Objective of the lecture

1-Theory of pure bending, assumptions, and relations.

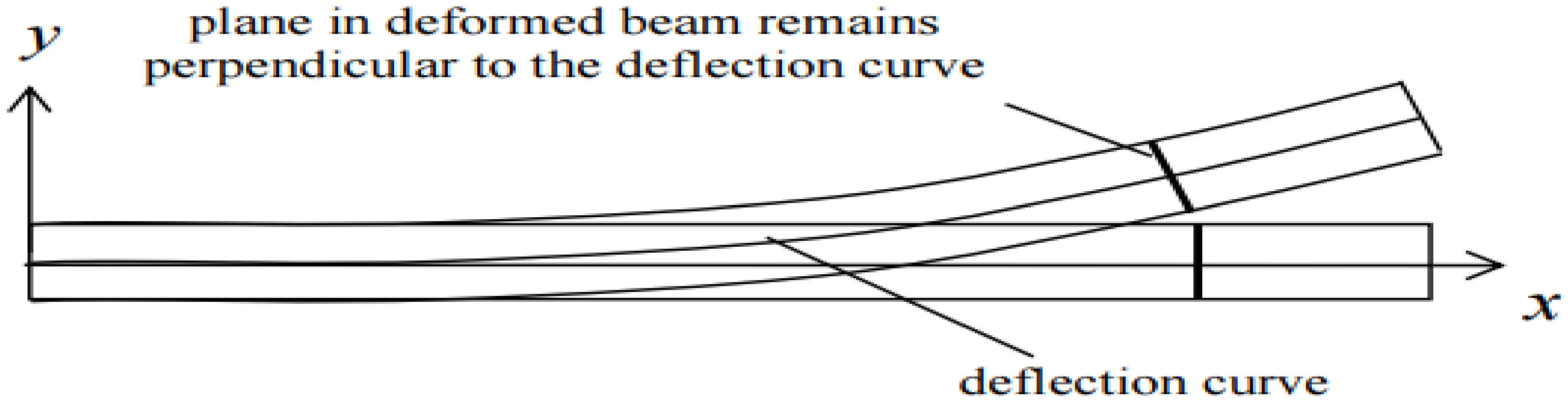
2-Why do we estimate the Product of Inertia?

3-Mohr circle of inertia.

Prepared by Eng.Maged Kamel.

Assumptions

1-Plain section perpendicular to the neutral axis before deformation, remains perpendicular to the neutral axis after deformation.

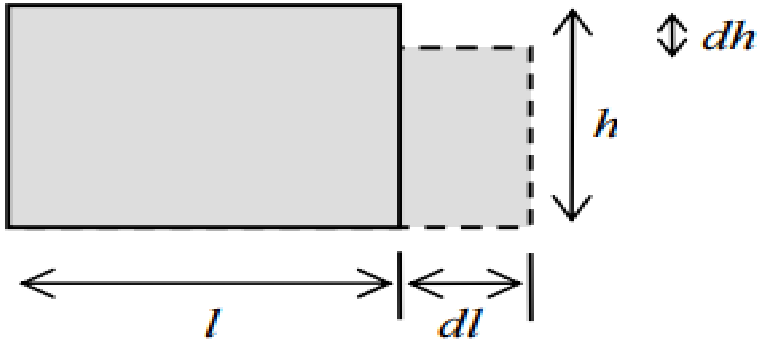


2-Deformation due to bending is small

3-The beam is initially straight and all longitudinal filaments bend into circular arcs with a common center of curvature.

Prepared by Eng.Maged Kamel.

4- Deformation in the vertical direction, i.e. the transverse strain ε_{yy}, may be neglected in deriving an expression for the longitudinal strain ε_{xx}. This assumption is summarised in the deformation shown in Fig. 7.4.20, which shows an element of length l and height h undergoing transverse and longitudinal strain.



$$\varepsilon_{xx} = \frac{dl}{l}, \quad \varepsilon_{yy} = -\frac{dh}{h} \approx 0$$

Figure 7.4.20: transverse strain is neglected in the elementary beam theory

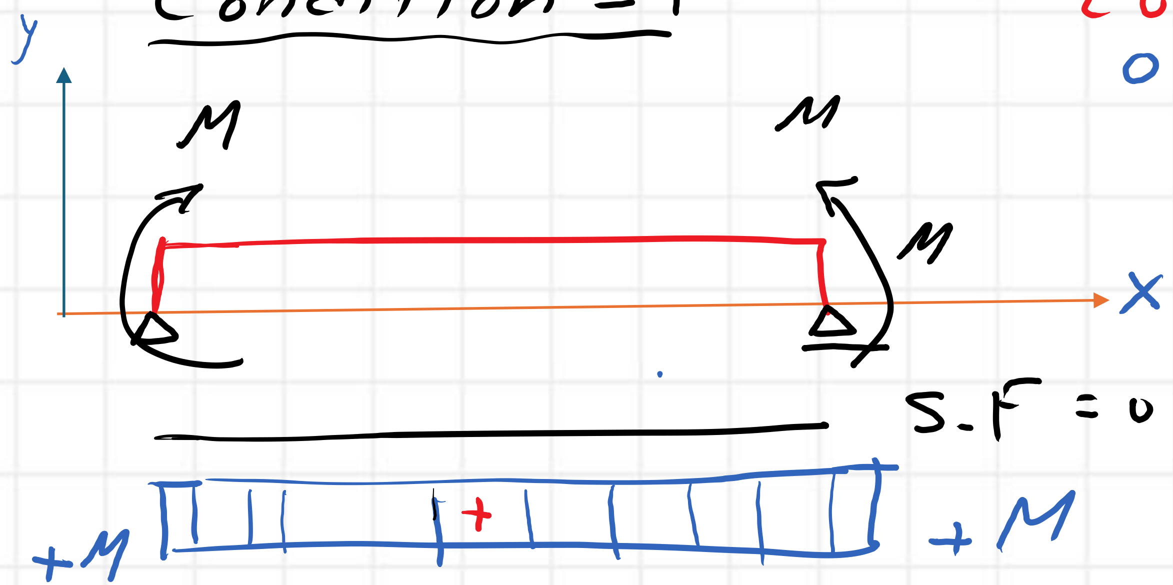
5-The radius of curvature is large as compared to the dimensions of the cross-section.

6-The value of Young's Modulus of Elasticity is same in tension and compression.

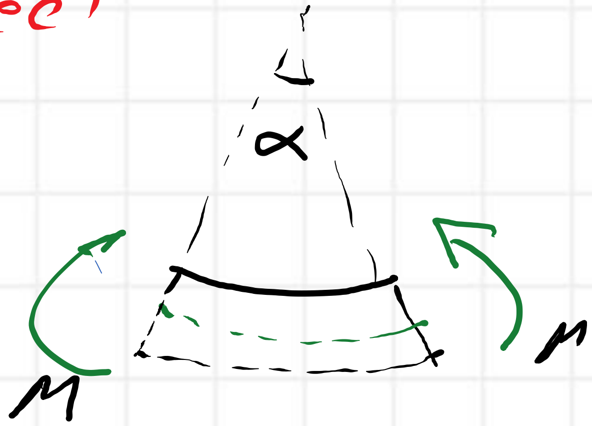
Prepared by Eng.Maged Kamel.

What is pure bending?

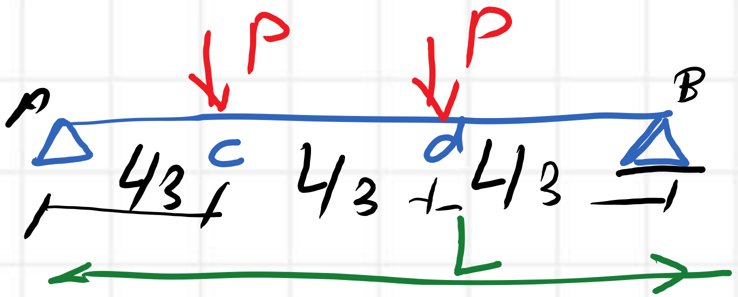
Condition - 1

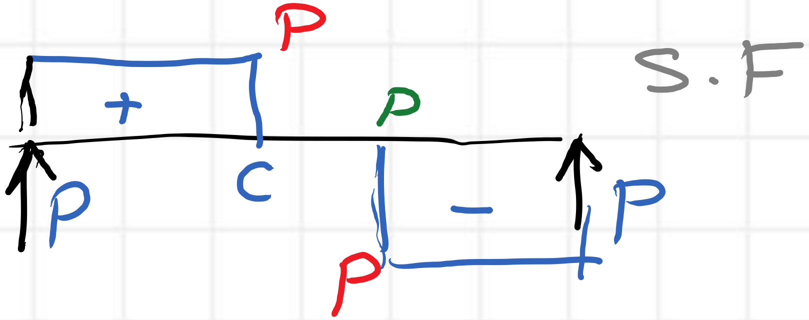


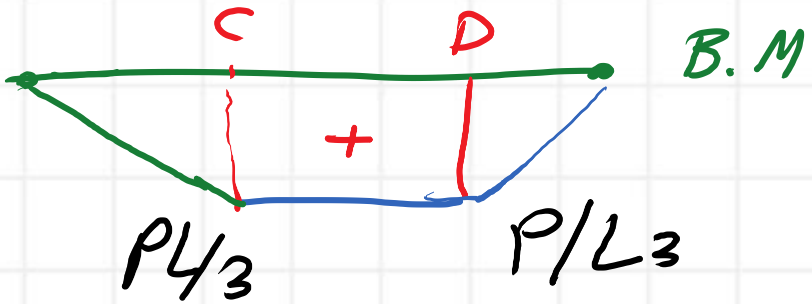
Beam subject to moment on both sides



Condition - 2







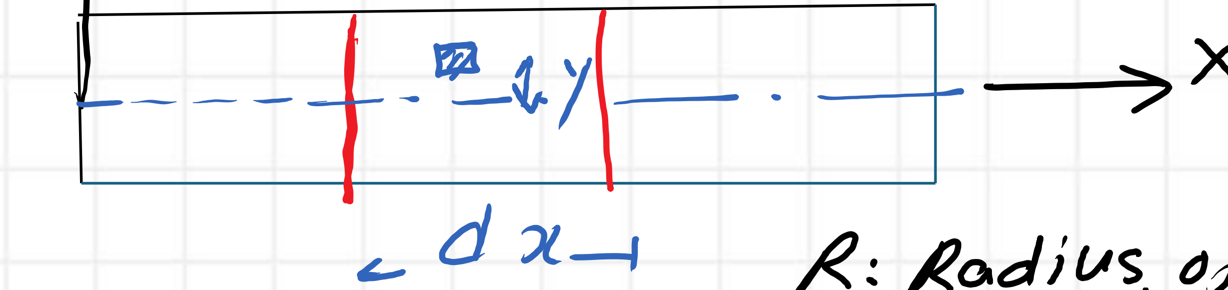
For part c-d

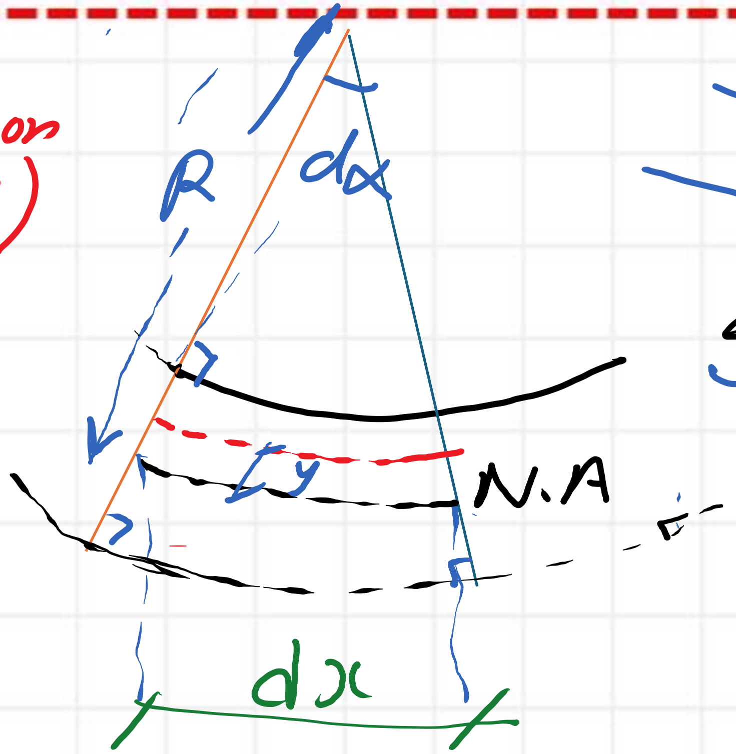
S.F = 0

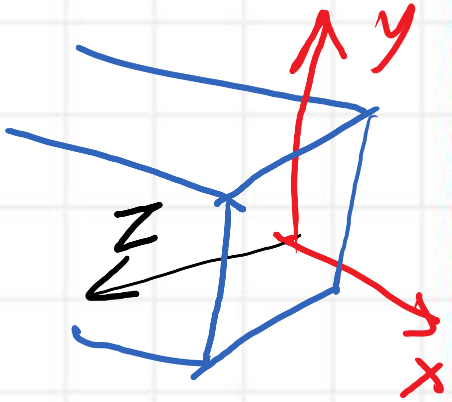
$M = Pl/3$ Pure moment

Prepared by Eng.Maged Kamel.

Beam intially straight (assumption #3)







R: Radius of curvature

90°

assumption #1

$dx = R\,d\alpha$ original length

Length at $y = (R-y)\,d\alpha$

original length $= dx = R\,d\alpha$ → y above x

Deformation $= (R-y)\,d\alpha - R\,d\alpha = d\alpha\,(R-y-R)$

Deformation $= -y\,d\alpha$

strain $= \dfrac{\text{deformation}}{\text{original length}} = -y\,d\alpha / R\,d\alpha = -y/R$

since $E = \dfrac{f}{\epsilon}$

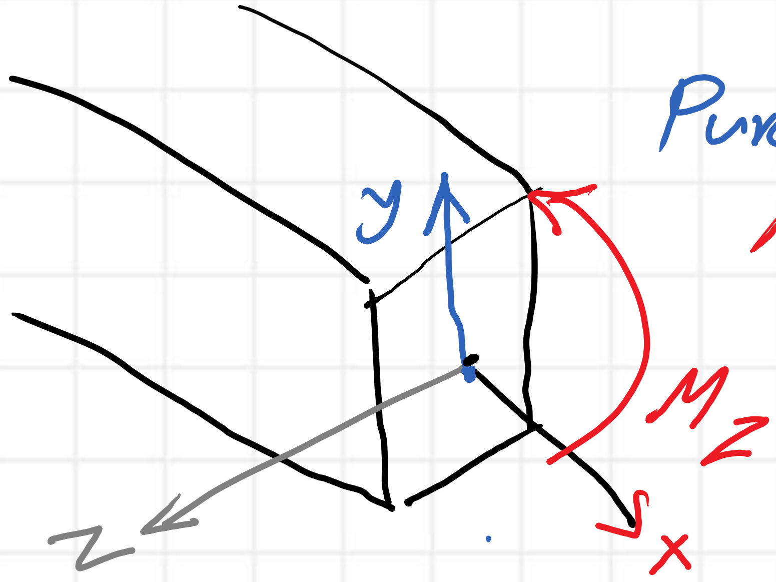
Prepared by Eng.Maged Kamel.

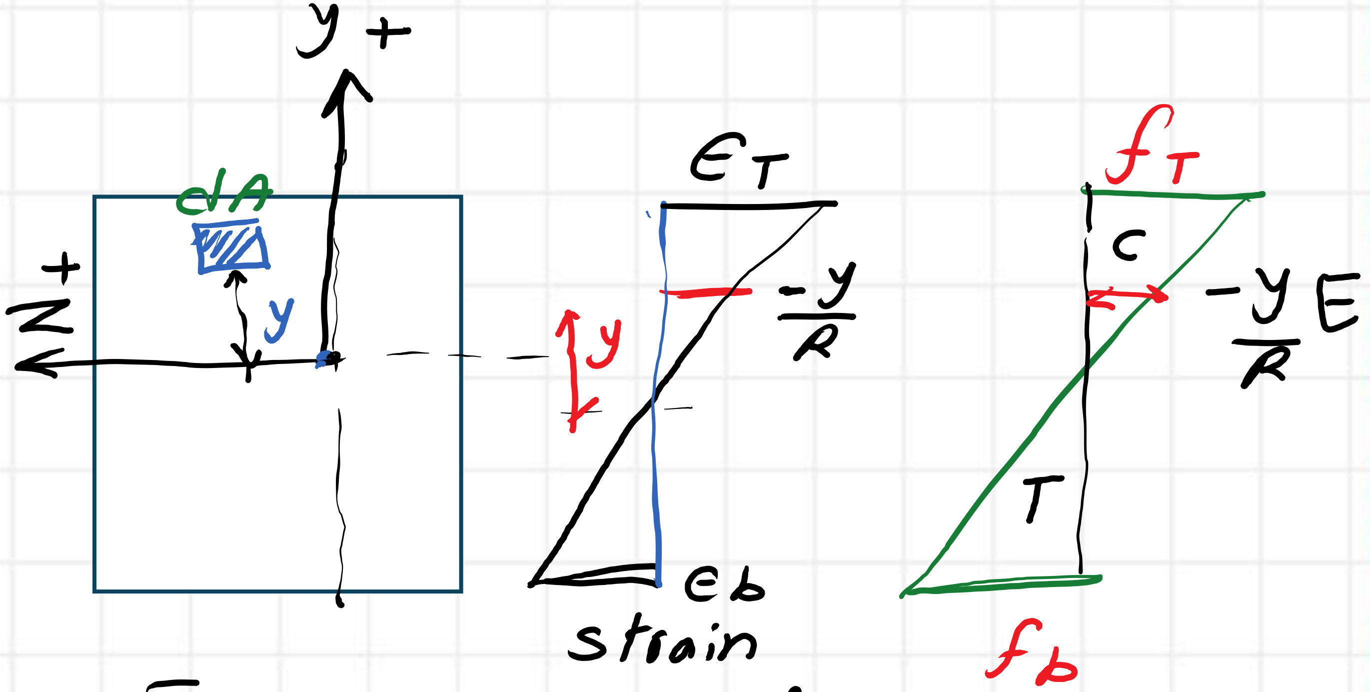
$$\boxed{f = \frac{-y}{R}\cdot E}$$

assumptions 4 & 6

Pure bending

Rotate around Z





$$\Sigma F = 0$$

$$\int dA \cdot f = 0$$

$$\int dA \cdot \left(-\frac{y}{R}E\right) = 0$$

Since $-\frac{E}{R} \neq 0$ $\quad \int dA \cdot y = 0$ First moment of Area = 0

N.A Passes by c.g

Prepared by Eng.Maged Kamel.