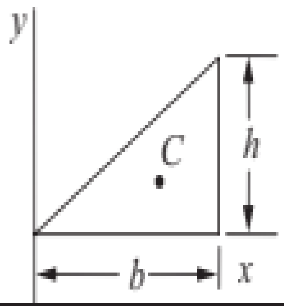
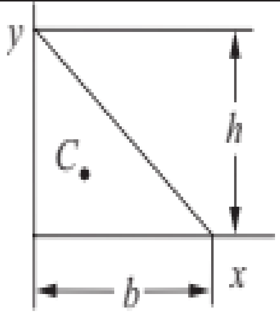
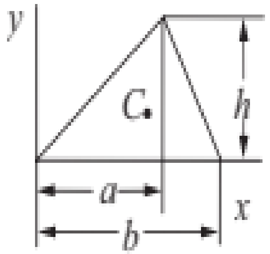
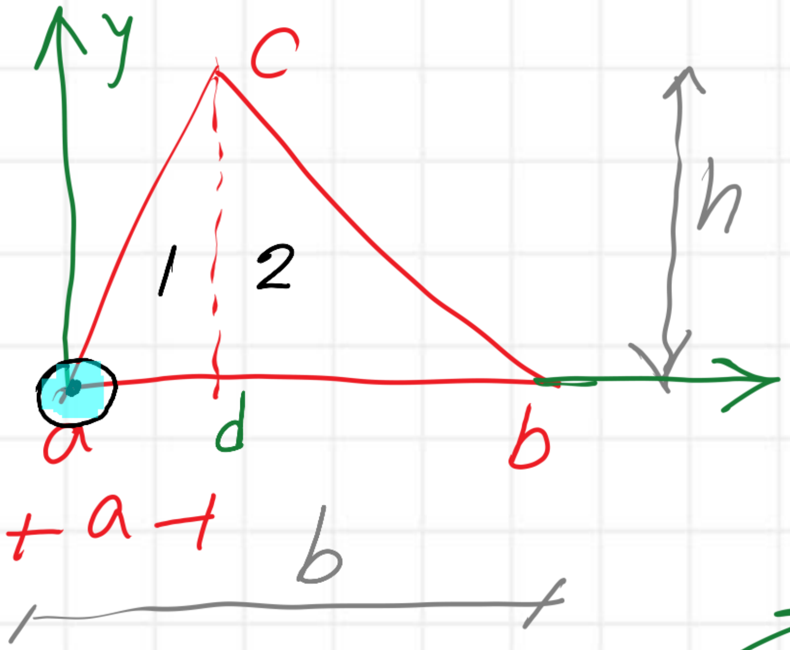
Ixy Product of inertia for a triangle

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = bh/2$ $x_c = 2b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/4$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/2$	$I_{x_c y_c} = Abh/36 = b^2h^2/72$ $I_{xy} = Abh/4 = b^2h^2/8$
	$A = bh/2$ $x_c = b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/6$	$I_{x_c y_c} = -Abh/36 = -b^2h^2/72$ $I_{xy} = Abh/12 = b^2h^2/24$
	$A = bh/2$ $x_c = (a+b)/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = [bh(b^2 - ab + a^2)]/36$ $I_x = bh^3/12$ $I_y = [bh(b^2 + ab + a^2)]/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = (b^2 - ab + a^2)/18$ $r_x^2 = h^2/6$ $r_y^2 = (b^2 + ab + a^2)/6$	$I_{x_c y_c} = [Ah(2a-b)]/36$ $= [bh^2(2a-b)]/72$ $I_{xy} = [Ah(2a+b)]/12$ $= [bh^2(2a+b)]/24$

at cg

General

Product of Inertia I_{xy} at a



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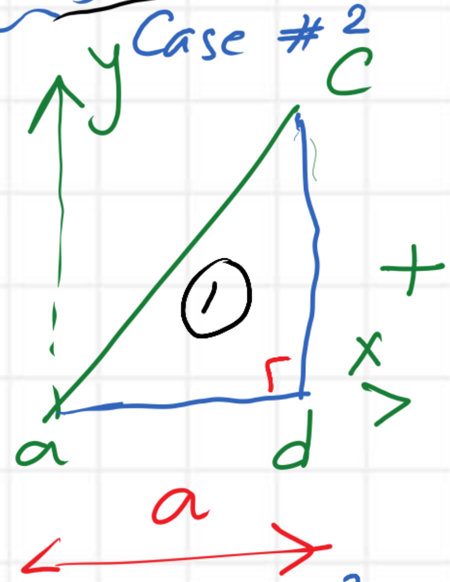
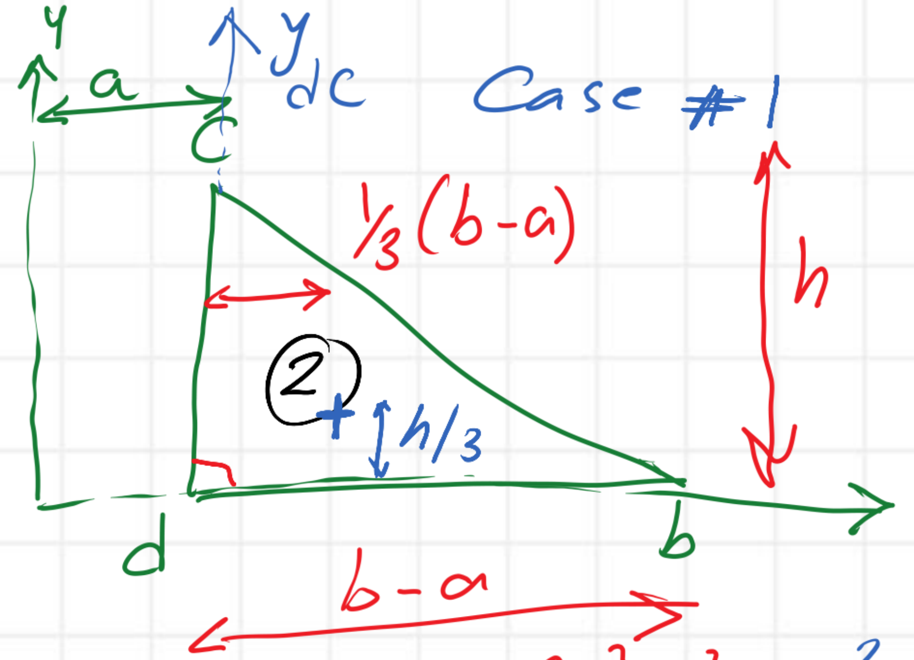


$$I_{xy} = \frac{h^2 a^2}{8}$$

+



$$I_{xy} = \frac{Base^2 \cdot h^2}{24} = \frac{(b-a)^2 h^2}{24}$$

$$I_{xy\,c.g} = -\frac{(b-a)^2}{72} \cdot h^2$$

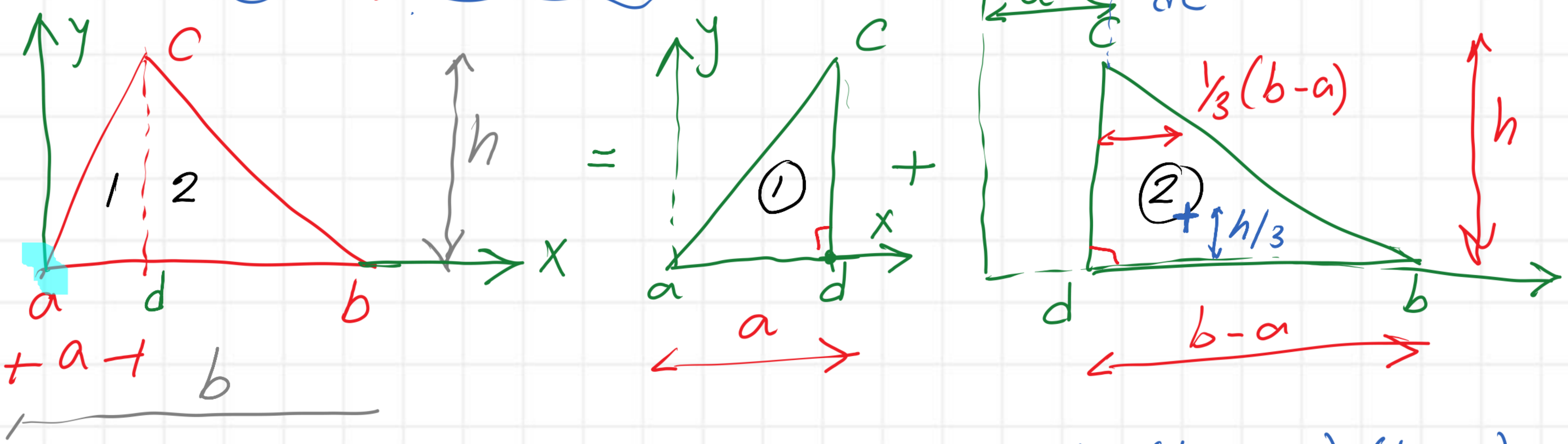
Our Final I_{xy}

$$= (I_{xy})_{\text{Triangle No.1}} + (I_{xy\,c.g})_{\text{Triangle No.2}} + A_2\,(x_{c.g}\; y_{c.g})$$

$$= \frac{h^2 a^2}{8} + \left(-\frac{(b-a)^2}{72}\cdot h^2 + \frac{1}{2}(b-a)h\left(\frac{2a+b}{3}\right)\left(\frac{h}{3}\right)\times\left(\frac{4}{4}\right)\right)$$

$$+ \frac{(b-a)h^2}{72}\left[-(b-a) + 8a + 4b\right] = \frac{h^2 a^2}{8} + \frac{(b-a)h^2}{72}(3b + 9a)$$

Product of Inertia I_{xy}

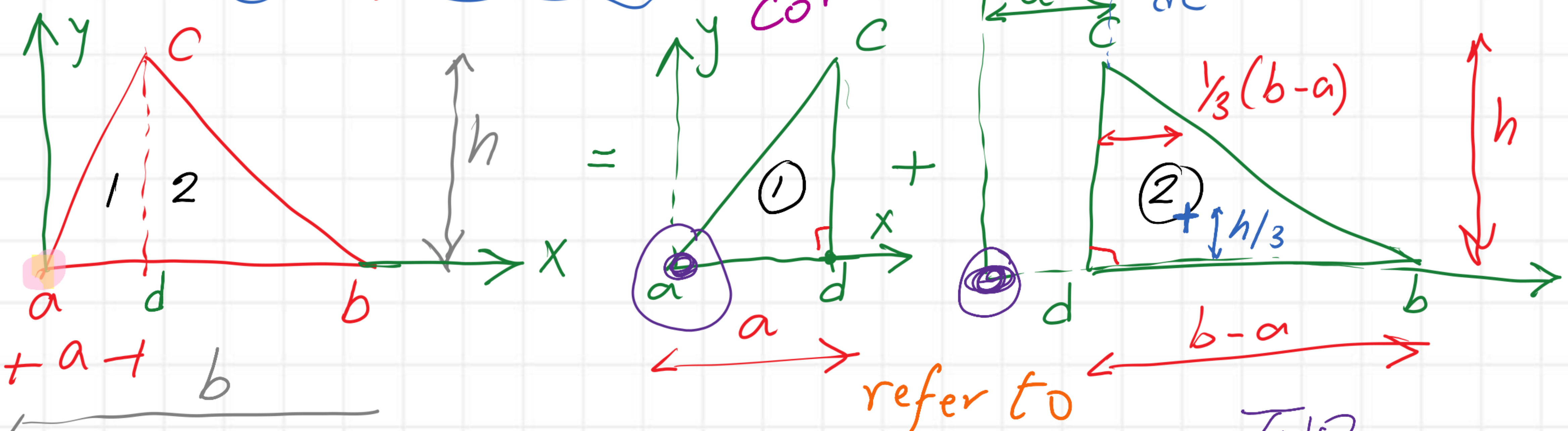


$$I_{xy} = \frac{h^2a^2}{8} + \frac{(b-a)h^2}{72}(3b+9a) = \frac{h^2a^2}{8} + \frac{3}{72}h^2(b+3a)(b-a)$$

$$= \frac{h^2a^2}{8} + \frac{h^2}{24}(b^2+2ab-3a^2) = \frac{h^2(3a^2+b^2+2ab-3a^2)}{24}$$

$$I_{xy} = \frac{h^2}{24}(b^2+2ab) = \frac{bh^2}{24}(b+2a) \text{ or } \frac{1}{2}\frac{bh}{12}\cdot h(2a+b)$$

Product of Inertia I_{xy} at corner



refer to

$$I_{xy} = \frac{bh^2}{24}(2a+b) \Rightarrow$$ Table

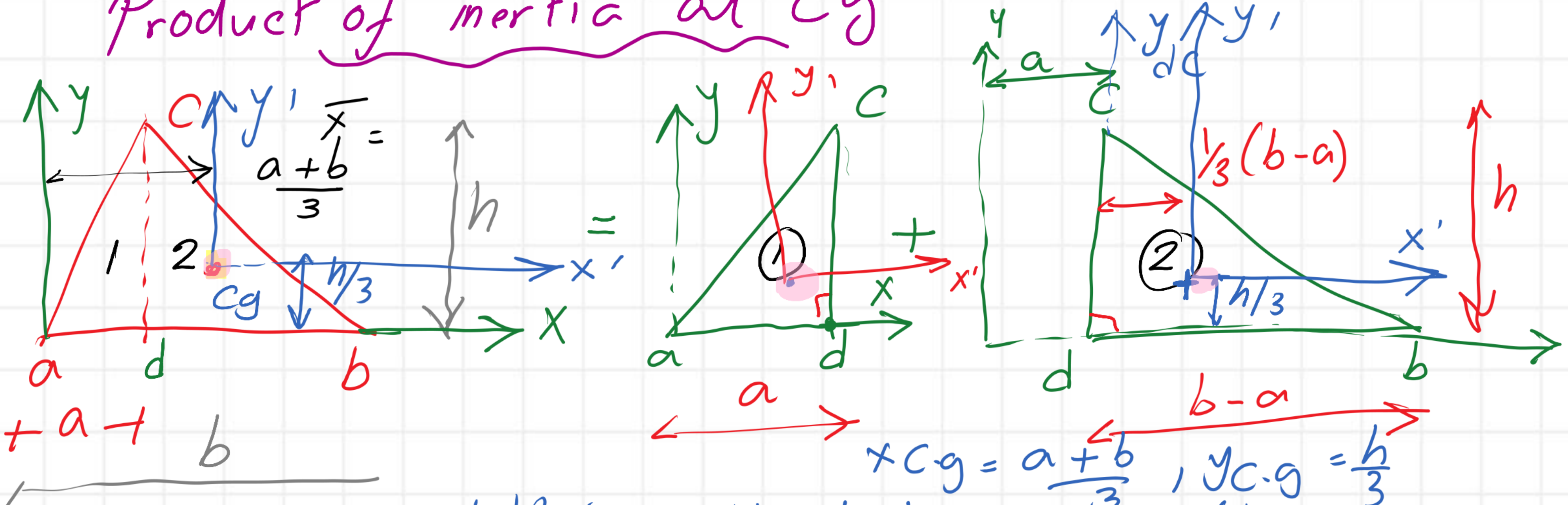
Two Expression

For $A_T = \frac{1}{2}bh \Rightarrow I_{xy} = \frac{1}{2}bh\left(\frac{1}{12}\right)(2a+b)(h)$

$$I_{xy} = A_T\left(\frac{1}{12}\right)(2a+b)(h)$$

$\Rightarrow$ Same as table

Product of inertia at Cg



$x_{c.g} = \frac{a+b}{3}$, $y_{c.g} = \frac{h}{3}$

I_{xy} at c.g $\rightarrow \frac{3\, b h^2}{24(3)}(2a+b) - \frac{1}{2} b h \left(\frac{a+b}{3}\right)\left(\frac{h}{3}\right)\left(\frac{4}{4}\right)$

$I_{xy_{c.g}} = \frac{b h^2}{72}\left[6a + 3b - 4a - 4b\right] = \frac{b h^2}{72}\left[2a - b\right]$

$A = \frac{1}{2} b h \Rightarrow I_{xy_c} = \frac{A}{36} h (2a - b)$

Two Expression