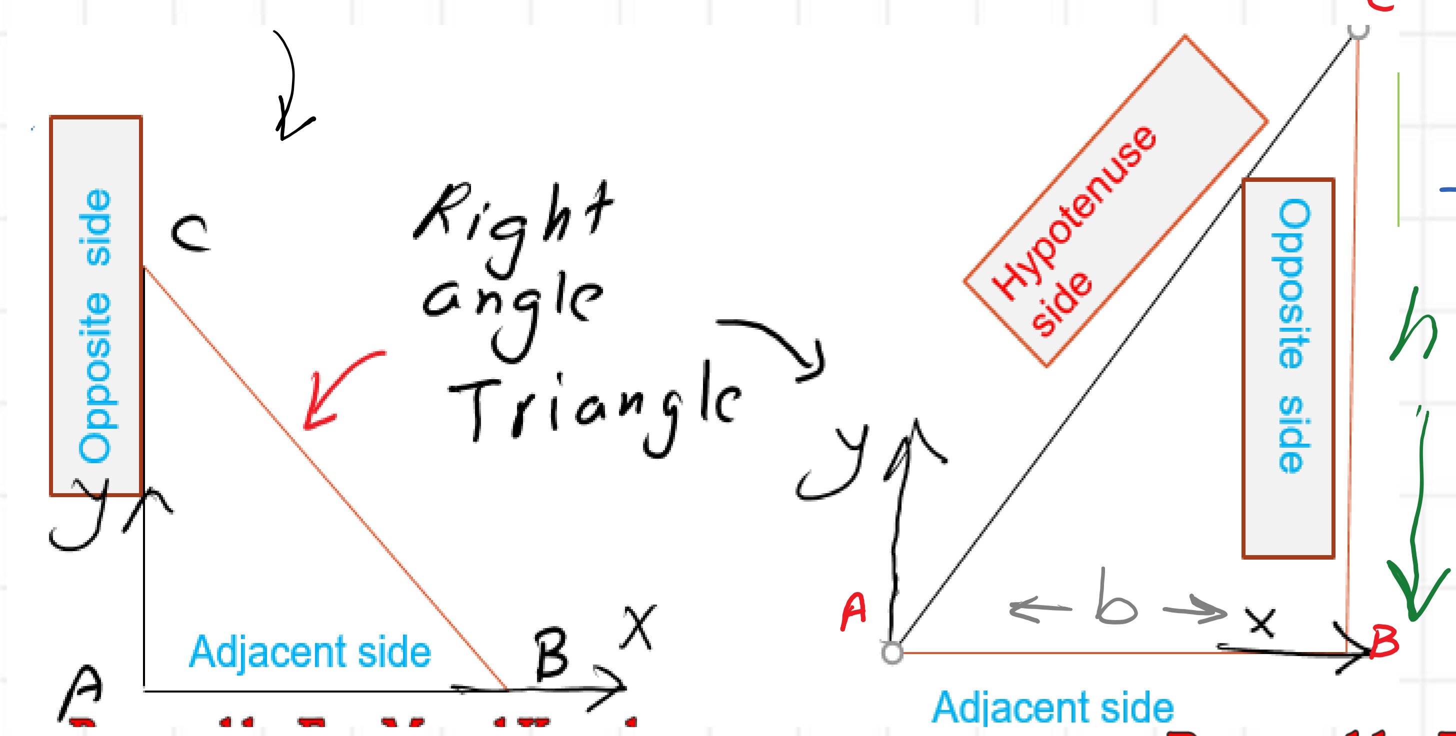
Objective of the lecture

1- How to determine the Ix for the right angle triangle by using:

a- Horizontal strip parallel to the external x axis → at the adjacent base

b- Vertical strip perpendicular to the external x axis. → at the adjacent base

This is the case #1



This is the Case #2

Moment for inertia – I_x

Case-2 : opposite is to the right

Strip is horizontal

$y = \frac{h}{b}x \rightarrow$ then $x = by/h$

width $= b-x$

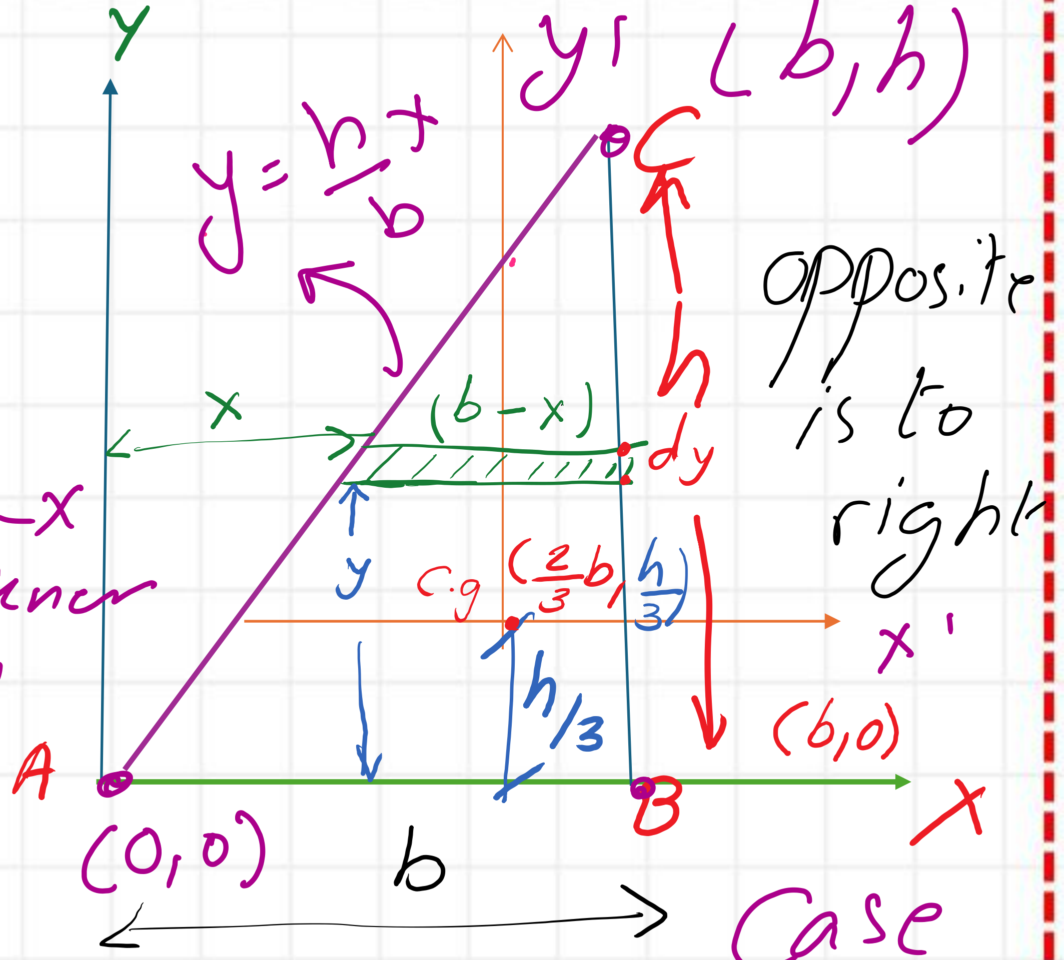
thickness dy

$dA = (b-x)\,dy$

$dI_x = (b-x)\,dy\,(y^2)$

$dI_x = \left(b - \frac{by}{h}\right) y^2\,dy = \left(\frac{hb - by}{h}\right) y^2\,dy$

$dI_x = \frac{b}{h}(h-y)\,y^2\,dy$



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Continue with $\int dI_x$

Find I_x & I_{xG}

$$\int dI_x = \int^{h} \frac{b}{h}\left(h y^2 dy - y^3 dy\right)$$

$$I_x = \frac{b}{h}\left(h \frac{y^3}{3}\Big|_0^h - \frac{y^4}{4}\Big|_0^h\right) = \frac{b}{h}\left(\frac{h^4}{3} - \frac{h^4}{4}\right)$$

$$I_x = \frac{b}{h}\left(\frac{1}{12}\right) h^4 \Rightarrow I_x = \frac{b}{12} h^3$$

For I_{xG} $\Rightarrow$ $I_{xG} = I_x - A \cdot y_{bar}^{2}$ $y_b = h/3$

$$I_{xG} = \frac{bh^3}{12} - \frac{1}{2}(bh)\left(\frac{h^2}{9}\right)$$

$$I_{xG} = \frac{1.5\,bh^3}{18} - \frac{bh^3}{18} = \frac{1}{36} bh^3$$

$\Rightarrow$ Parallel axes

Moment of inertia – Case #2

Case #2 → opposite to the right

Using a VL strip

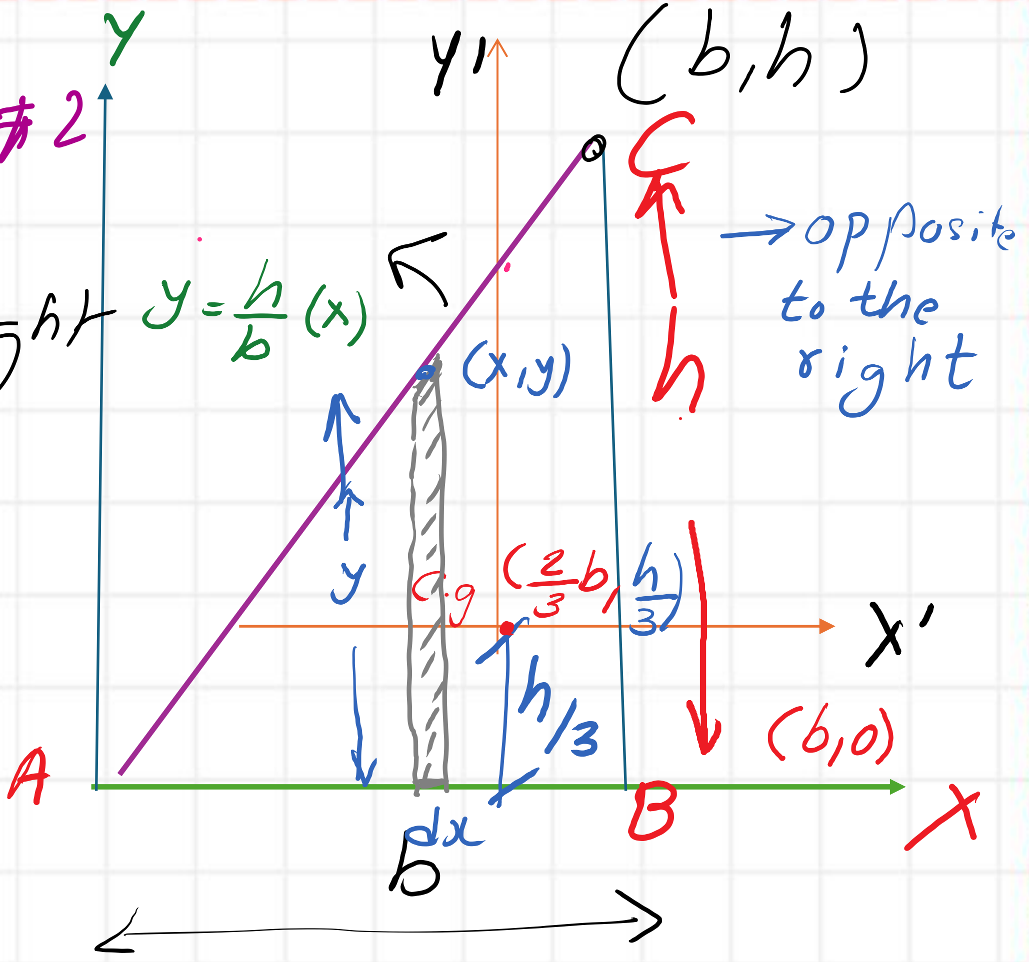
$y = \frac{h}{b} x$, width: dx, height: y } strip

$dI_x = dx \frac{(y)^3}{3}$

$y^3 = \frac{h^3}{b^3} x^3 \Rightarrow \int dI_x = \int_0^b \frac{1}{3} \frac{h^3}{b^3} x^3 \, dx$

$I_x = \frac{1}{3} \frac{h^3}{b^3} \frac{x^4}{4} \Big|_0^b = \frac{1}{3} \frac{h^3}{b^3} \left(\frac{1}{4}\right) b^4$

$= \frac{1}{3} h^3 b$



Same as using a HL strip

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Radius of Gyration K_x

$$K_x^2 = I_x / A = \frac{bh^3}{12}\left(\frac{1}{\frac{1}{2}bh}\right) \Rightarrow \frac{h^2}{6}$$

$$K_x = \sqrt{\frac{h^2}{6}} = \frac{h}{\sqrt{6}}$$

Radius of Gyartion about x-axis at the base

Find $K_{x\text{-}G}$

$$I_{xg} = \frac{bh^3}{36}$$

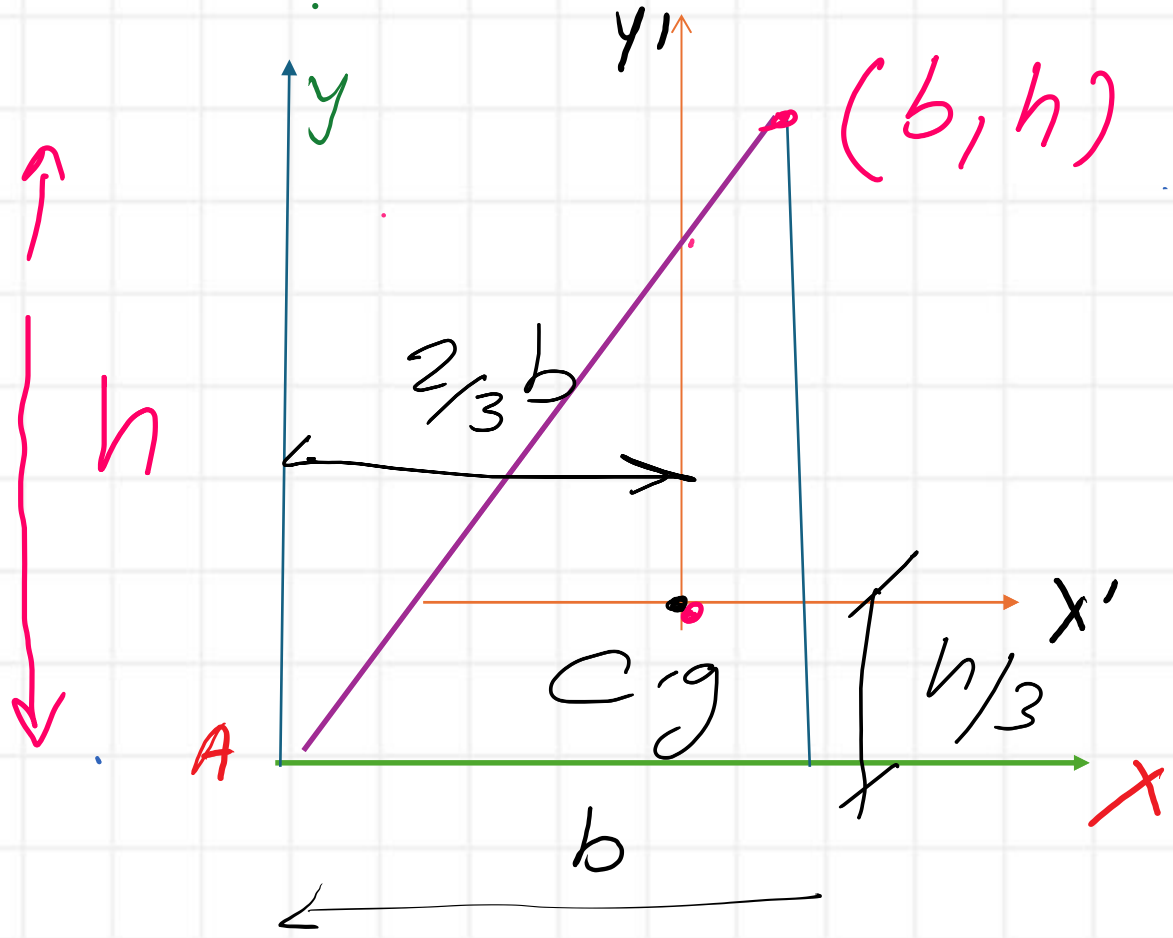
$$K_G^2 = I_{xG}/A$$

$$I_{xg} = \frac{bh^3}{36}$$

$$A = \frac{1}{2}bh$$

$$K_{xG}^2 = \frac{bh^3}{36}\left(\frac{1}{\frac{1}{2}bh}\right) = \frac{h^2}{18}$$

$$K_{xG} = \frac{h}{2\sqrt{3}}$$



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