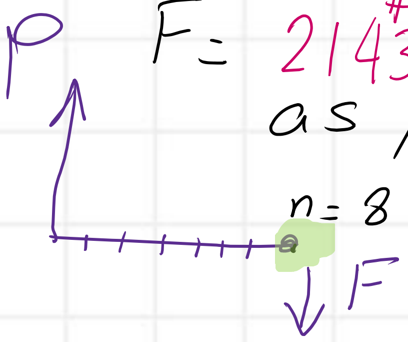
| $F$ | $P$ | A firm borrows \$1,000 for eight years. How much must it repay in a lump sum at the end of the eighth year? | What is the future equivalent at the end of eight years of \$1,000 at the beginning of those eight years? |   | $F = P(F/P, 10\%, 8)$ $= \$1{,}000(2.1436)$ $= \$2{,}143.60$ |




$F = \$2143.60$, $i = 10\%$, $n = 8$. Taking moment at $n = 8$ as pivot point $\Rightarrow P(1+i)^n - F(1+i)^0 = 0$

$P(1.10)^8 - 2143.60(1) = 0$

$P = 2143.60/(1.10)^8 = \$1000$