Prof. Timothy Philpot

Chapter 7-4

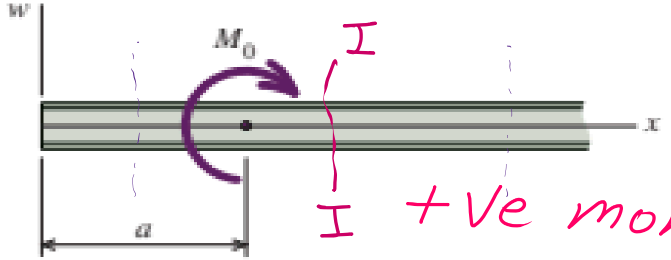
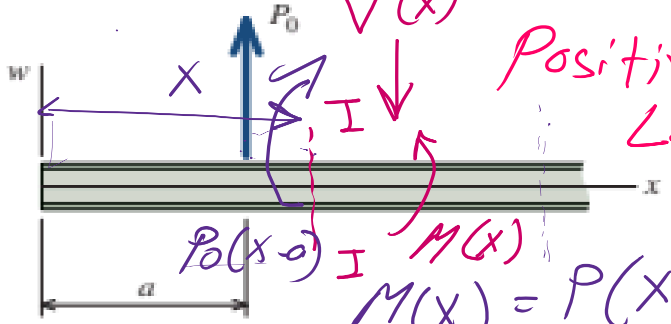
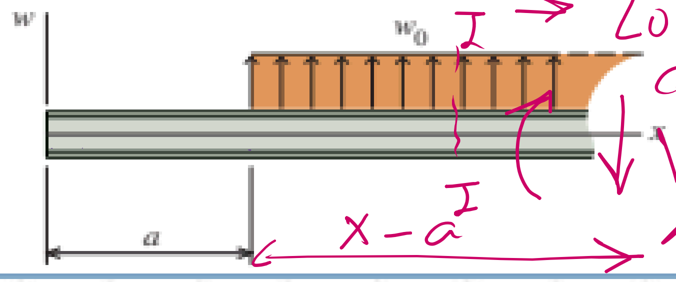
Mechanics of Materials_ An Integrated Learning System

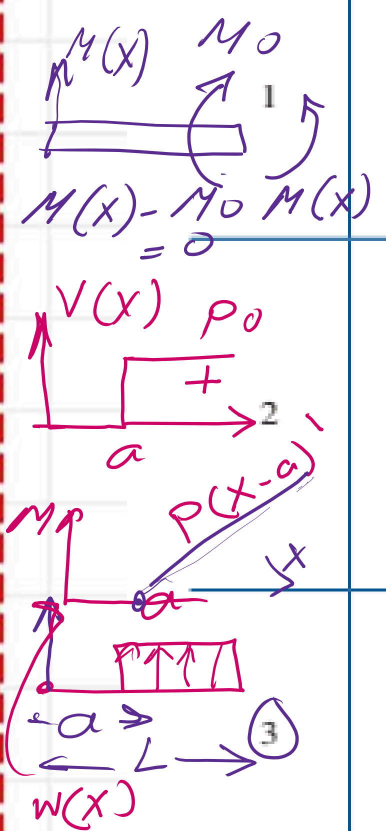
$\Sigma X = 0$ & $\Sigma Y = 0$

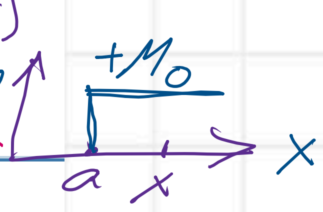
+ve

$\Sigma M = 0$

Table 7.2 Basic Loads Represented by Discontinuity Functions

Case	Load on Beam	Discontinuity Expressions
1	 I +ve moment	$w(x) = M_0\langle x-a\rangle^{-2}$ $V(x) = M_0\langle x-a\rangle^{-1}$ $M(x) = M_0\langle x-a\rangle^{0} \Rightarrow$ step function Macaulay's Function
2	 Positive Load $P_0(x-a)$ I $M(x)$ $M(x) = P(x-a)^1$	$w(x) = P_0\langle x-a\rangle^{-1} = P_0\langle x-a\rangle^{-1}$ $V(x) = P_0\langle x-a\rangle^{0}$ $\quad P_0\langle x-a\rangle^{0}$ $M(x) = P_0\langle x-a\rangle^{1}$ $\quad P_0\langle x-a\rangle^{1}$
3	 I → Load continues $V(x)$ $M(x)$	$w(x) = w_0\langle x-a\rangle^{0}$ $V(x) = w_0\langle x-a\rangle^{1}$ $M(x) = \frac{w_0}{2}\langle x-a\rangle^{2}$ } Macaulay's function $w_0(x-a)\left(\frac{x-a}{2}\right) = M$





Load Continues

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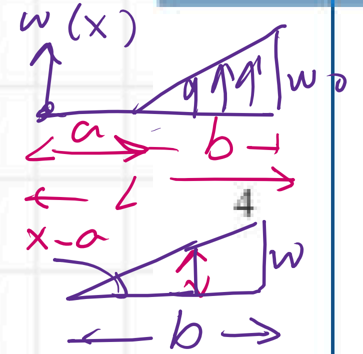
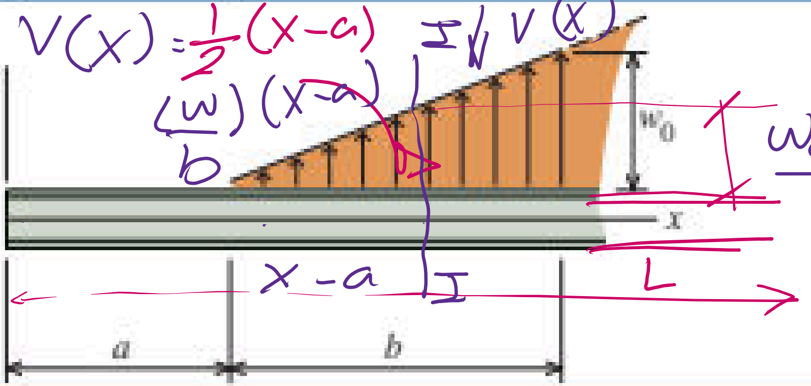
4





$V(x) = \frac{1}{2}(x-a)\,\frac{(w)}{b}(x-a)$

$\frac{w_0(x-a)}{b}$

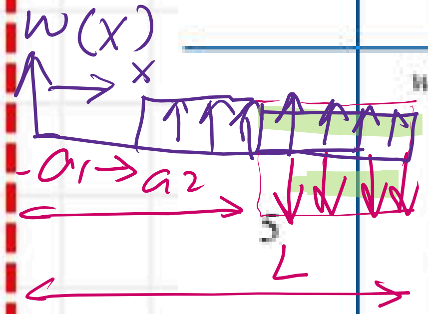
$$w(x) = \frac{w_0}{b}\langle x-a\rangle^1$$

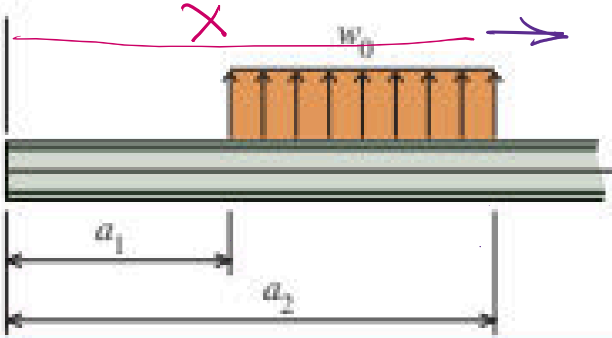
$$V(x) = \frac{w_0}{2b}\langle x-a\rangle^2$$

$$M(x) = \frac{w_0}{6b}\langle x-a\rangle^3$$

Macaulay is

5





Limited width

Deduct

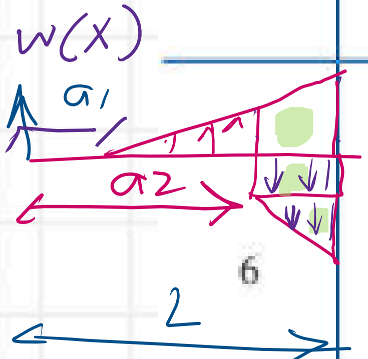
$$w(x) = w_0\langle x-a_1\rangle^0 - w_0\langle x-a_2\rangle^0$$

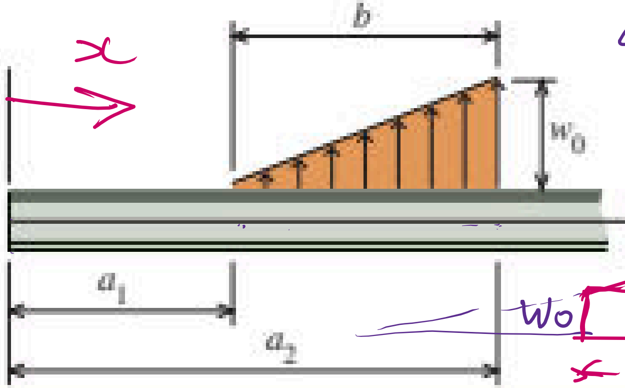
$$V(x) = w_0\langle x-a_1\rangle^1 - w_0\langle x-a_2\rangle^1$$

$$M(x) = \frac{w_0}{2}\langle x-a_1\rangle^2 - \frac{w_0}{2}\langle x-a_2\rangle^2$$

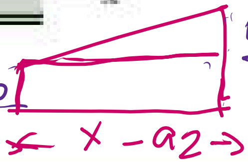
Macaulay's

6



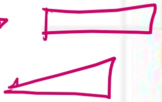


Limited width



$\frac{w_0(x-a_2)}{b} + w_0$

Deduct   deduct



$$w(x) = \frac{w_0}{b}\langle x-a_1\rangle^1 - \frac{w_0}{b}\langle x-a_2\rangle^1 - w_0\langle x-a_2\rangle^0$$

$$V(x) = \frac{w_0}{2b}\langle x-a_1\rangle^2 - \frac{w_0}{2b}\langle x-a_2\rangle^2 - w_0\langle x-a_2\rangle^1$$

$$M(x) = \frac{w_0}{6b}\langle x-a_1\rangle^3 - \frac{w_0}{6b}\langle x-a_2\rangle^3 - \frac{w_0}{2}\langle x-a_2\rangle^2$$

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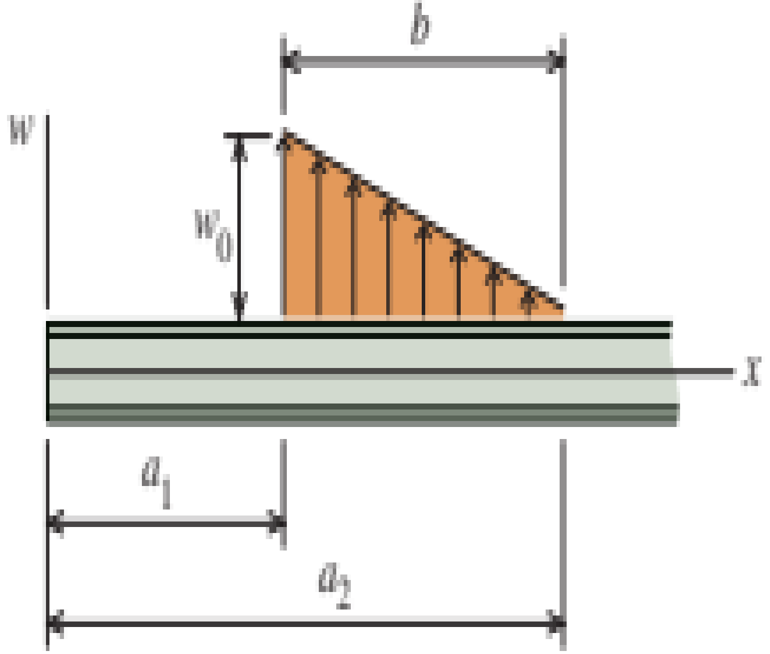
7		$w(x) = w_0\langle x-a_1\rangle^0 - \frac{w_0}{b}\langle x-a_1\rangle^1 + \frac{w_0}{b}\langle x-a_2\rangle^1$ $V(x) = w_0\langle x-a_1\rangle^1 - \frac{w_0}{2b}\langle x-a_1\rangle^2 + \frac{w_0}{2b}\langle x-a_2\rangle^2$ $M(x) = \frac{w_0}{2}\langle x-a_1\rangle^2 - \frac{w_0}{6b}\langle x-a_1\rangle^3 + \frac{w_0}{6b}\langle x-a_2\rangle^3$