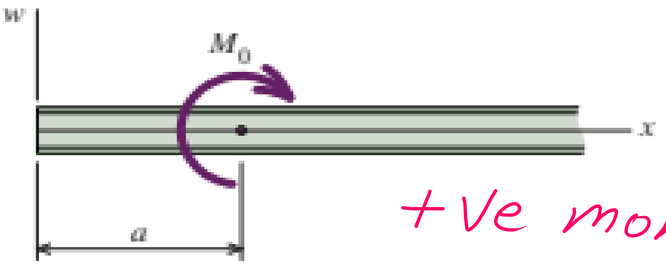
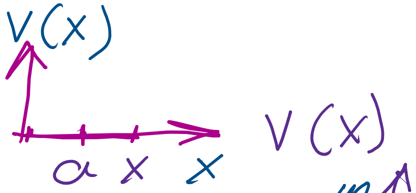
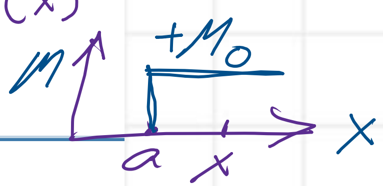
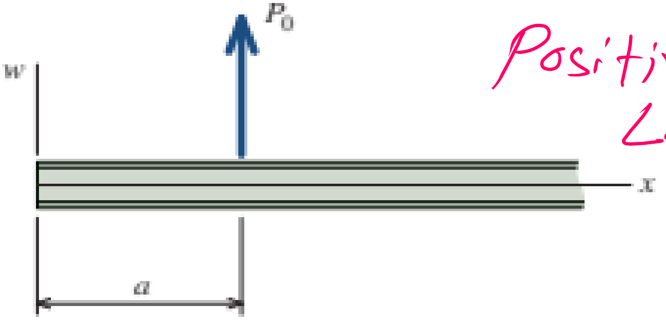
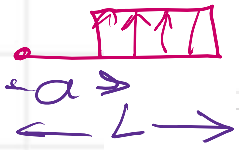
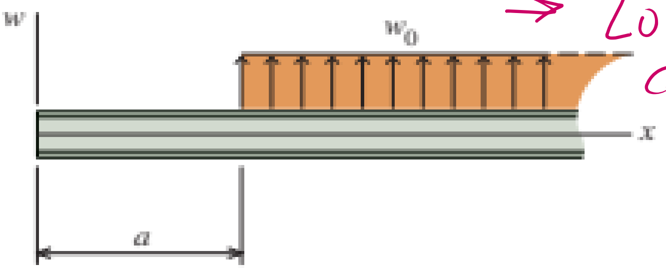
Mechanics of Materials_ An Integrated Learning System

Prof. Timothy Philpot

Chapter 7-4

Table 7.2 Basic Loads Represented by Discontinuity Functions

Case	Load on Beam	Discontinuity Expressions
1	 +ve moment	$w(x) = M_0\langle x-a\rangle^{-2}$ $V(x) = M_0\langle x-a\rangle^{-1}$ $M(x) = M_0\langle x-a\rangle^{0}$  
2	 Positive Load	$w(x) = P_0\langle x-a\rangle^{-1}$ $= P_0\langle x-a\rangle^{-1}$ $V(x) = P_0\langle x-a\rangle^{0}$ $P_0\langle x-a\rangle^{0}$ $M(x) = P_0\langle x-a\rangle^{1}$ $P_0\langle x-a\rangle^{1}$
 3	 → Load continues	$w(x) = w_0\langle x-a\rangle^{0}$ $V(x) = w_0\langle x-a\rangle^{1}$ $M(x) = \frac{w_0}{2}\langle x-a\rangle^{2}$ } Macaulay

Load Continues

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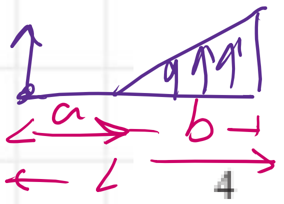
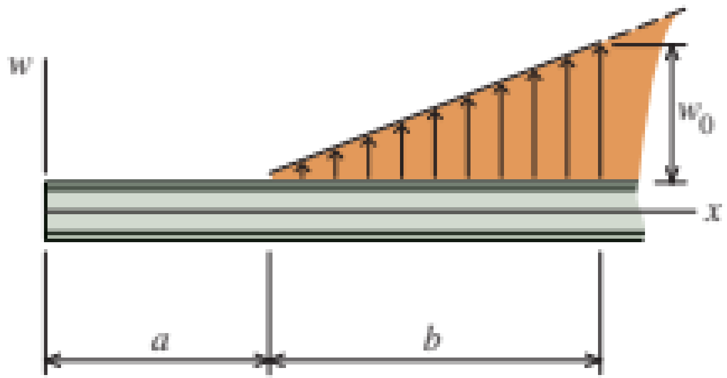
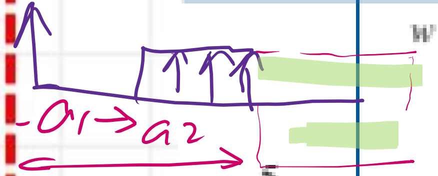
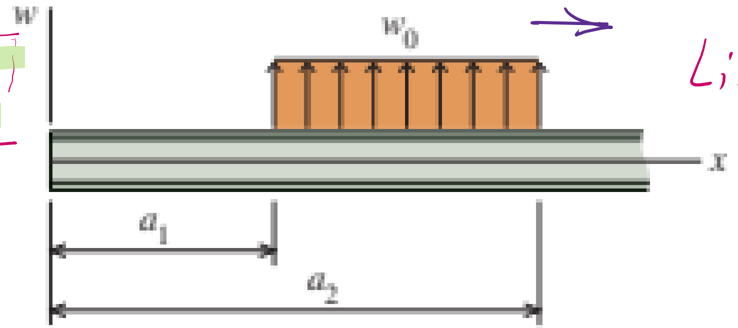
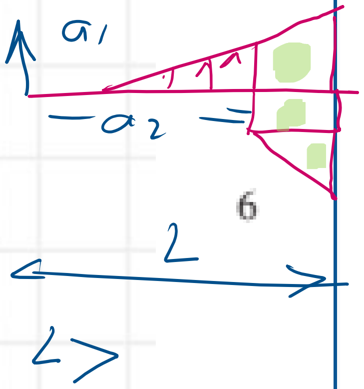
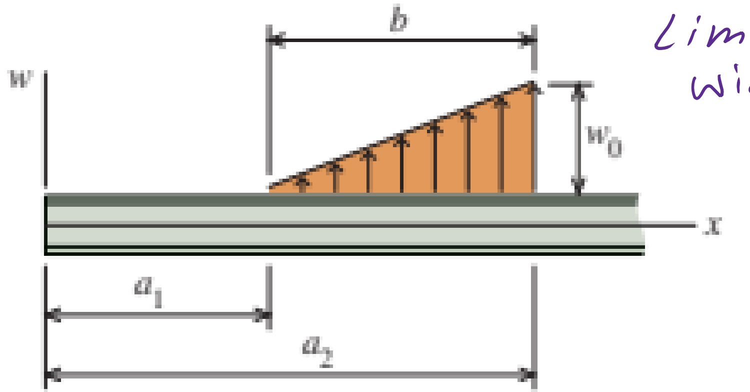
 4		$w(x) = \frac{w_0}{b}\langle x - a\rangle^1$ $V(x) = \frac{w_0}{2b}\langle x - a\rangle^2$ $M(x) = \frac{w_0}{6b}\langle x - a\rangle^3$	Macaulay is
 5		Deduct $w(x) = w_0\langle x - a_1\rangle^0 - w_0\langle x - a_2\rangle^0$ $V(x) = w_0\langle x - a_1\rangle^1 - w_0\langle x - a_2\rangle^1$ $M(x) = \frac{w_0}{2}\langle x - a_1\rangle^2 - \frac{w_0}{2}\langle x - a_2\rangle^2$	Limited width Macaulay's
 6		Deduct deduct $w(x) = \frac{w_0}{b}\langle x - a_1\rangle^1 - \frac{w_0}{b}\langle x - a_2\rangle^1 - w_0\langle x - a_2\rangle^0$ $V(x) = \frac{w_0}{2b}\langle x - a_1\rangle^2 - \frac{w_0}{2b}\langle x - a_2\rangle^2 - w_0\langle x - a_2\rangle^1$ $M(x) = \frac{w_0}{6b}\langle x - a_1\rangle^3 - \frac{w_0}{6b}\langle x - a_2\rangle^3 - \frac{w_0}{2}\langle x - a_2\rangle^2$	Limited width

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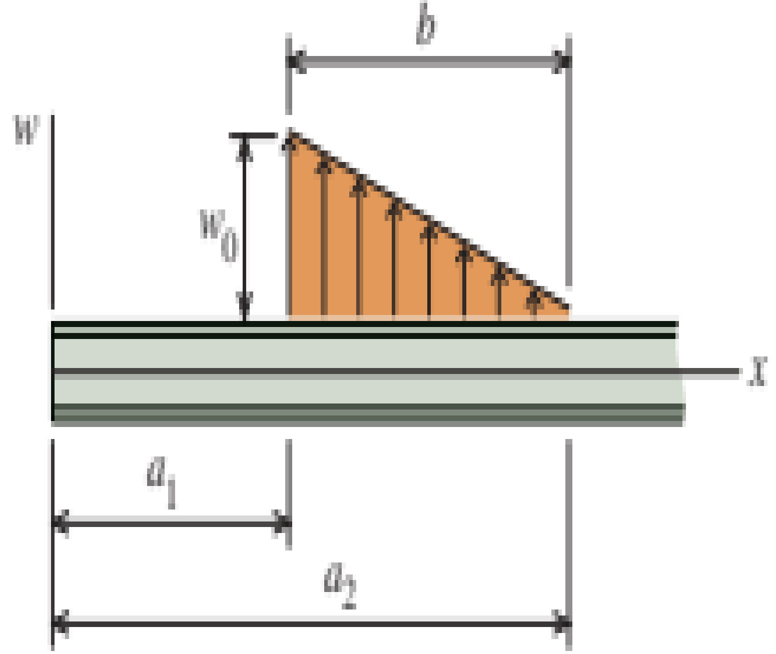
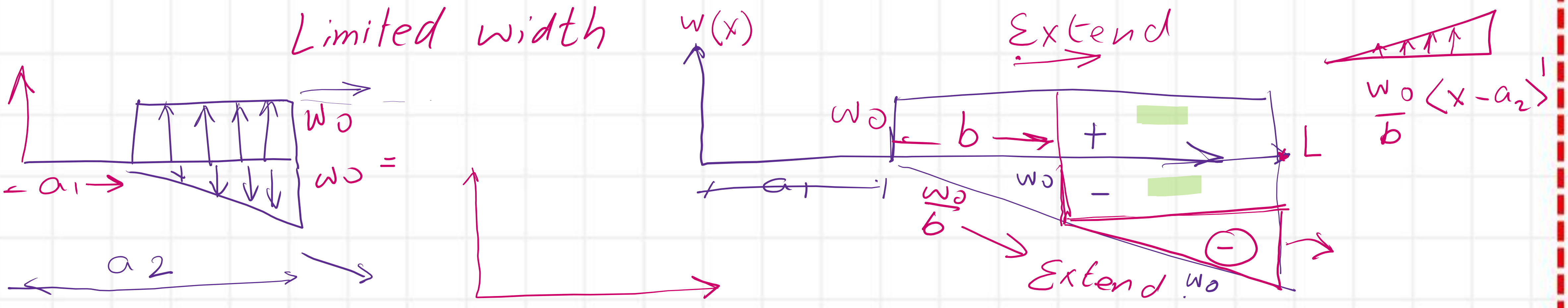
7		$w(x) = w_0\langle x - a_1\rangle^0 - \dfrac{w_0}{b}\langle x - a_1\rangle^1 + \dfrac{w_0}{b}\langle x - a_2\rangle^1$ $V(x) = w_0\langle x - a_1\rangle^1 - \dfrac{w_0}{2b}\langle x - a_1\rangle^2 + \dfrac{w_0}{2b}\langle x - a_2\rangle^2$ $M(x) = \dfrac{w_0}{2}\langle x - a_1\rangle^2 - \dfrac{w_0}{6b}\langle x - a_1\rangle^3 + \dfrac{w_0}{6b}\langle x - a_2\rangle^3$



P7.32–P7.42 For the beams and loadings shown in Figures P7.32–P7.42,

(a) use discontinuity functions to write the expression for $w(x)$; include the beam reactions in this expression.

(b) integrate $w(x)$ twice to determine $V(x)$ and $M(x)$.

(c) use $V(x)$ and $M(x)$ to plot the shear-force and bending-moment diagrams.

Cases 1, 2, 5

(A) C.L

(B) Moment

(C) U.L with Limited width

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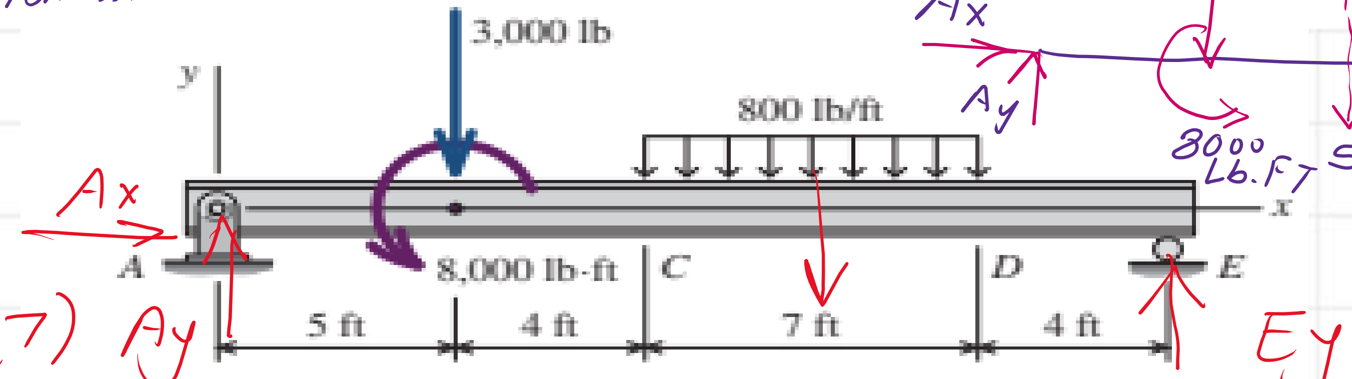
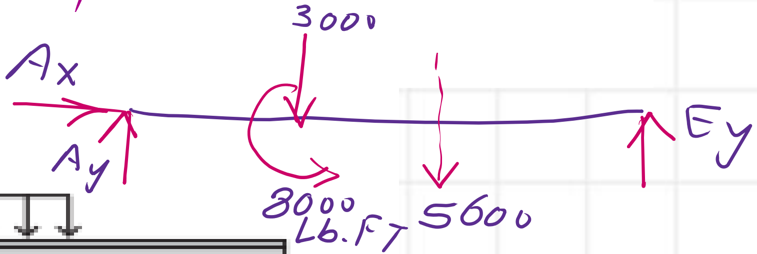


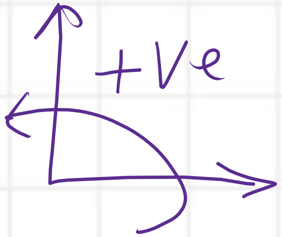
FIGURE P7.38



$\Sigma X = 0 \Rightarrow A_x = 0$

$\Sigma Y = 0$

$A_y + E_y = 3000 + 800(7)$
$= 8600$ Lb



$\Sigma M_E = 0$

$-A_y(20) + 8000 + 3{,}000(15) + 5600(7.5) = 0$

$A_y = \dfrac{(8000 + 45000 + 42000)}{20} = 4750$ Lb

$E_y = 8600 - 4750 = 3850$ Lb

Prepared by Eng.Maged Kamel.

Cases $\underline{1, 2, 5}$

Case ① Concentrated moment

Case ② Concentrated Load

Case 5 : Partially Loaded U.L

0 → 5'
5 → 9'
9 → 16'
16 → 20'

$$w(x) = 4750\langle x-0\rangle^{-1} - 3000\langle x-5\rangle^{-1} - 8000\langle x-5\rangle^{-2} - 800\langle x-9\rangle^{0} + 800\langle x-16\rangle^{0} + 3850\langle x-20\rangle^{-1}$$

ignored

4750 Lb

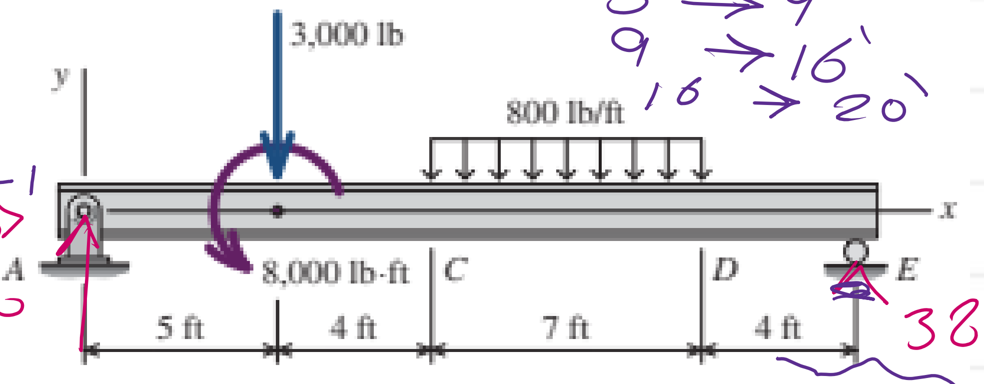
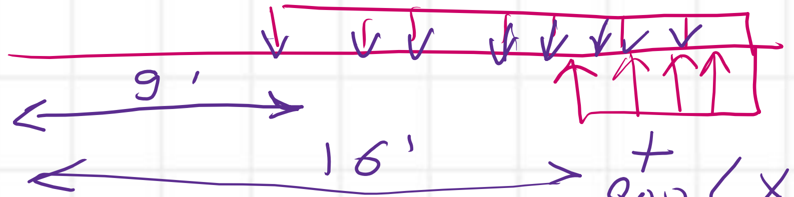


FIGURE P7.38

$-800\langle x-9\rangle^{0}$

=



$+800\langle x-16\rangle^{0}$

$$V(x) = 4750\langle x-0\rangle^{0} - 3000\langle x-5\rangle^{0} - 8000\langle x-5\rangle^{-1} - 800\langle x-9\rangle^{1} + 800\langle x-16\rangle^{1} + \underbrace{3850\langle x-20\rangle^{0}}_{\text{ignored}}$$

Prepared by Eng.Maged Kamel.

P7.32–P7.42 For the beams and loadings shown in Figures P7.32–P7.42,

(a) use discontinuity functions to write the expression for $w(x)$; include the beam reactions in this expression.

(b) integrate $w(x)$ twice to determine $V(x)$ and $M(x)$.

(c) use $V(x)$ and $M(x)$ to plot the shear-force and bending-moment diagrams.

$$M(x) = 4750\langle x-0\rangle^1 - 3000\langle x-5\rangle^1 - 8000\langle x-5\rangle^0 - \frac{800}{2}\langle x-9\rangle^2 + \frac{800}{2}\langle x-16\rangle^2 + [3850\langle x-20\rangle^1]$$

+ve

Parts

a, b

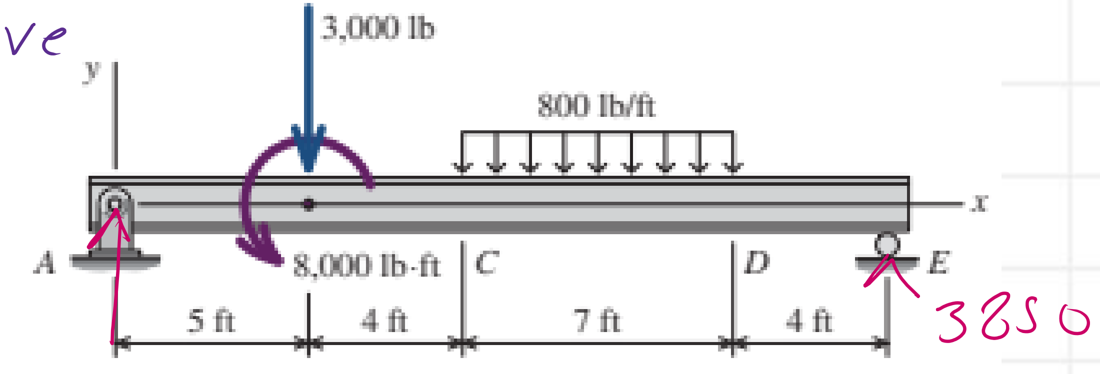
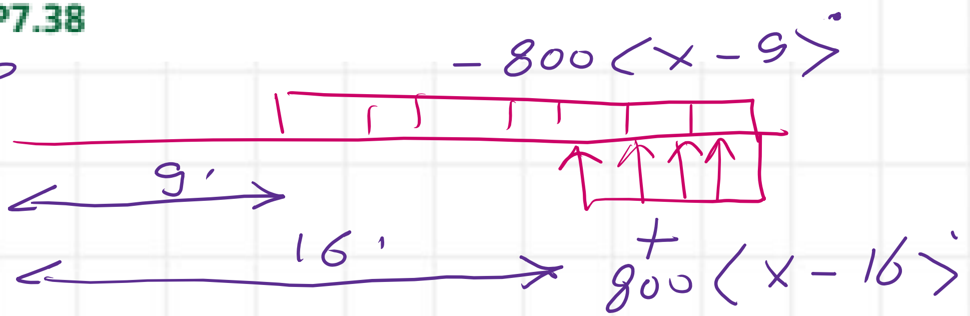


FIGURE P7.38

4750 Lb



Prepared by Eng.Maged Kamel.

$$V(x) = 4750\langle x-0\rangle^0 - \underset{C.L}{3000}\langle x-5\rangle^0 - \underbrace{8000\langle x-5\rangle^{-1}}_{\text{ignored}}^{C.M} - 800\langle x-9\rangle^1$$

$$+ 800\langle x-16\rangle^1 + 3850\langle x-20\rangle^0$$

$0 \rightarrow \leq 5$ $x \geq 5'$, $< 9'$

$V(x=0^+) = 4750(1) - 0 - 0 - 0 + 0 = 4750$ Lb

$V(x=3) = 4750(1) - 0 - 0 = 4750$ Lb

$V(x=5^-) = 4750(1) = 4750$ Lb

4750 Lb

$V(x=5^+) = 4750 - 3000 = 1750$ Lb

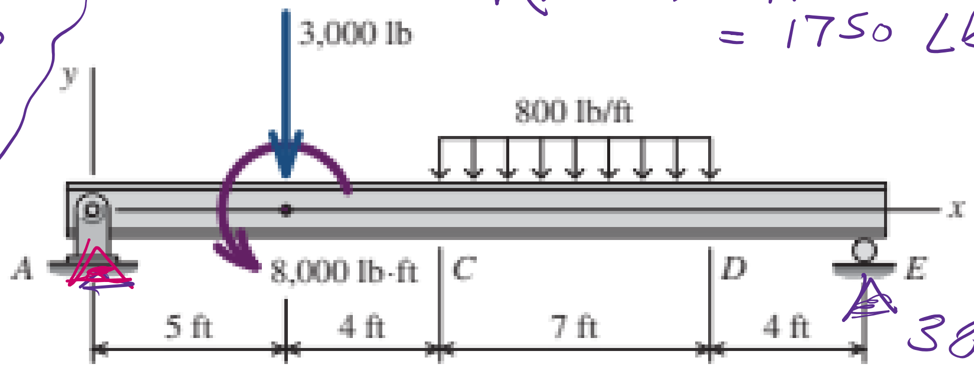


FIGURE P7.38

3850 Lb

$V(x=7) = 4750(7)^0 - 3000(2)^0 = 1750$ Lb

$V(x=9) = 4750(9)^0 - 3000(4)^0 = 1750$ Lb

$V(x=10) = 4750(10)^0 - 3000(5)^0 - 800(1)^1 = 1750 - 800 = 950$ Lb

$\downarrow$ 800 Lb drop

$V(x=11) = 4750(11)^0 - 3000(6)^0 - 800(2)^1 = 1750 - 1600 = 150$ Lb

$x \geq 9'$, $\leq 16'$

$4750 - 3000 - 800(x-9) = 0$

$800(x-9) = 1750 \Rightarrow x_{max} = 11.1875'$

Prepared by Eng.Maged Kamel.

$$V(x) = 4750\langle x-0\rangle^0 - 3000\langle x-5\rangle^0 - 8000\langle x-5\rangle^{-1} - 800\langle x-9\rangle^1 + 800\langle x-16\rangle^1 + 3850\langle x-20\rangle^0$$

ignored

$$V(x=12) = 4750(12)^0 - 3000(7)^0 - 800(3)^1 + 0 = 1750 - 2400 = -650 \text{ Lb}$$

$$V(x=15) = 4750(15)^0 - 3000(10)^0 - 800(6)^1 + 0 = 1750 - 4800 = -3050 \text{ Lb}$$

$$V(x=16') = 4750(16)^0 - 3000(11)^0 - 800(7)^1 = 1750 - 5600 = -3850 \text{ Lb}$$

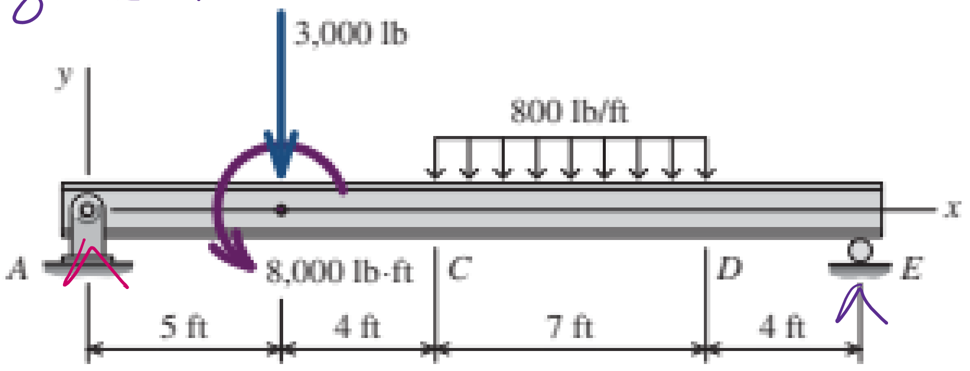


FIGURE P7.38

4750 Lb (x=0')

3850 Lb (x=20')

$$V(x=18') = 4750(18)^0 - 3000(13)^0 - 800(9)^1 + 800(2)^1 = 1750 - 7200 + 1600 = -3850 \text{ Lb}$$

$$V(x=20') = 4750(20)^0 - 3000(15)^0 - 800(11)^1 + 800(4)^1 = 1750 - 8800 + 3200 = -3850 \text{ Lb} = D_y$$

Prepared by Eng.Maged Kamel.

$$M(x) = 4750\langle x-0\rangle^{1} - 3000\langle x-5\rangle^{1} - 8000\langle x-5\rangle^{0} - \frac{800}{2}\langle x-9\rangle^{2}$$
$$+ \frac{800}{2}\langle x-16\rangle^{2} + 3850\langle x-20\rangle^{0}$$

$M(x=0^{+}) = 4750(0) - 0 - 0 - 0 + 0 = 0$ Lb.FT

$M(x=3) = 4750(3) - 0 - 0 = 14250$ lb.FT

$M(x=5) = 4750(5) - 0 - 0 = 23750$ Lb.FT

$x \geq 5'$

$5^{+} = 23750 - 8000(+)^{0} = 15750$ Lb.FT

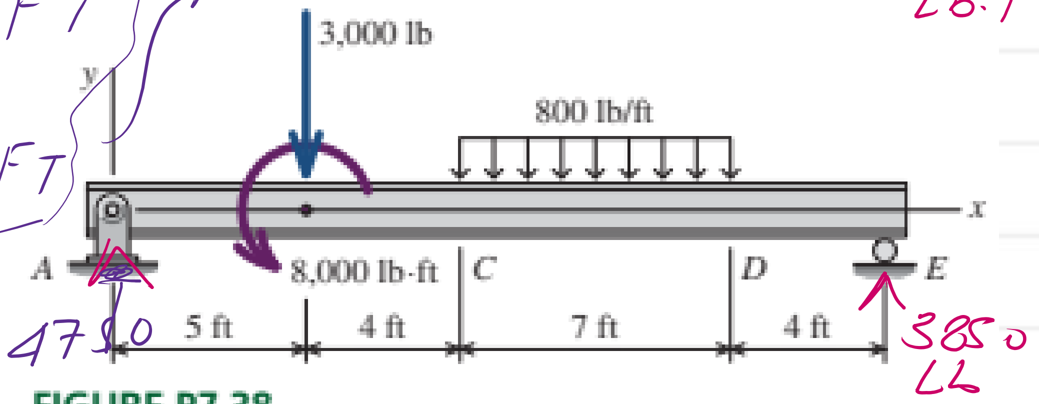


FIGURE P7.38

$M(x=7) = 4750(7) - 3000(2) - 8000(2)^{0}$

$M(x=7) = 33250 - 6000 - 8000$
$= 19250$ Lb.FT

$M(x=9) = 4750(9) - 3000(4) - 8000(4)^{0} = 22750$ lb.FT

$M(x=11) = 4750(11) - 3000(6) - 8000(6)^{0} - \frac{800}{2}(2)^{2} = 24650$ Lb.FT

Prepared by Eng.Maged Kamel.

$$M(x) = 4750\langle x-0\rangle^1 - 3000\langle x-5\rangle^1 - 8000\langle x-5\rangle^0 - \frac{800}{2}\langle x-9\rangle^2 + \frac{800}{2}\langle x-16\rangle^2 + 3850\langle x-20\rangle^1$$

$$M(x=12') = 4750(12)^1 - 3000(7)^1 - 8000(7)^0 - 400(3)^2 + 0 = +24400 \text{ lb.FT}$$

$$M(x=14') = 4750(14)^1 - 3000(9)^1 - 8000(9)^0 - 400(5)^2 + 0 = 21500 \text{ lb.FT}$$

$$M(x=16') = 4750(16)^1 - 3000(11)^1 - 8000(11)^0 - 400(7)^2 + 0 = 15400 \text{ lb.FT}$$

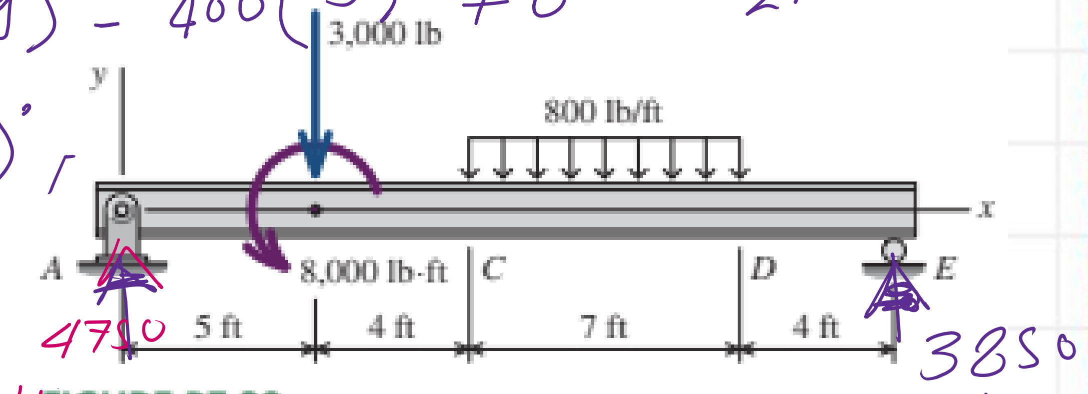


FIGURE P7.38

$$M(x=18') = 4750(18)^1 - 3000(13)^1 - 8000(13)^0 - 400(9)^2 + 400(2)^2 \longrightarrow = 7700 \text{ lb.FT}$$

$$M(x=20') = 4750(20)^1 - 3000(15)^1 - 8000(15)^0 - 400(11)^2 + 400(4)^2 = 0$$

Prepared by Eng.Maged Kamel.

0 → 5'
5' → 9'
9' → 16'
16' → 20'

	A	B	C
1	Ay+Ey	8600	Lb
2			
3			
4	Ay	4750	Lb
5	Ey	3850	lb
6			
7			
8	Ey	3850	lb
9			
10			
11			
12	A	B	C
13	Distance x (FT)	Shear V(x)-Lb	Moment in lb-Ft
14	0	4750	0
16	2	4750	9500
17	3	4750	14250
18	4	4750	19000
19	5	4750	23750
20	5	1750	15750
21	6	1750	17500
22	7	1750	19250
23	8	1750	21000
24	9	1750	22750
25	10	950	24100
26	11	150	24650
27	11.1875	0	24664.06
28	12	-650	24400
29	13	-1450	23350
30	14	-2250	21500

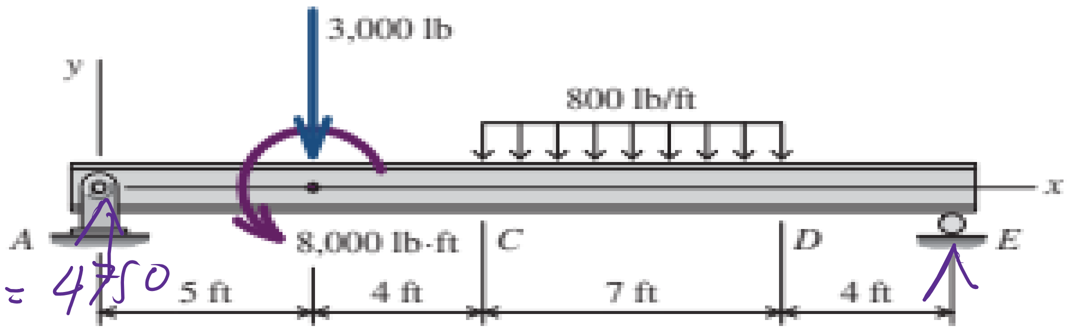


FIGURE P7.38

Ay = 4750

3850 Lb

Drop at x = 5'

Change in slope at x = 9'

x = 5'

= No =

B19 =IF(AND(0<=A19,A19<=5),4750,IF(AND(5<A19,A19<=9),4750-3000,IF(AND(9<A19,A19<=16),4750-3000-800*(A19-9),IF(AND(16<A19,A19<=20),4750-3000-800*(A19-9)+800*(A19-16),0))))

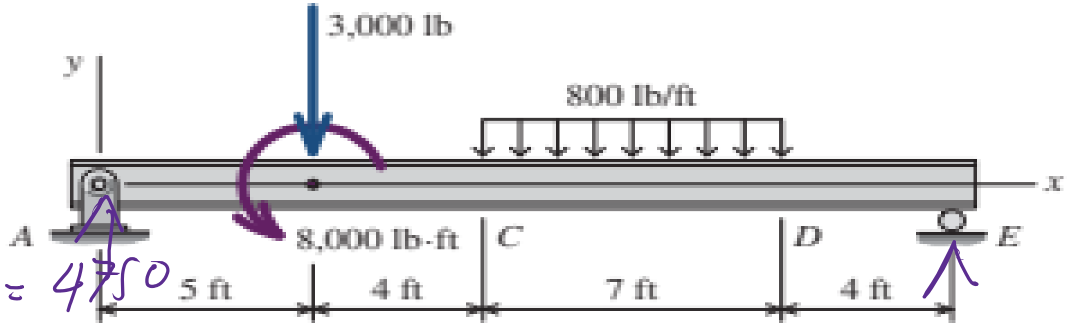
x = 5'

No = =

B20 =IF(AND(0<=A20,A20<5),4750,IF(AND(5<=A20,A20<=9),4750-3000,IF(AND(9<A20,A20<=16),4750-3000-800*(A20-9),IF(AND(16<A20,A20<=20),4750-3000-800*(A20-9)+800*(A20-16),0))))

	A	B	C
1	Ay+Ey	8600	Lb
2			
3			
4	Ay	4750	Lb
5	Ey	3850	lb
6			
7			
8	Ey	3850	lb
9			
10			
11			
12	A	B	C
13	Distance x (FT)	Shear V(x)-Lb	Moment in lb-Ft
14	0	4750	0
16	2	4750	9500
17	3	4750	14250
18	4	4750	19000
19	5	4750	23750
20	5	1750	15750
21	6	1750	17500
22	7	1750	19250
23	8	1750	21000
24	9	1750	22750
25	10	950	24100
26	11	150	24650
27	11.1875	0	24664.06
28	12	-650	24400
29	13	-1450	23350
30	14	-2250	21500

$0 \to 5'$
$5' \to 9'$
$9' \to 16'$
$16' \to 20'$



Ay = 4750

FIGURE P7.38

3850 Lb

Drop at $x = 5'$

Change in slope $x = 9'$

$x = 9'$

B_{24}

=IF(AND(0<=A24,A24<=5),4750,IF(AND(5<A24,A24<=9),4750-3000,IF(AND(9<A24,A24<=16),4750-3000-800*(A24-9),IF(AND(16<A24,A24<=20),4750-3000-800*(A24-9)+800*(A24-16),0))))

=

V(X)

S.F.D

Shearing Force diagram

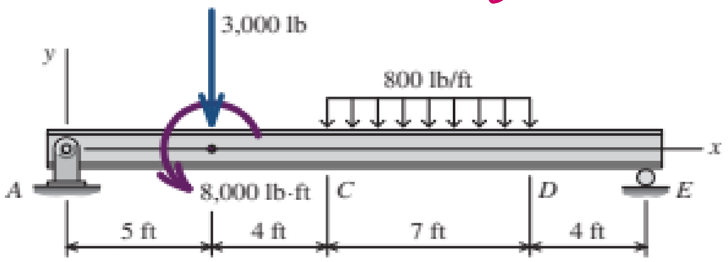


FIGURE P7.38

$V(X) =$

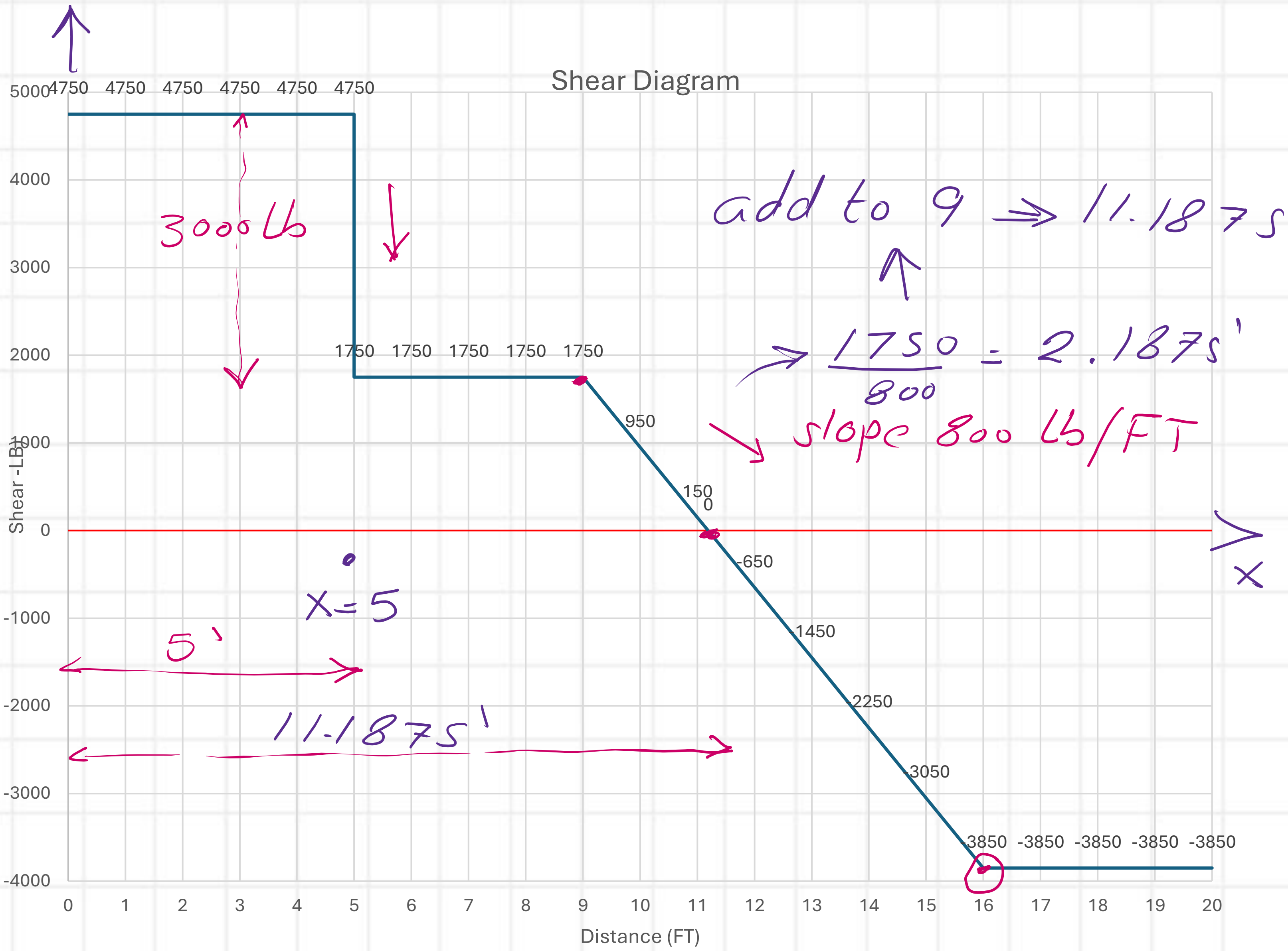
$[+1750-(800)(7)]$

$1750 - 5600 = -3850$ Lb

@ $X = 16'$

$X_{max} = 11.1875'$

$V(X) = 0$



	A	B	C
16	2	4750	9500
17	3	4750	14250
18	4	4750	19000
19	5	4750	23750
20	5	1750	15750
21	6	1750	17500
22	7	1750	19250
23	8	1750	21000
24	9	1750	22750
25	10	950	24100
26	11	150	24650
27	11.1875	0	24664.06
28	12	-650	24400
29	13	-1450	23350
30	14	-2250	21500

$\to X_{max} \Rightarrow 24664.06$ Lb.FT

Moment

$0' \to 5'$
$5' \to 9'$
$9' \to 16'$
$16' \to 20'$

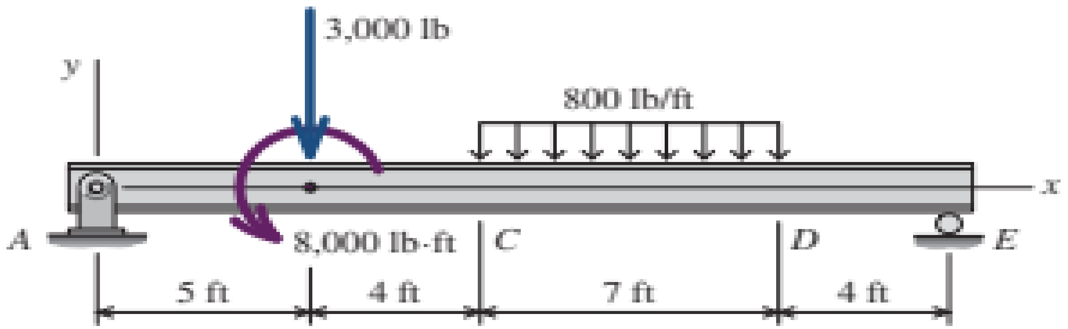


FIGURE P7.38

$x \leq 5$

$x = 5'$

Drop at 5' $x \leq 5'$ $x \geq 5'$

C_{19}

```
=IF(AND(0<=A19,A19<=5),4750*A19,IF(AND(5<A19,A19<=9),4750*A19-3000*(A19-5)-8000,IF(AND(9<A19,A19<=16),4750*A19-3000*(A19-5)-8000-800*0.5*(A19-9)^2,IF(AND(16<A19,A19<=20),4750*A19-3000*(A19-5)-8000-800*0.5*(A19-9)^2+800*0.5*(A19-16)^2,0))))
```

Lower cell

$x = 5'$

$\geq 5'$

C_{20}

```
=IF(AND(0<=A20,A20<5),4750*A20,IF(AND(5<=A20,A20<=9),4750*A20-3000*(A20-5)-8000,IF(AND(9<A20,A20<=16),4750*A20-3000*(A20-5)-8000-800*0.5*(A20-9)^2,IF(AND(16<A20,A20<=20),4750*A20-3000*(A20-5)-8000-800*0.5*(A20-9)^2+800*0.5*(A20-16)^2,0))))
```

Bending Moment Diagram

B.M.D

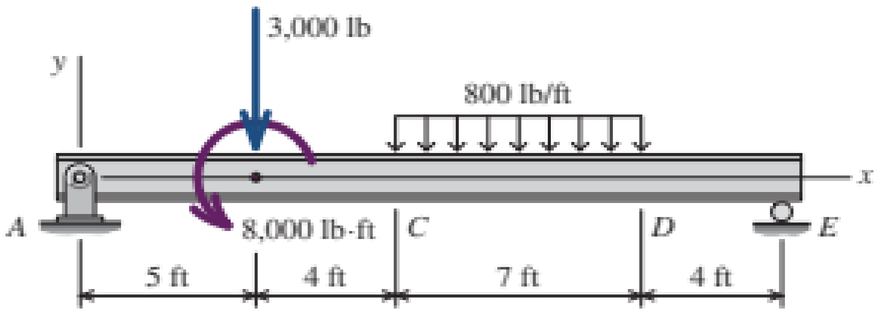


FIGURE P7.38

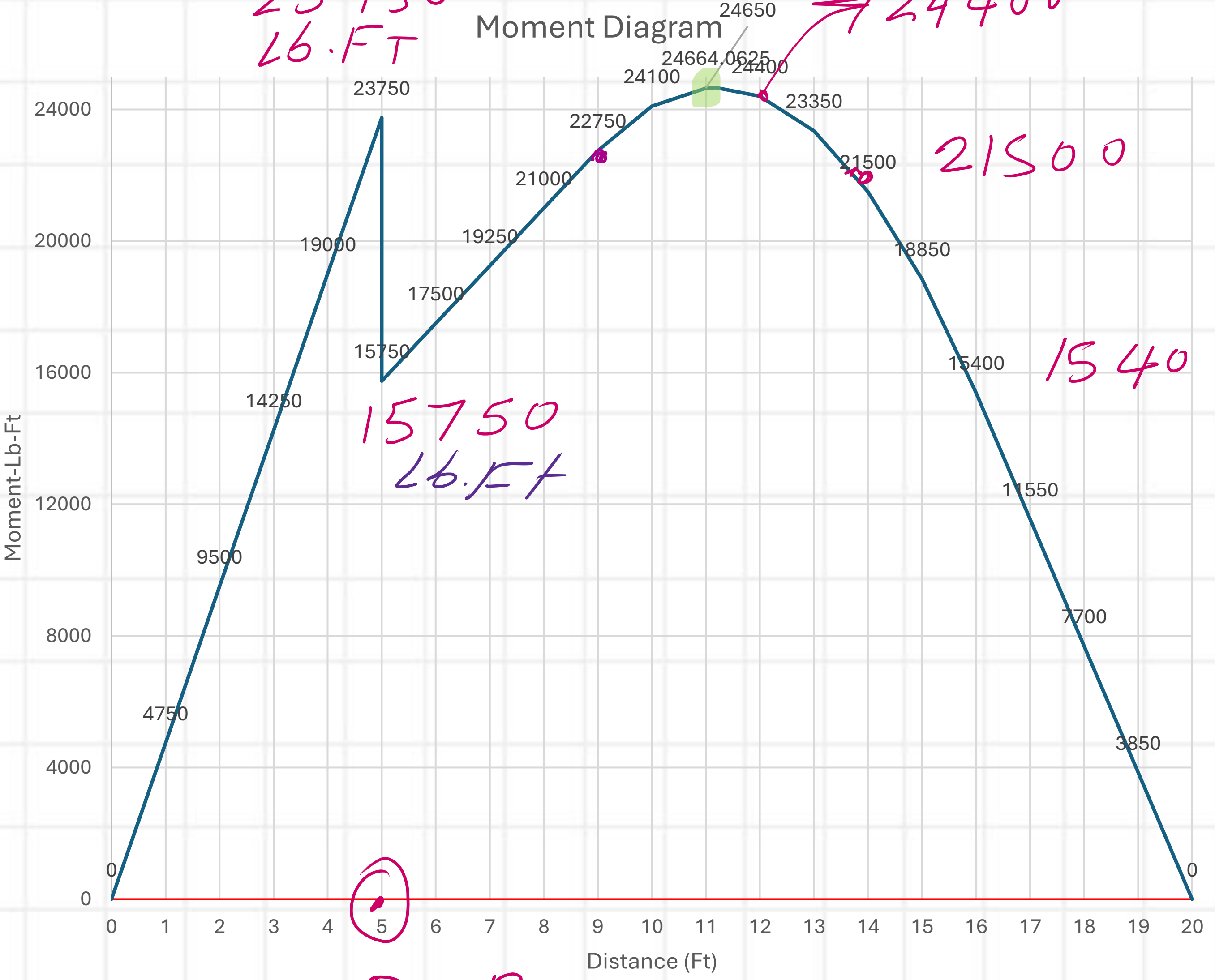
$X_{max} = 11.1875'$

$M_{max} = 24664$ Lb.FT

Highlighted

23750 Lb.FT

Lb.FT 24400



21500

15400

15750 Lb.Ft

Drop by 8000 Lb.FT