To understand the following points

1- Definitions.

2- Matrices properties.

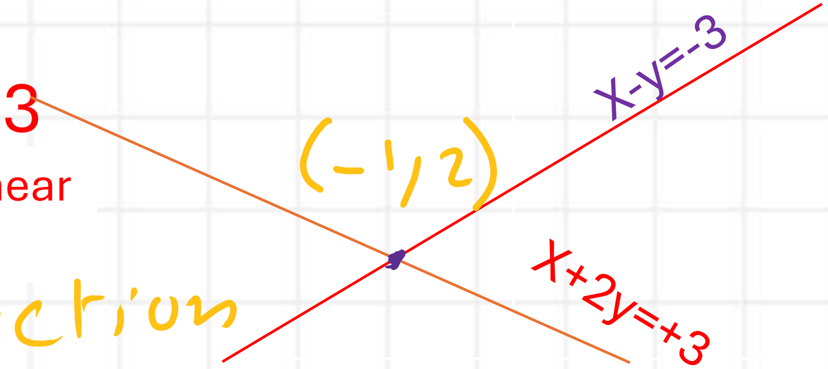
3- Gaussian elimination.

4-Crammer’s rule.

What is Matrix ? *Matrix is a rectangular array of numbers and bounded by the brackets*

Suppose we have $X - y = -3$ and $x + 2y = +3$

System of linear equations



To get the point of intersection

From the first Equation

$y = x + 3$

Substitute by the value of y, in the second Equation

Matrix (2x2) means two rows and two columns

We have $x + 2(x + 3) =$

$x + 2x + 6 = 3$

$3x = 3 - 6 = -3$

$x = -1$

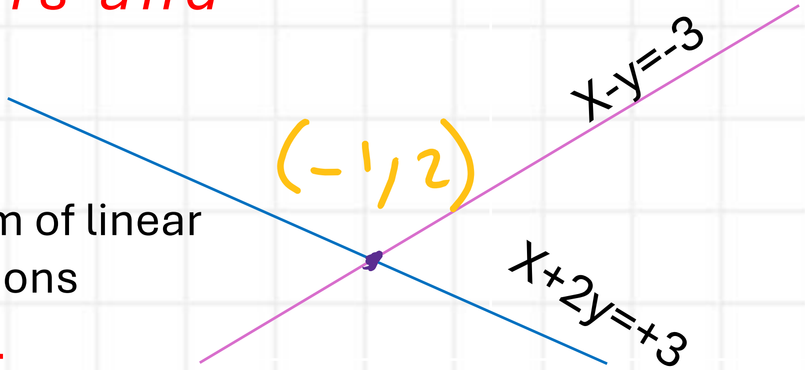
Back to

$y = x + 3$

What is Matrix ?

Matrix is a rectangular array of numbers and bounded by the brackets

Suppose we have X - y = - 3 & X +2 y =+ 3



The two lines can be written un the general form.

$a_1x + b_1y = c_1 \rightarrow$ First Line

$a_2x + b_2y = c_2 \rightarrow$ 2nd Line

Matrix (2x2) means two rows and two columns

a_1, a_2 Coefficients

$\left.\begin{matrix} c_1 \\ c_2 \end{matrix}\right\}$ Constants

For the first line: a1=1, b1=-1 and c1= -3

For the second line: a2=1, b2= 2 and c2= +3

Matrix is a rectangular array of numbers and bounded by the brackets

Matrix can be formed as follows

$AX = B$

$$\begin{pmatrix} 1 & +1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$$

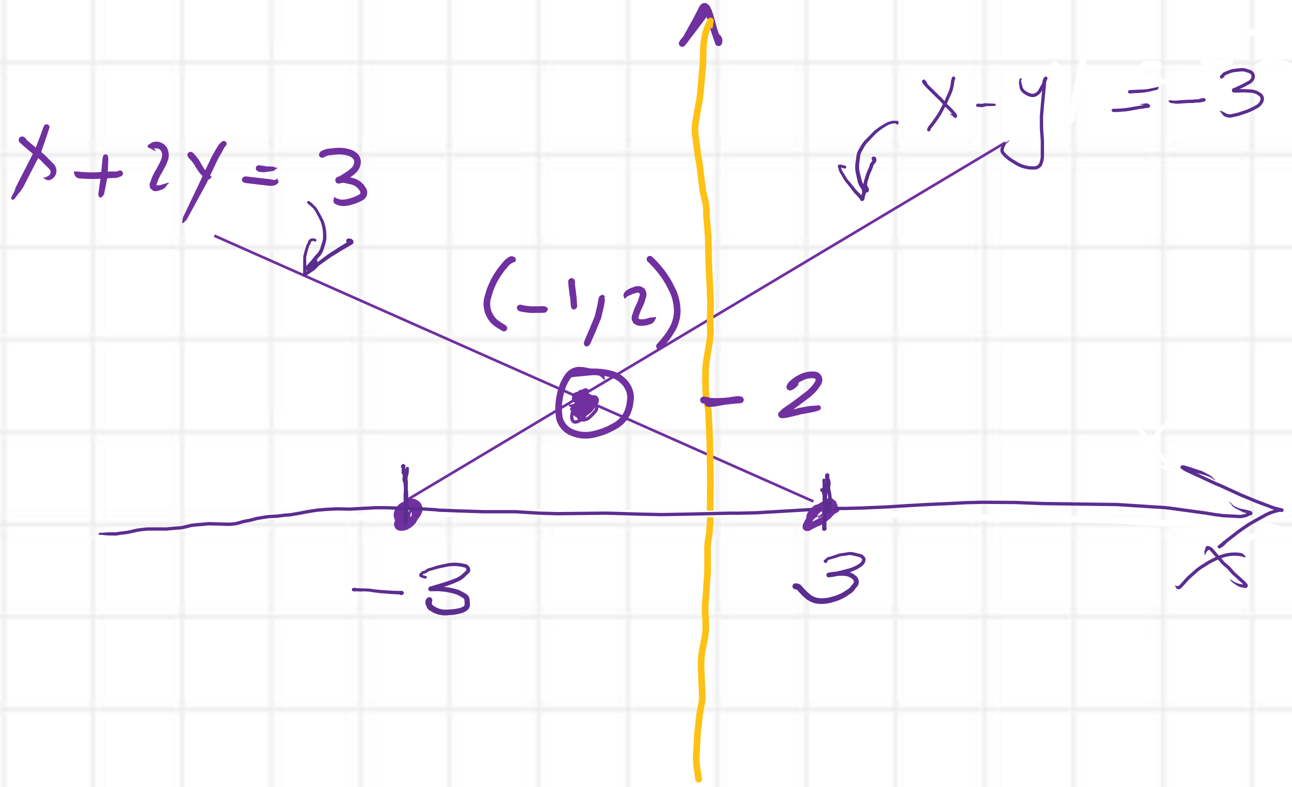
Constants

Matrix coefficient

Vector of column matrix

$x - y = -3$

$x + 2y = +3$



What is Augmented Matrix ?

Two Equations

$$x - y = -3 \quad \text{I}$$
$$x + 2y = 3 \quad \text{II}$$

$$\left(\begin{array}{cc|c} 1 & -1 & -3 \\ 1 & +2 & 3 \end{array}\right)$$

Coefficient of Variable at the Left

Vertical bar Seperator

Constants at the right

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \\ a_{m1} & a_{m2} & a_{m3} & a_{mn} \end{pmatrix}$$

matrix m rows by n Column

$2 \times 2 \Rightarrow$ Two rows Two Columns

Square matrix where Number of rows = Number of Columns

represented as $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}_{2\times 2}$

Row matrix

$A = \begin{pmatrix} 2 & 6 & 7 & 9 \end{pmatrix} (1 \times 4)$

Null or Zero matrix

Matrix with all elements are zeros

$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{2\times 3}$

Diagonal matrix

Matrix with all non diagonal are zeros.

$\begin{pmatrix} 3 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & 2 \end{pmatrix}_{3\times 3}$

Scalar matrix

Matrix with all diagonals are a scalar

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}_{3\times 3}$$

Scalar = 2

Symmetric matrix

Matrix with all values of I,j

aij=aji

$a_{21} = a_{12}$

$$\begin{pmatrix} 2 & -3 & 5 \\ -3 & 6 & 7 \\ 5 & 7 & -8 \end{pmatrix}$$

$a_{21} = -3 = a_{12}$

$a_{31} = 5 = a_{13}$

$a_{32} = 7 = a_{23}$

Skew Symmetric matrix

a21 =-a12

a23 =-a32

$$\begin{pmatrix} 2 & 3 & -5 \\ -3 & 6 & +7 \\ 5 & 7 & 8 \end{pmatrix}$$

$a_{21} = -3 = -a_{12}$

$a_{31} = 5 = -a_{13}$

$a_{32} = -7 = -a_{23}$

Unit matrix-I

Matrix with all diagonals are unity and other elements are zeros

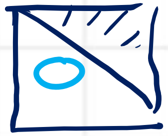
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Triangular matrix (Echelon form)

Matrix consists of one of the two forms

Upper Triangular

$$U = \begin{pmatrix} 1 & 3 & 2 \\ 0 & 4 & 1 \\ 0 & 0 & 6 \end{pmatrix} \text{ (3x3)}$$



$$\begin{pmatrix} a_{ij} & \text{for } i < \text{or } = J \\ 0 & \text{for } I > J \end{pmatrix}$$

Value

$$L = \begin{pmatrix} 2 & 0 & 0 \\ 4 & 2 & 0 \\ 5 & 8 & 2 \end{pmatrix}$$

Lower Triangular

a_{ij} For $i \leq J$ $= 0$

$i > y \rightarrow$ Value

$a_{31} = 5$

$a_{32} = 8$

$i > J$

$3 > 2$

$3 > 1$

$\left.\begin{matrix} i = 1 \\ < 1 \end{matrix}\right\}$ a_{ij} has a value

$$\begin{pmatrix} a_{11} = 1 \\ a_{12} = 3 \\ a_{13} = 2 \end{pmatrix}$$

$i > j$

$2 > 1$ $a_{21} = 0$

$3 > 1$ $a_{31} = 0$

While for

$i < j$ $1 < 3$

$a_{13} = 0$

$a_{23} = 0$

Transpose of matrix

Case of a square matrix

$m = n$

$$A = \begin{pmatrix} 2 & 5 & 4 \\ 7 & 6 & 8 \\ 2 & 3 & 6 \end{pmatrix}_{3\times3} \rightarrow$$ First row $\rightarrow$ First Column

$$A^T = \begin{pmatrix} 2 & 7 & 5 \\ 5 & 6 & 3 \\ 4 & 8 & 6 \end{pmatrix}_{3\times3}$$

2nd row $\longrightarrow$ 2nd Column

3rd row $\longrightarrow$ 3rd Column

For a non square matrix $(m \times n) \Rightarrow (n \times m)$ new Dimension

$$B = \begin{pmatrix} 5 & 8 & 7 \\ 1 & 4 & 3 \end{pmatrix}_{\substack{m\ n \\ 2\times3}}$$

$$B^T = \begin{bmatrix} 5 & 1 \\ 8 & 4 \\ 7 & 3 \end{bmatrix}_{3\times2}$$

Orthogonal matrix

MATrix A is orthogonal

If the product of matrix and its transpose is an identity matrix

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}_{2\times 2} \Rightarrow A^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}_{2\times 2}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$A \cdot A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}_{2\times 2} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}_{2\times 2}$$

$$AA^T = \begin{pmatrix} \cos\theta\cos\theta + \sin\theta\sin\theta & -\cos\theta\sin\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow I$$

Equality of matrices

B & C are Equal if have same order

$$B = \begin{pmatrix} A & -2 & B \\ A & C & 1 \end{pmatrix} \quad C = \begin{pmatrix} 3 & X & -4 \\ 3 & -1 & 4 \end{pmatrix}$$

$B = C$

Dim (2×3) B $\quad (2 \times 3)$ C

Then

$A = 3$

$X = -2$

$B = -4$

$C = -1$

Cue Math

Example 1: Check if the matrices $\begin{bmatrix} 2 & 3 & 6 \\ 4 & 5 & 7 \\ 1 & -3 & -12 \end{bmatrix}$ and $\begin{bmatrix} -1 & 3 & 6 \\ 4 & 5 & 7 \\ 1 & -3 & -12 \end{bmatrix}$ are equal using equality of matrices definition.

A B

Dim of $A = 3 \times 3$, $B = 3 \times 3$

$a_{11} = 2 \quad a_{12} = 3$

$b_{11} = -1 \quad b_{12} = 3$

$a_{11} \neq b_{11}$ A & B are not Equal

1-Matrix addition & subtraction

If $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & -1 & 4 \end{pmatrix}_{2x3}$ $B = \begin{pmatrix} 1 & +2 & 1 \\ 1 & 3 & -4 \end{pmatrix}_{2x3}$

$$A + B = \begin{pmatrix} 1+1 & -2+2 & 3+1 \\ 2+1 & -1+3 & 4-4 \end{pmatrix}_{2\times 3}$$

$$A + B = \begin{pmatrix} 2 & 0 & 4 \\ 3 & 2 & 0 \end{pmatrix}_{2\times 3}$$

$$A - B = \begin{pmatrix} 1-1 & -2-2 & 3-1 \\ 2-1 & -1-3 & 4+4 \end{pmatrix} = \begin{pmatrix} 0 & -4 & 2 \\ 1 & -4 & 8 \end{pmatrix}$$

Matrix operations

-2-Scalar multiplication

iF A

$$A = \begin{pmatrix} 2 & -3 & 1 \\ 3 & 1 & -2 \\ 1 & -1 & 4 \end{pmatrix}_{3\times 3}$$

C = -2

$$CA = -2\begin{pmatrix} 2 & 3 & 1 \\ 3 & 1 & -2 \\ 1 & 1 & 4 \end{pmatrix} = \begin{pmatrix} -2(2) & -2(-3) & -2(1) \\ -2(3) & -2(1) & -2(-2) \\ -2(1) & -2(-1) & -2(4) \end{pmatrix}$$

$$CA = \begin{pmatrix} -4 & +6 & -2 \\ -6 & -2 & 4 \\ -2 & 2 & -8 \end{pmatrix}$$

Matrix operations

If A= $\begin{pmatrix} 2 & 3 & 5 \end{pmatrix}$ 1×3

Final dimension

$B = \begin{pmatrix} 1 \\ 4 \\ -6 \end{pmatrix}$ 3×1

$= (1 \times 1)$

Check that No of Columns = number of rows

$$\begin{pmatrix} 2 & 3 & 5 \end{pmatrix} \begin{bmatrix} 1 \\ 4 \\ -6 \end{bmatrix} = \left(2(1) + 3(4) + 5(-6) \right) = 2 + 12 - 30 = -16$$

$= (-16)$

while $(A_{2 \times 1})(B_{1 \times 3})$

$\Rightarrow (2 \times 3)$