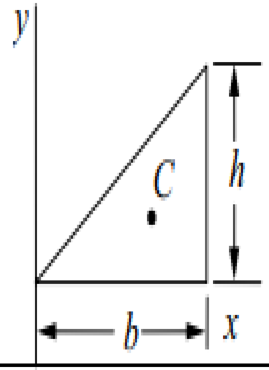
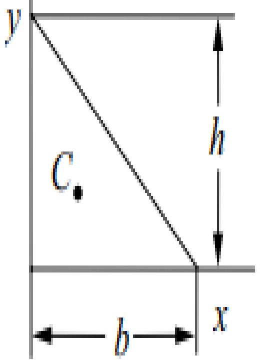
Case - 2

I_y at C_g

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = bh/2$ $x_c = 2b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/4$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/2$	$I_{x_c y_c} = Abh/36 = b^2h^2/72$ $I_{xy} = Abh/4 = b^2h^2/8$
	$A = bh/2$ $x_c = b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/6$	$I_{x_c y_c} = -Abh/36 = -b^2h^2/72$ $I_{xy} = Abh/12 = b^2h^2/24$

I_y

$r_y^2 = b^2/2$

I_y : moment of inertia –

Case-2 → opposite to the right

Use a VL strip

dx & y Dimension

$$y = \frac{h}{b} x$$

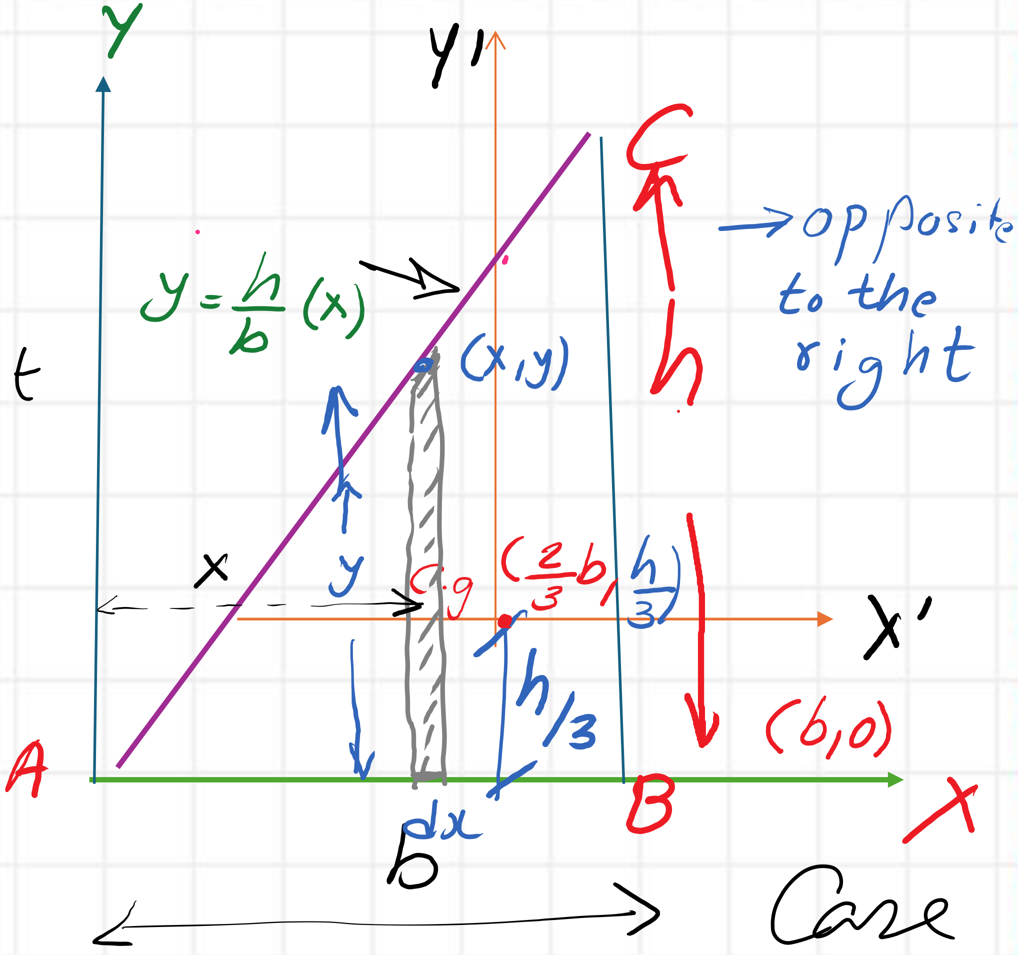
$$dI_y = (dx\, y)(x^2)$$

$$dI_x = \left(\frac{h}{b}\right) x\, (x^2)\, dx$$

$$\int dI_x = \int_{x=0}^{x=b} \frac{h}{b} x^3 dx \Rightarrow \frac{h}{b} \frac{x^4}{4} \Big|_0^b = \frac{h\, b^4}{4b} = \frac{hb^3}{4} \rightarrow \frac{b^3 h}{4}$$

Case #2

$$K_y^2 = \frac{b^3 h}{4\,(0.5\, bh)} = \frac{b^2}{2}$$



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Other option For I_y → other y-axis

Case - 2 → opposite to the right

Area of strip

$dA = dx \cdot y \rightarrow y = \frac{(b-x)\,h}{b}$

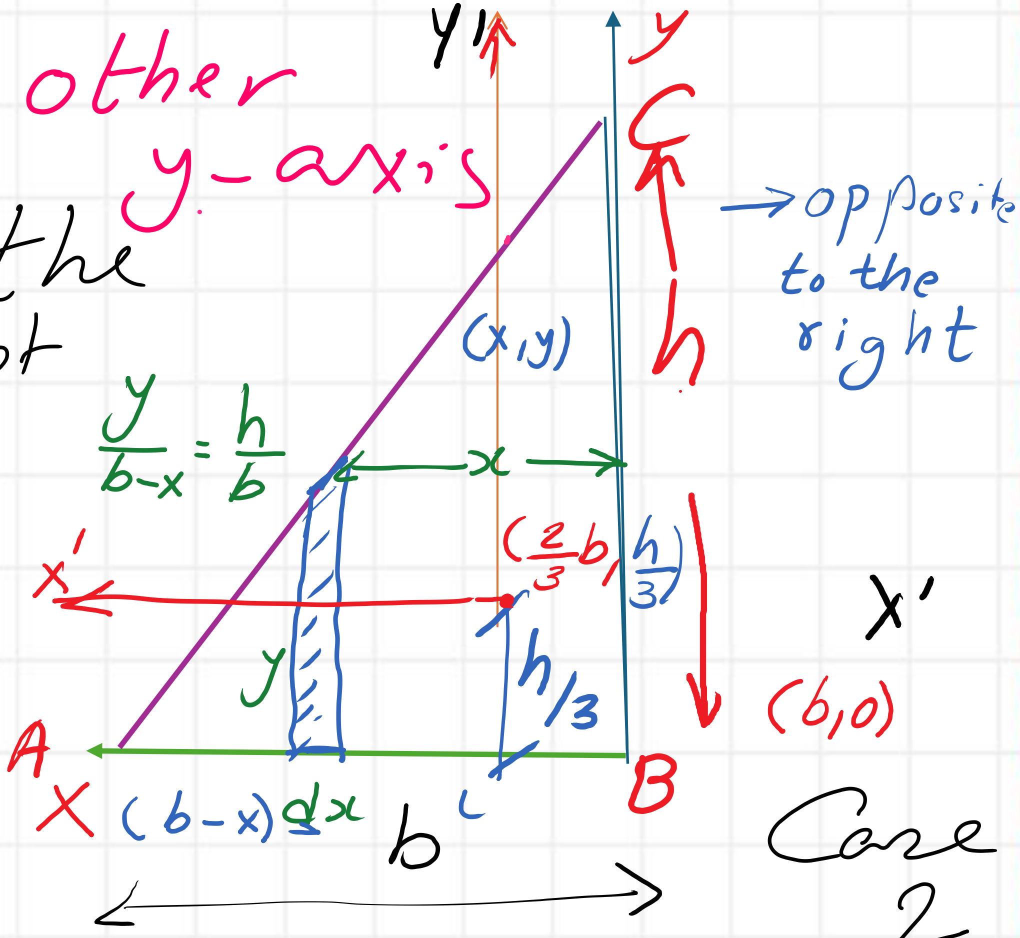
$dI_x = (y\,dx)\,x^2$

$\int dI_y = \int_0^b \frac{(b-x)}{b} h\, x^2\, dx$

$I_y = \frac{h}{b}\int_0^b (b x^2 dx - x^3 dx)$

$I_y = \frac{h}{b}\left(\left(b\frac{x^3}{3}\Big|_0^b - \frac{b}{b}\left(\frac{x^4}{4}\Big|_0^b\right)\right)\right)$

$I_y = \frac{h}{b}\left(\frac{b^4}{3} - \frac{b^4}{4}\right) = \frac{hb^4}{12b} = hb^3/12$



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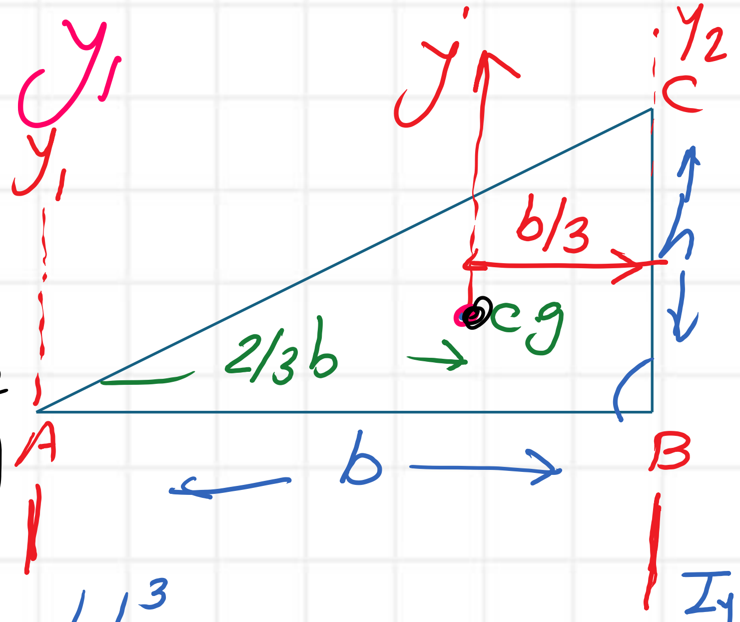
For $I_{y\,cg}$ → From y_1



$I_{y_1} = hb^3/4$

$$I_{ycg} = \frac{hb^3}{4} - \frac{1}{2}bh\left(\frac{2}{3}b\right)^2$$

$$= \frac{4.5hb^3 - 4hb^3}{18}$$

$$I_{y_1} = \frac{hb^3}{4}$$

$$I_{y_2} = \frac{hb^3}{12}$$

$$I_{y\,c.g} = \frac{1}{36}hb^3$$

Same

$$K^2_{y\,cg} = I_{y\,cg}/A = \frac{hb^3}{36}\left(\frac{1}{\frac{1}{2}bh}\right) = \frac{b^2}{18}$$

While For I_{yG} from right

$$I_{yg} = \frac{hb^3}{12} - \frac{1}{2}(b)(h)\left(\frac{b}{3}\right)^2 = \frac{hb^3}{12} - \frac{hb^3}{18} = \frac{hb^3(1.5-1)}{18} = \frac{hb^3}{36}$$

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Polar moment of inertia - Case - 2 at Left Corner)

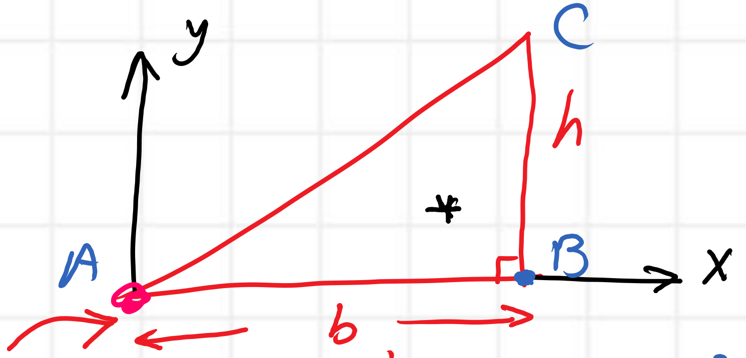
AT Corner (x, y Intersection)

Recall

$I_x : bh^3/12$

$I_y : hb^3/4$

$$J_o = I_x + I_y = \frac{bh^3}{12} + \frac{hb^3}{4} = \frac{4bh}{12}(h^2+b^2) = \frac{1}{3} bh(h^2+b^2)$$



$I_x = \frac{bh^3}{12}$

$I_y = \frac{hb^3}{4}$

$I_x = \frac{bh^3}{12}$

$I_{y_{BC}} = \frac{hb^3}{12}$

$J_o = \frac{1}{12} hb(h^2+b^2)$

While J_{oG} at Centroid

J_{oG}

$I_{xg} : bh^3/36$

$I_{yg} : hb^3/36$

add $J_{oG} = bh(h^2+b^2)/36$

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Polar moment of inertia - Case - 2 at point B, right - Corner

AT Corner (x, y Intersection)

Recall

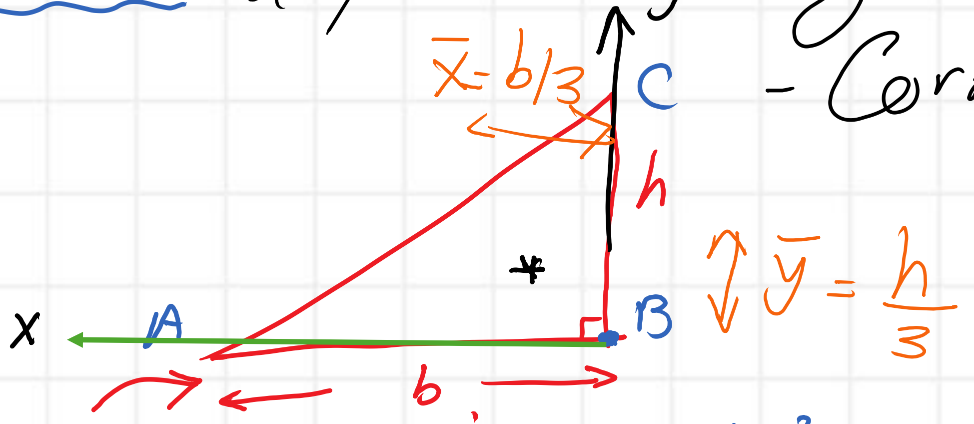
$I_x : bh^3/12$

$I_y : hb^3/12$

$J_o = I_x + I_y$

$= \frac{bh^3}{12} + \frac{hb^3}{12}$

$J_o = \frac{1}{12} bh(h^2+b^2)$



$I_x = \frac{bh^3}{12}$

$I_y = \frac{hb^3}{4}$

$I_x = \frac{bh^3}{12}$

$I_{y_{bc}} = \frac{hb^3}{12}$

$J_o = \frac{1}{12} hb(h^2+b^2)$

while J_{oG} at Centroid $\bar{x} = \frac{b}{3}$ & $\bar{y} = h/3$

$$J_{oG} = J_o - A\bar{x}^2_{c.g} - A\bar{y}^2_{cg} = \frac{1}{12} hb(h^2+b^2) - A(x^2_{c.g} + y^2_{c.g})$$

$$= \frac{1}{12} hb(h^2+b^2) - \frac{1}{2} bh\left(\frac{b^2}{9} + \frac{h^2}{9}\right)$$

$$J_{oG} = \frac{hb}{12}\left(h^2+b^2 - \frac{2}{3}b^2 - \frac{2}{3}h^2\right) = \frac{hb}{36}(h^2+b^2)$$ same value

Prepared by Eng.Maged Kamel.