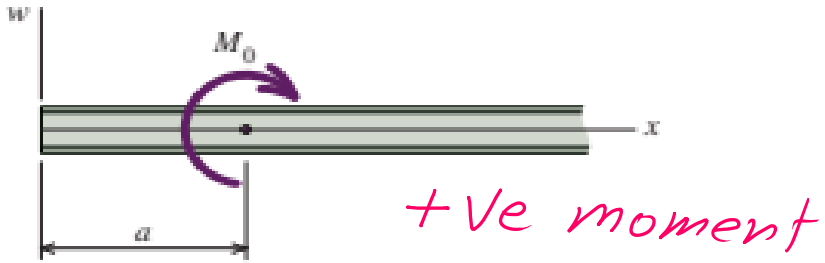
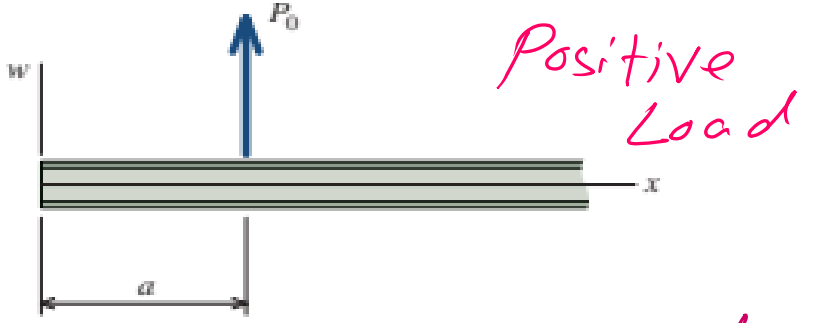
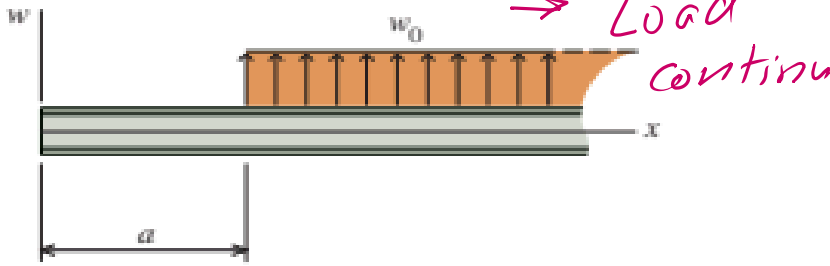
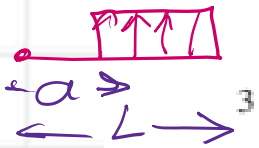
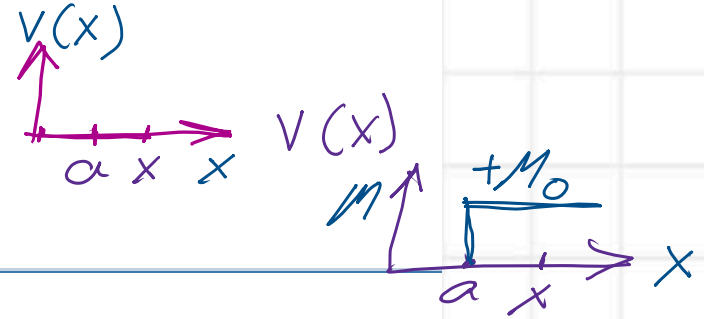
Mechanics of Materials_ An Integrated Learning System

Prof. Timothy Philpot
Chapter 7-4

**Table 7.2 Basic Loads Represented by Discontinuity Functions**

| Case | Load on Beam | Discontinuity Expressions |
|---|---|---|
| 1 |   +ve moment | $w(x) = M_0\langle x-a\rangle^{-2}$ 2<br>$V(x) = M_0\langle x-a\rangle^{-1}$ 1<br>$M(x) = M_0\langle x-a\rangle^{0}$ 0 |
| 2 |   Positive Load | $w(x) = P_0\langle x-a\rangle^{-1}$ = $P_0\langle x-a\rangle^{-1}$<br>$V(x) = P_0\langle x-a\rangle^{0}$ $P_0\langle x-a\rangle^{0}$<br>$M(x) = P_0\langle x-a\rangle^{1}$ $P_0\langle x-a\rangle^{1}$ |
| 3 |   → Load continues | $w(x) = w_0\langle x-a\rangle^{0}$<br>$V(x) = w_0\langle x-a\rangle^{1}$<br>$M(x) = \frac{w_0}{2}\langle x-a\rangle^{2}$ } Macaulay |



P 7.56 3 edition

**P7.32–P7.42** For the beams and loadings shown in Figures P7.32–P7.42,

(a) use discontinuity functions to write the expression for $w(x)$; include the beam reactions in this expression.

(b) integrate $w(x)$ twice to determine $V(x)$ and $M(x)$.

(c) use $V(x)$ and $M(x)$ to plot the shear-force and bending-moment diagrams.

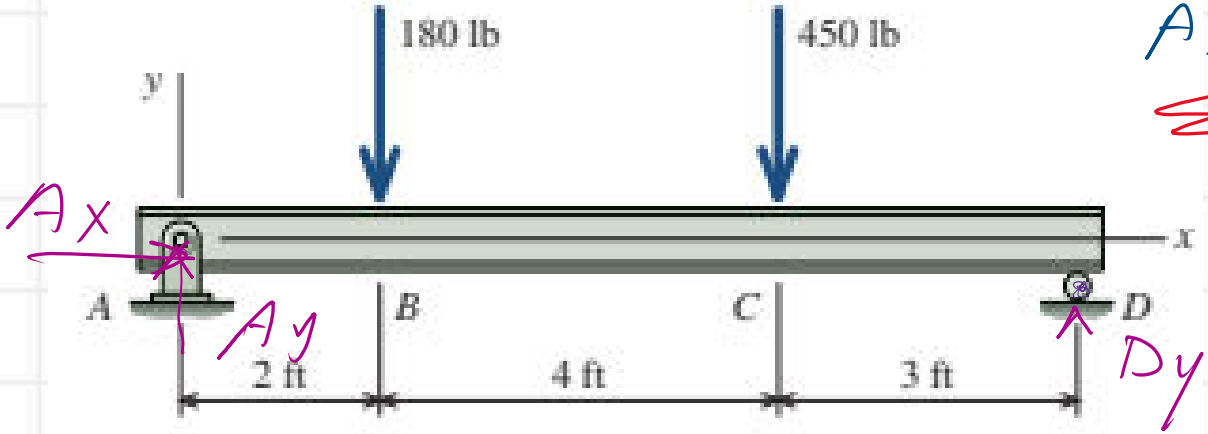



FIGURE P7.32

$\Sigma X = 0$

$A_X = 0$

$\Sigma Y = 0$

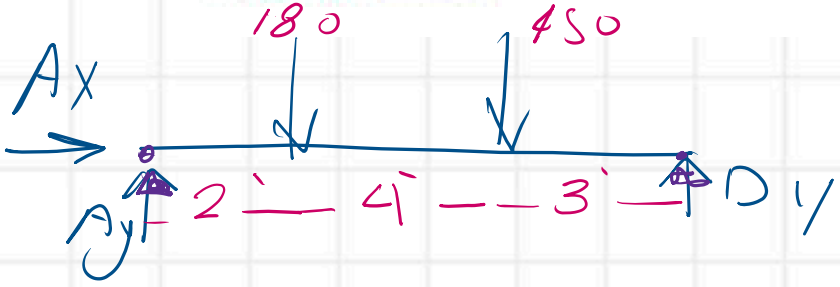
$A_y + D_y = 180 + 450 = 630$ Lb

$\Sigma M_D = 0$

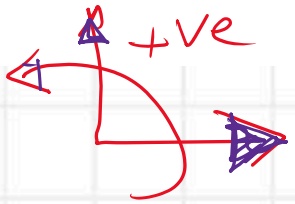
$-A_y(9) + 180(7) + 450(3) = 0$

$9A_y = 2610$ Lb $\Rightarrow A_y = 290$ Lbs

$D_y = 630 - 290 = 340$ Lb




F.B.D




Prepared by Eng.Maged Kamel.

**P7.32–P7.42** For the beams and loadings shown in Figures P7.32–P7.42,

(a) use discontinuity functions to write the expression for $w(x)$; include the beam reactions in this expression.

(b) integrate $w(x)$ twice to determine $V(x)$ and $M(x)$.

(c) use $V(x)$ and $M(x)$ to plot the shear-force and bending-moment diagrams.

Case-2

$$w(x) = 290\langle x-0\rangle^{-1} - 180\langle x-2\rangle^{-1} - 450\langle x-6\rangle^{-1} + 340\langle x-9\rangle^{-1}$$

ignored

+ve ↑

-ve ↓

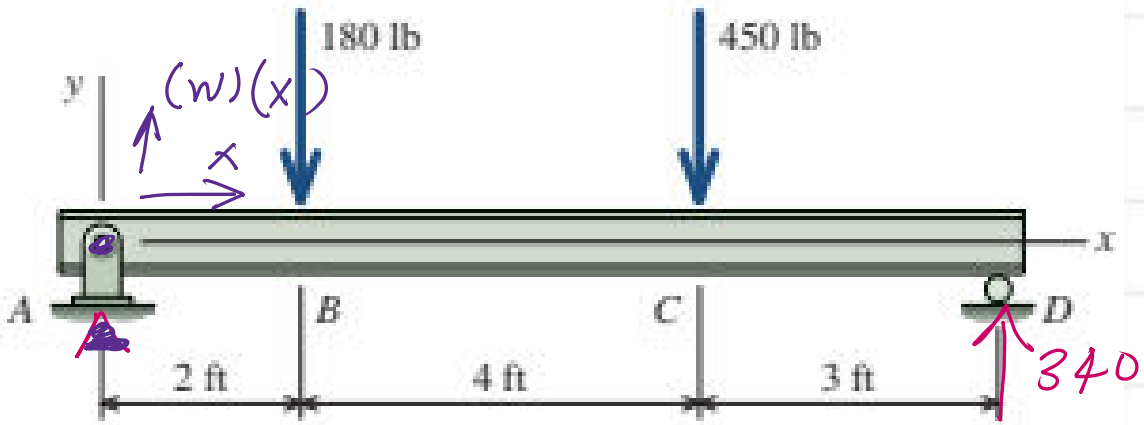



FIGURE P7.32

$$V(x) = 290\langle x-0\rangle^{0} - 180\langle x-2\rangle^{0} - 450\langle x-6\rangle^{0} + 340\langle x-9\rangle^{0}$$

$$M(x) = 290\langle x-0\rangle^{1} - 180\langle x-2\rangle^{1} - 450\langle x-6\rangle^{1} + 340\langle x-9\rangle^{1}$$

Parts (a) & (b)

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**P7.32–P7.42** For the beams and loadings shown in Figures P7.32–P7.42,

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(b) integrate $w(x)$ twice to determine $V(x)$ and $M(x)$.

(c) use $V(x)$ and $M(x)$ to plot the shear-force and bending-moment diagrams.

$$V(x) = 290\langle x-0\rangle^0 - 180\langle x-2\rangle^0 - 450\langle x-6\rangle^0 + 340\langle x-9\rangle^0$$

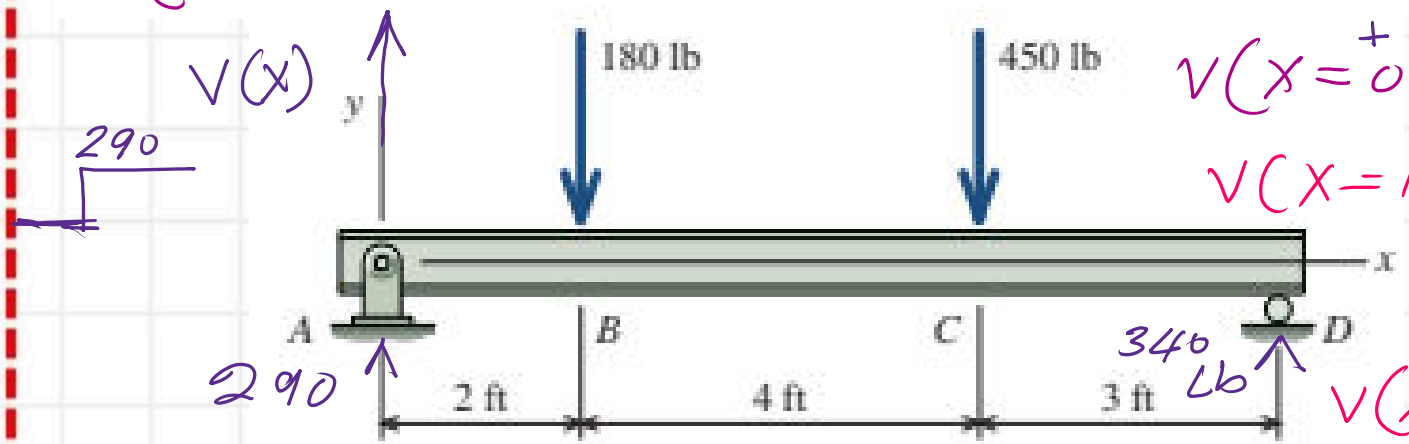


FIGURE P7.32

$V(x=0^+) = 290(1) - 0 - 0 = 290$ Lb (-ve, -ve)

$V(x=1) = 290(1) - 0 - 0 = 290$ Lb

$V(x=2) = 290(1) - 0 - 0 = 290$ Lb

$V(x=2^+) = 290(2) - 180(2^+ - 2) = 110$ Lb

$V(x=3) = 290(1) - 180(3-2)^0 - 0 = 290 - 180 = 110$ Lb

$V(x=4) = 290(1) - 180(4-2)^0 - 0 = 290 - 180 = 110$ Lb

$V(x=5) = 290(1) - 180(5-2)^0 - 0 = 290 - 180 = 110$ Lb

Prepared by Eng.Maged Kamel.

**P7.32–P7.42** For the beams and loadings shown in Figures P7.32–P7.42,

(a) use discontinuity functions to write the expression for $w(x)$; include the beam reactions in this expression.

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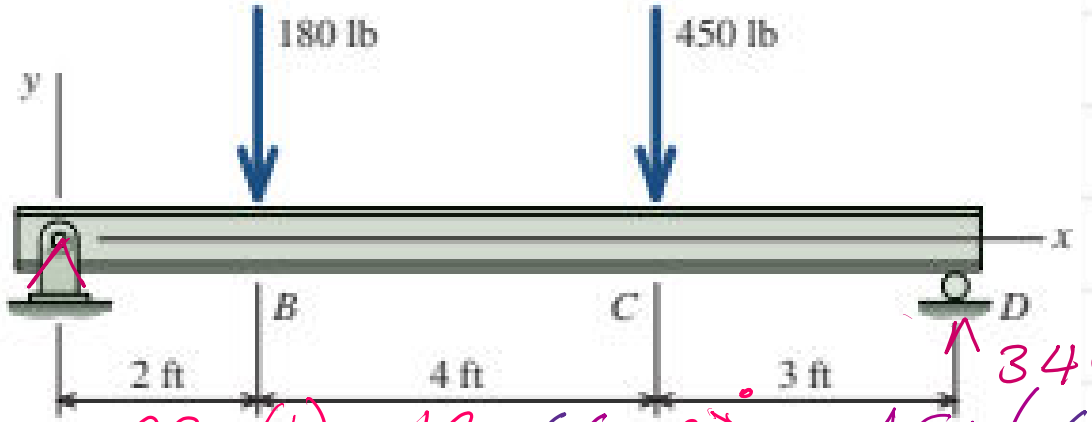
$$V(x) = 290\langle x-0\rangle^0 - 180\langle x-2\rangle^0 - 450\langle x-6\rangle^0 + 340\langle x-9\rangle^0$$




290 Lb

340 Lb

Drop

FIGURE P7.32

$V(x=6^-) = 290(1) - 180(6-2)^0 - 450(6-6)^0 = 290 - 180 - 0 = -110$ Lb

$V(x=6^+) = 290(1) - 180(6-2)^0 - 450(6^+-6)^0 = 290 - 180 - 450 = -340$ Lb

$V(x=7) = 290(1) - 180(7-2)^0 - 450(7-6)^0 = 290 - 180 - 450 = -340$

$V(x=8) = 290(1) - 180(8-2)^0 - 450(8-6)^0 = 290 - 180 - 450 = -340$

$V(x=9) = 290(1) - 180(9-2)^0 - 450(9-6)^0 = -340$ Lb

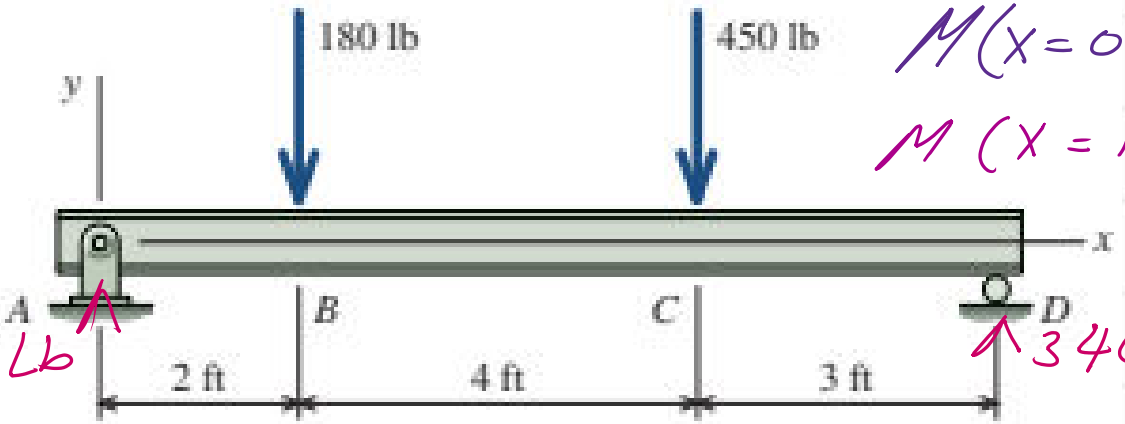
**P7.32–P7.42** For the beams and loadings shown in Figures P7.32–P7.42,

(a) use discontinuity functions to write the expression for $w(x)$; include the beam reactions in this expression.

(b) integrate $w(x)$ twice to determine $V(x)$ and $M(x)$.

(c) use $V(x)$ and $M(x)$ to plot the shear-force and bending-moment diagrams.

$M(x) = 290\langle x-0\rangle^1 - 180\langle x-2\rangle^1 - 450\langle x-6\rangle^1 + 340\langle x-9\rangle^1$




290 Lb

340 Lb

FIGURE P7.32

$M(x=0) = 290(0) - 0 - 0 = 0$

$M(x=1) = 290(1) - 0 - 0 = 290$ Lb.FT

$M(x=2) = 290(2) - 0 - 0 = 580$ Lb.ft

$M(x=3) = 290(3) - 180(3-2) - 0 = 870 - 180 = 690$ Lb.FT

$M(x=4) = 290(4) - 180(4-2) - 0 = 1160 - 360 = 800$ Lb.FT

$M(x=6) = 290(6) - 180(6-2) - 0 = 1020$ Lb.FT

**P7.32–P7.42** For the beams and loadings shown in Figures P7.32–P7.42,

(a) use discontinuity functions to write the expression for $w(x)$; include the beam reactions in this expression.

(b) integrate $w(x)$ twice to determine $V(x)$ and $M(x)$.

(c) use $V(x)$ and $M(x)$ to plot the shear-force and bending-moment diagrams.

$$M(x) = 290\langle x-0\rangle^1 - 180\langle x-2\rangle^1 - 450\langle x-6\rangle^1 + 340\langle x-9\rangle^1$$

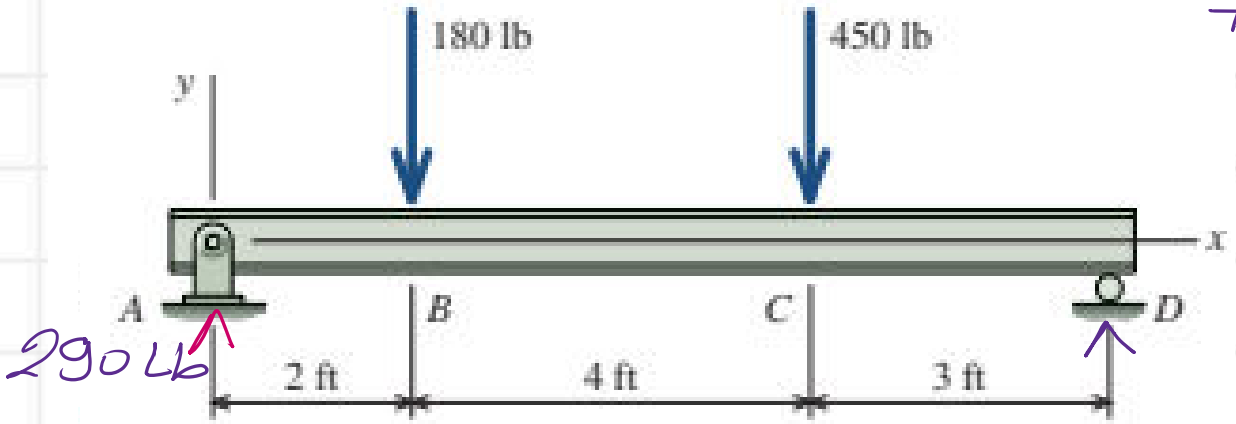



FIGURE P7.32

$M(X=7') = 290(7) - 180(7-2)^1 - 450(7-6)^1 = 2030 - 900 - 450 = 680$ lb.FT

$M(X=8') = 290(8) - 180(8-2)^1 - 450(8-6)^1 = 2320 - 1080 - 900 = 340$ Lb.FT

$M(X=9') = 290(9) - 180(9-2)^1 - 450(9-6)^1 = 2610 - 1260 - 1350 = 0$

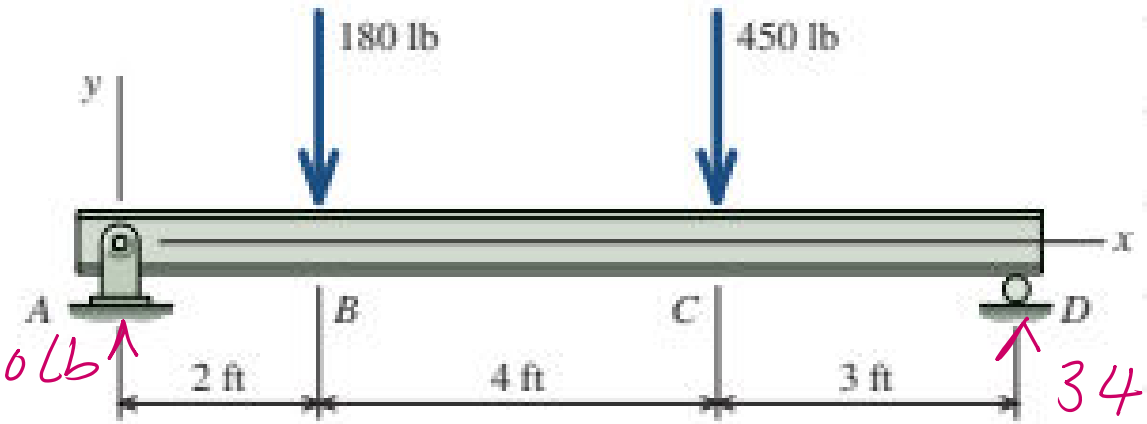
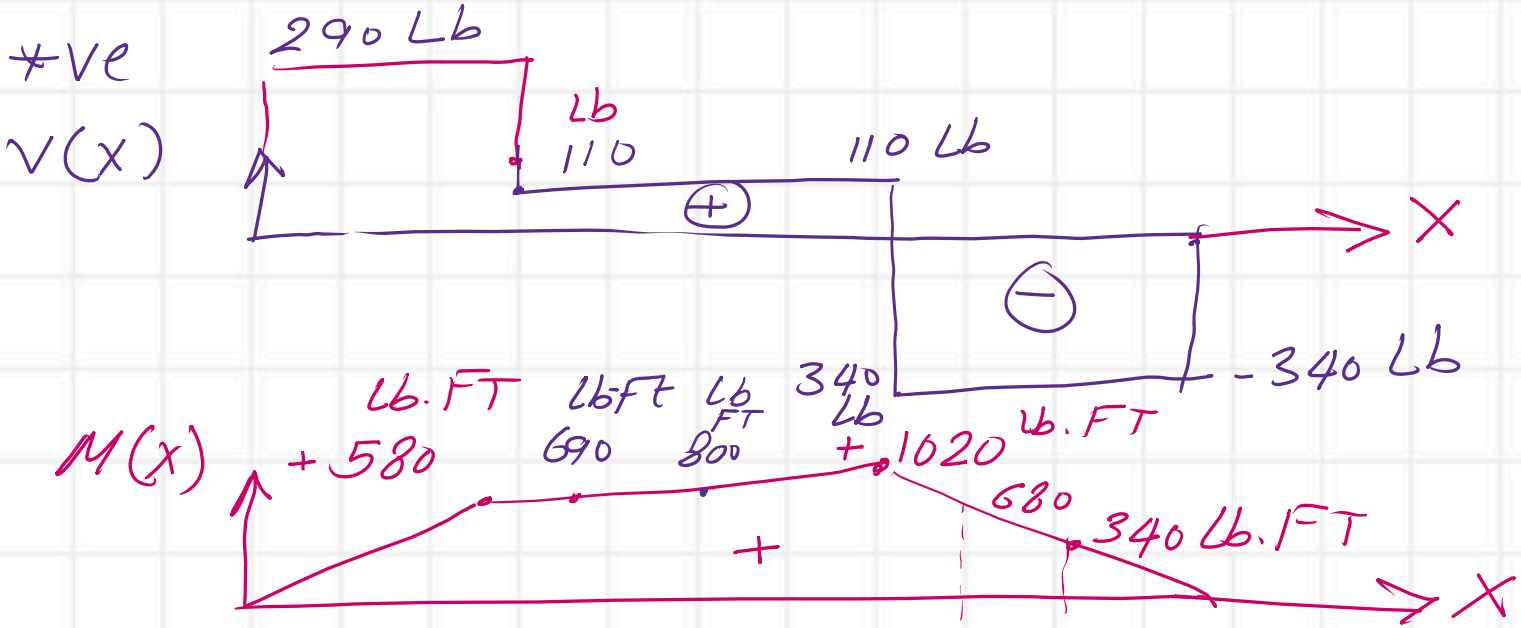

180 lb
450 lb
y
x
A
B
C
D
2 ft
4 ft
3 ft
290 Lb
340 Lb

FIGURE P7.32


+ve
V(x)
290 Lb
Lb
110
110 Lb
X
S.F.D
-340 Lb
340
Lb.FT
LbFt
Lb
FT
Lb
M(x)
+580
690
800
+1020
Lb.FT
680
340 Lb.FT
B.M.D
X

B14

```
=IF(AND(0<=A14,A14<2),290,IF(AND(2<=A14,A14<6),290-180,IF(AND(6<=A14,A14<=9),290-180-450,0)))
```

} Shear Expression ) 0 → 2, 2 → 6, 6 → 9

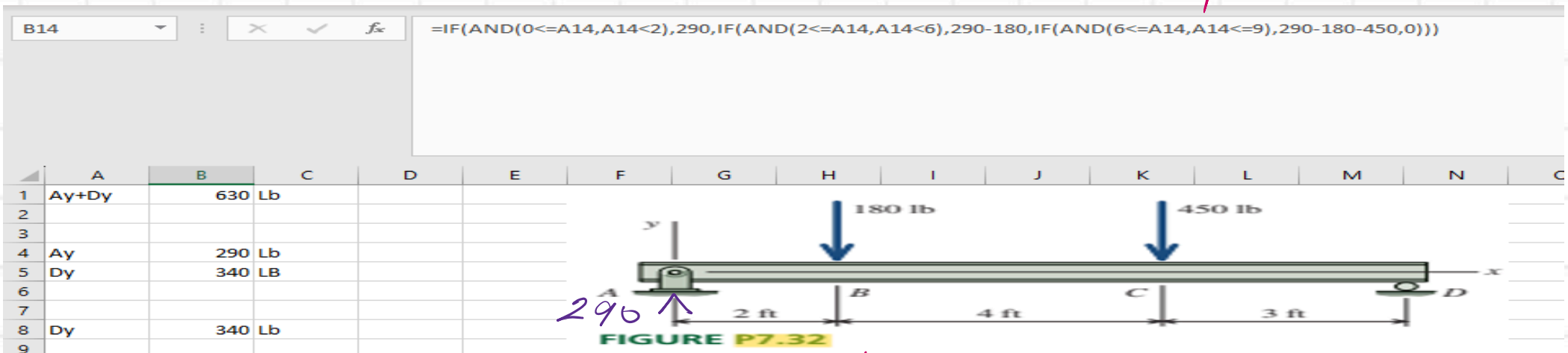



FIGURE P7.32

| Distance x (Ft) | Shear V(x)Lb | Moment in LB.Ft |
|---|---|---|
| 0 | 290 | 0 |
| 1 | 290 | 290 |
| 2 | 290 | 580 |
| 2 | 110 | 580 |
| 3 | 110 | 690 |
| 4 | 110 | 800 |
| 5 | 110 | 910 |
| 6 | 110 | 1020 |
| 6 | -340 | 1020 |
| 7 | -340 | 680 |
| 8 | -340 | 340 |
| 9 | -340 | 0 |

A14 A16 A17

0 → 2 one Equation is on

$x \geq 0 \Rightarrow 290$ $< 2$ one Drop at 2

$V(x) = 290 - 180$

$x \geq 2$ $< 6$

$V(x) = 290 - 180 - 450$

$x \geq 6$ $< 9$

```
=IF(AND(0<=A16,A16<=2),290,IF(AND(2<A16,A16<6),290-180,IF(AND(6<=A16,A16<9),290-180-450,0)))
```

```
=IF(AND(0<=A17,A17<2),290,IF(AND(2<=A17,A17<6),290-180,IF(AND(6<=A17,A17<9),290-180-450,0)))
```

} For ≥ 2 remove Equal sign from < 2

B14

=IF(AND(0<=A14,A14<2),290,IF(AND(2<=A14,A14<6),290-180,IF(AND(6<=A14,A14<=9),290-180-450,0)))

} Shear Expression ) 0→2, 2→6, 6→9

A14 value in FT

B14 =IF(AND(0<=A14,A14<2),290,IF(AND(2<=A14,A14<6),290-180,IF(AND(6<=A14,A14<=9),290-180-450,0)))

| | A | B | C |
|---|---|---|---|
| 1 | Ay+Dy | 630 | Lb |
| 2 | | | |
| 3 | | | |
| 4 | Ay | 290 | Lb |
| 5 | Dy | 340 | LB |
| 6 | | | |
| 7 | | | |
| 8 | Dy | 340 | Lb |
| 9 | | | |

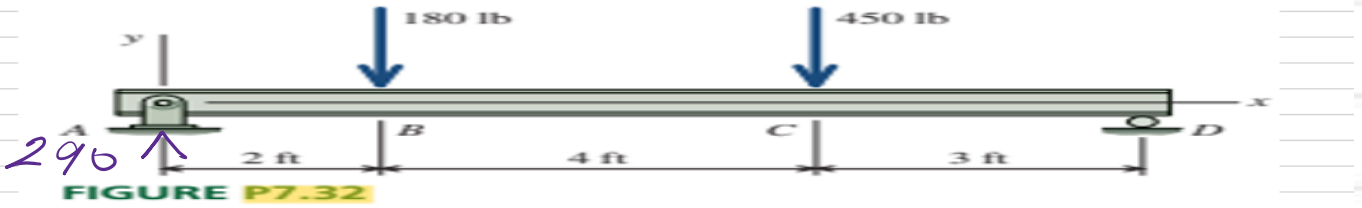


FIGURE P7.32

| Distance x (Ft) | Shear V(x)Lb | Moment in LB.Ft |
|---|---|---|
| 0 | 290 | 0 |
| 1 | 290 | 290 |
| 2 | 290 | 580 |
| 2 | 110 | 580 |
| 3 | 110 | 690 |
| 4 | 110 | 800 |
| 5 | 110 | 910 |
| 6 | 110 | 1020 |
| 6 | -340 | 1020 |
| 7 | -340 | 680 |
| 8 | -340 | 340 |
| 9 | -340 | 0 |

A14

0→2 one Equation is on

$x \geq 0 \Rightarrow 290$ one Drop at 6

$< 2$

$V(x) = 290 - 180$

$x \rightarrow 2 < 6$

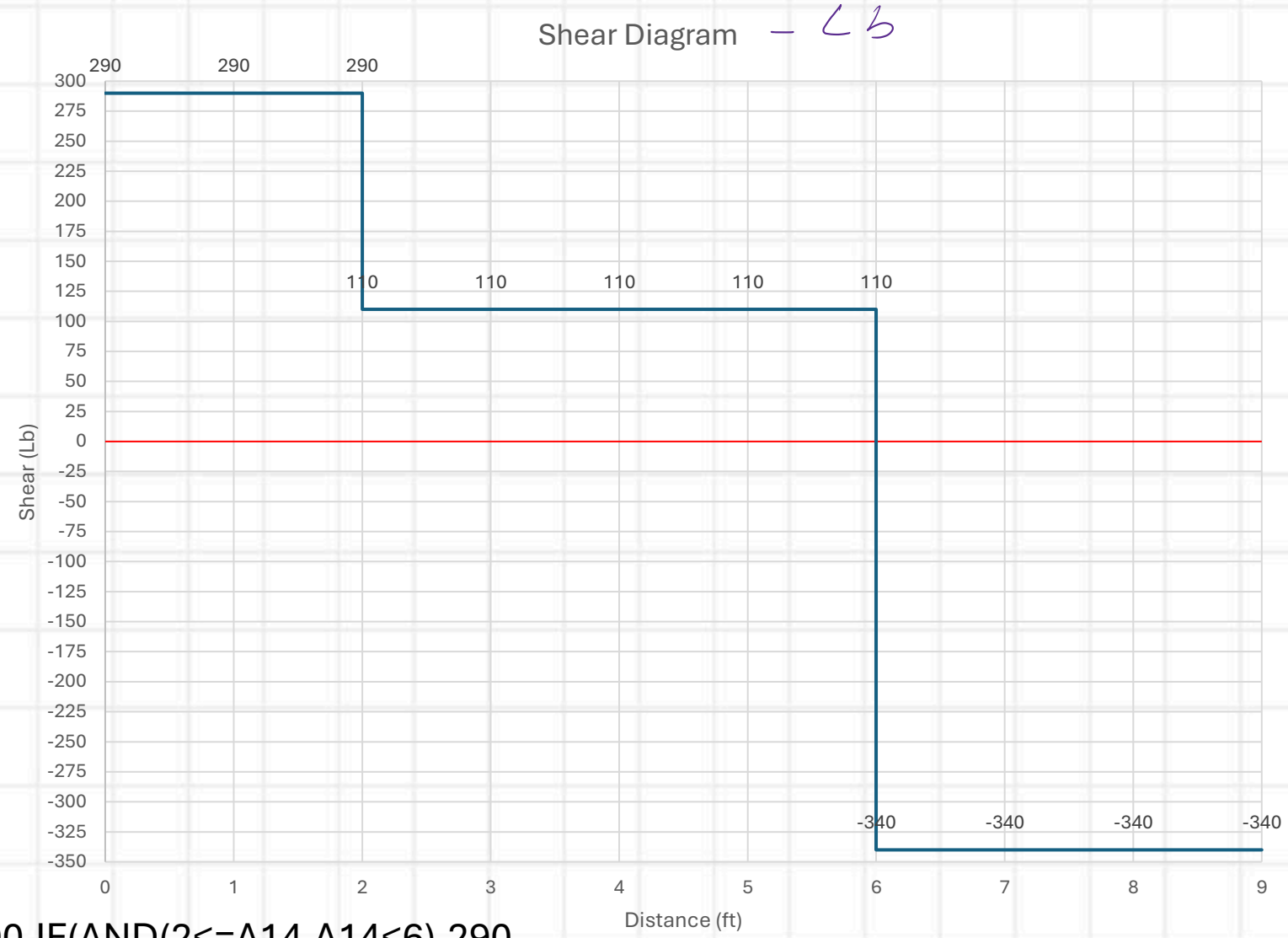
$x \geq 6 < 9$

$V(x) = 290 - 180 - 450$ Yellow 6

=IF(AND(0<=A21,A21<2),290,IF(AND(2<=A21,A21<=6),290-180,IF(AND(6<A21,A21<9),290-180-450,0)))

=IF(AND(0<=A22,A22<2),290,IF(AND(2<=A22,A22<6),290-180,IF(AND(6<=A22,A22<=9),290-180-450,0)))

} For ≥ 6 remove Equal sign from < 6




=IF(AND(0<=A14,A14<2),290,IF(AND(2<=A14,A14<6),290-180,IF(AND(6<=A14,A14<=9),290-180-450,0)))

C14

=IF(AND(0<=A14,A14<2),290*A14,IF(AND(2<=A14,A14<6),290*A14-180*(A14-2),IF(AND(6<=A14,A14<9),290*A14-180*(A14-2)-450*(A14-6),0)))

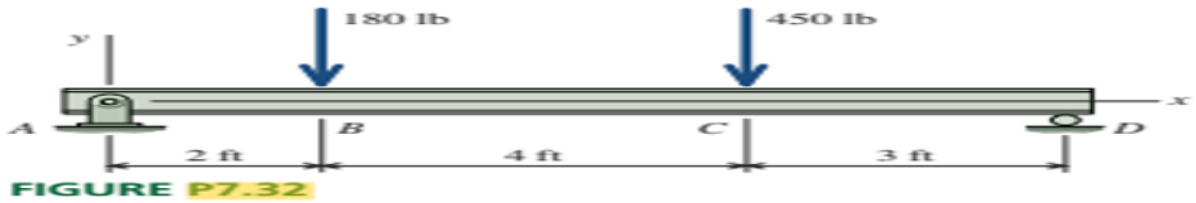
0 → 2
2 → 6
6 → 9

A14 value in FT

Moment Expression

B14 | =IF(AND(0<=A14,A14<2),290,IF(AND(2<=A14,A14<6),290-180,IF(AND(6<=A14,A14<=9),290-180-450,0)))

| | A | B | C |
|---|---|---|---|
| 1 | Ay+Dy | 630 | Lb |
| 2 | | | |
| 3 | | | |
| 4 | Ay | 290 | Lb |
| 5 | Dy | 340 | LB |
| 6 | | | |
| 7 | | | |
| 8 | Dy | 340 | Lb |



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| Distance x (Ft) | Shear V(x)Lb | Moment in LB.Ft |
|---|---|---|
| 0 | 290 | 0 |
| 1 | 290 | 290 |
| 2 | 290 | 580 |
| 2 | 110 | 580 |
| 3 | 110 | 690 |
| 4 | 110 | 800 |
| 5 | 110 | 910 |
| 6 | 110 | 1020 |
| 6 | -340 | 1020 |
| 7 | -340 | 680 |
| 8 | -340 | 340 |
| 9 | -340 | 0 |

A14

A17

C17

=IF(AND(0<=A17,A17<2),290*A17,IF(AND(2<=A17,A17<6),290*A17-180*(A17-2),IF(AND(6<=A17,A17<9),290*A17-180*(A17-2)-450*(A17-6),0)))

M(x)
x = 2

A22

C22

=IF(AND(0<=A22,A22<2),290*A22,IF(AND(2<=A22,A22<6),290*A22-180*(A22-2),IF(AND(6<=A22,A22<=9),290*A22-180*(A22-2)-450*(A22-6),0)))

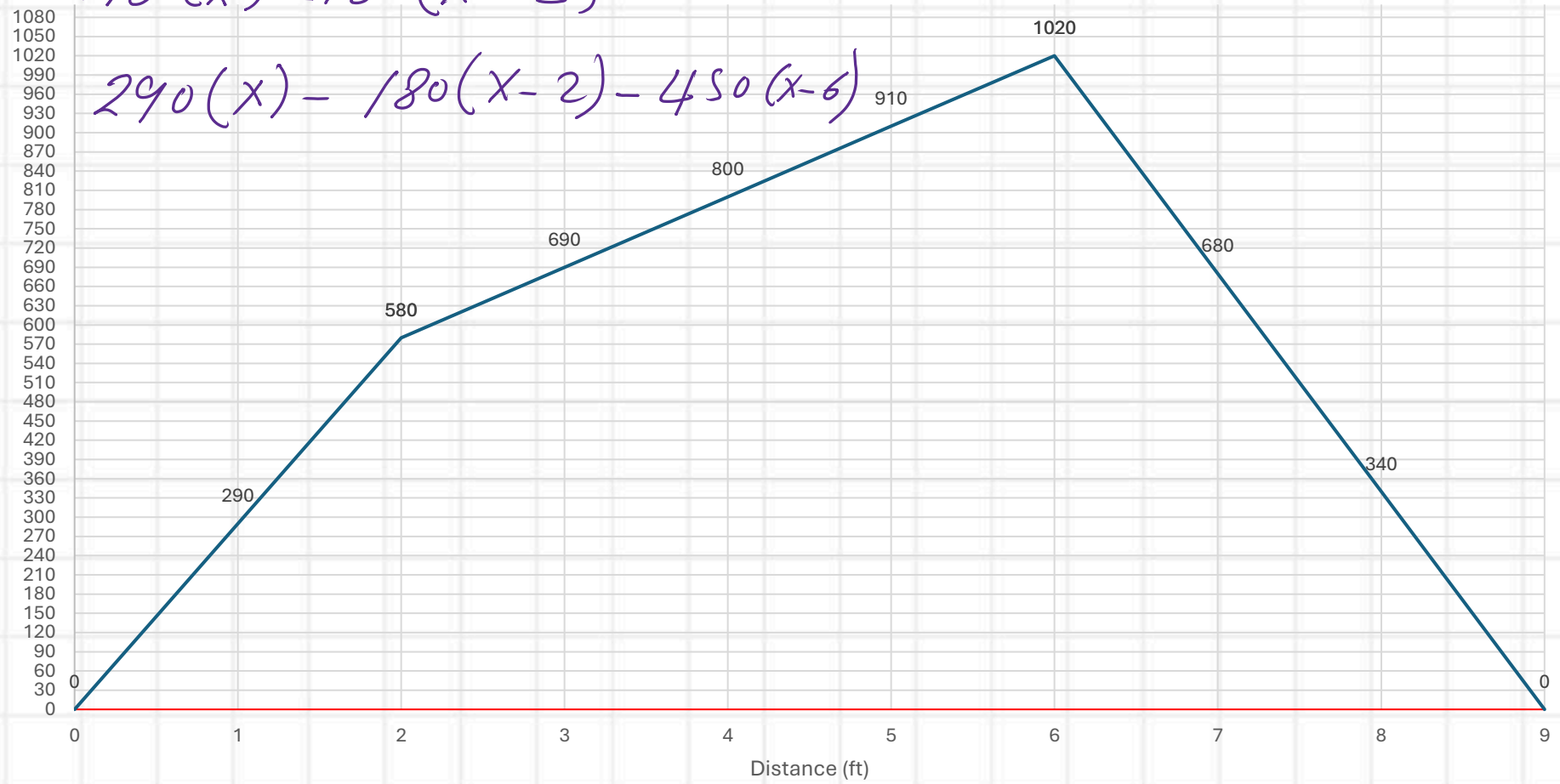
M(x)
x = 6

0 ≥ 2 $290(x)$

2 - 6 $290(x) - 180(x-2)$

6 - 9 $290(x) - 180(x-2) - 450(x-6)$

Moment Diagram – Lb. FT




Cell C14 A14 value in FT

```
=IF(AND(0<=A14,A14<2),290*A14,IF(AND(2<=A14,A14<6),290*A14-180*(A14-2),IF(AND(6<=A14,A14<9),290*A14-180*(A14-2)-450*(A14-6),0)))
```