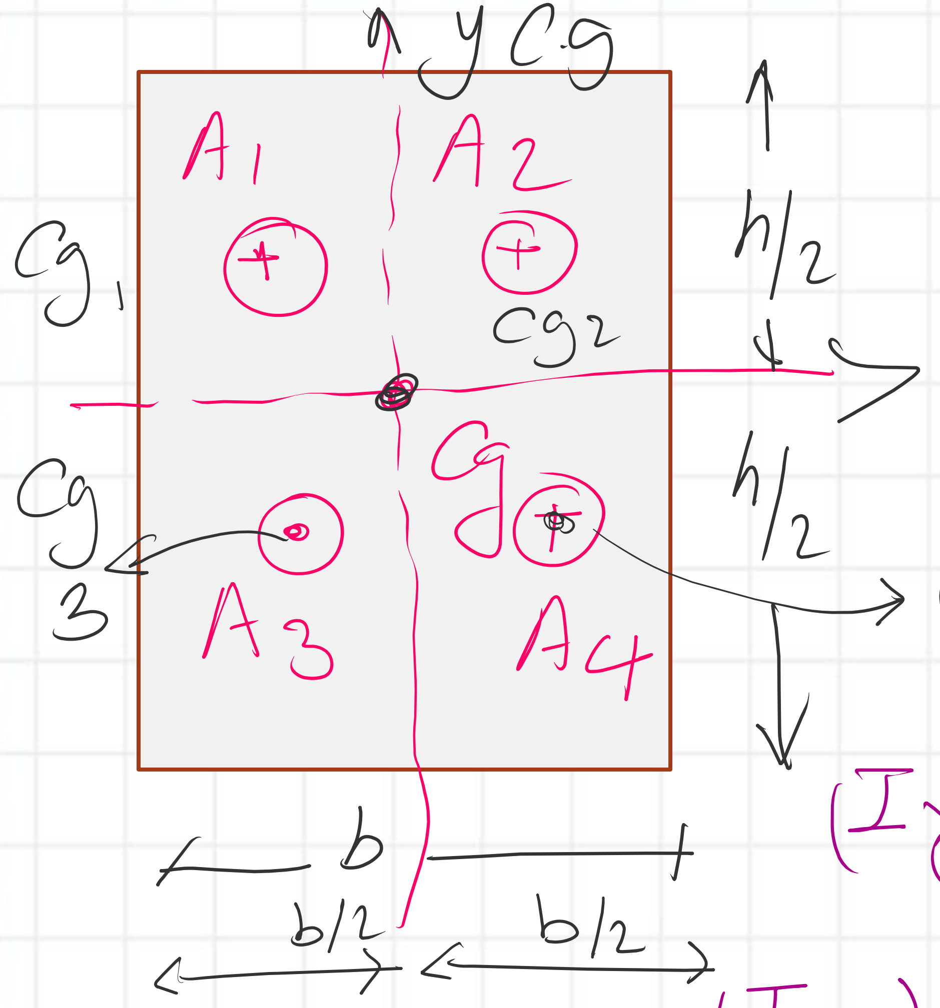

I_{xy} at Cg (origion of axes is at Cg)

For a rectangular section

Due to Symmetry $A_1 = A_2 = A_3 = A_4$

$$I_{xy\,Cg} = \sum_{A_1}^{A_4} I_{xy}$$

$$(I_{xy})_{A_1\,Cg} = A_1\left(-\frac{b}{4}\right)\left(\frac{h}{4}\right) = -A_1\frac{bh}{16}$$

$$(I_{xy})_{A_2\,Cg} = A_1\left(\frac{b}{4}\right)\left(\frac{h}{4}\right) = +A_1\frac{bh}{16}$$

$$(I_{xy})_{A_3\,Cg} = A_1\left(\frac{-b}{4}\right)\left(-\frac{h}{4}\right) = +A_1\frac{bh}{16}$$

$$(I_{xy})_{A_4\,Cg} = A_1\left(\frac{+b}{4}\right)\left(-\frac{h}{4}\right) = A_1\left(-\frac{bh}{16}\right)$$

$$\sum I_{xy\,Cg} = 0$$

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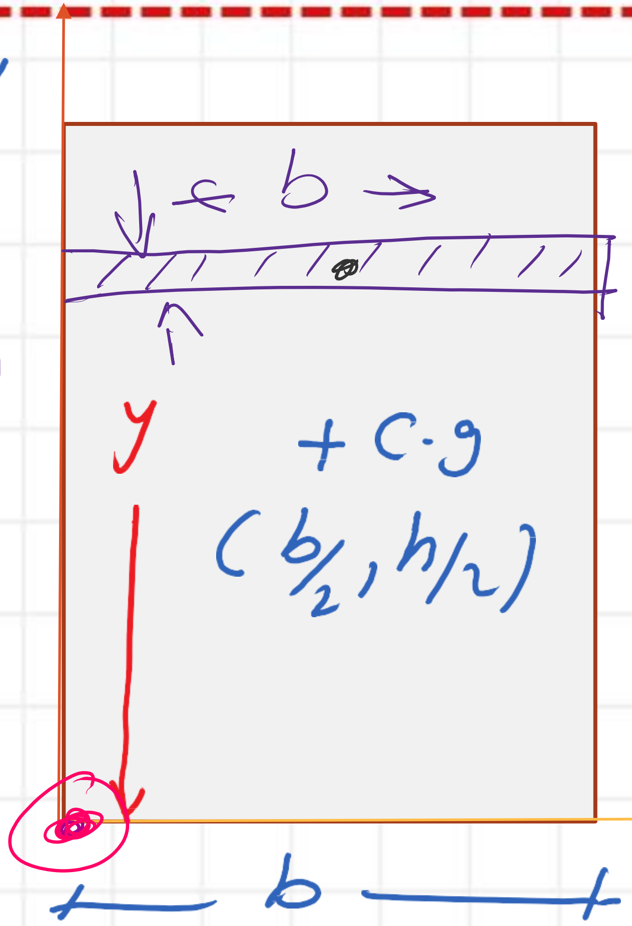
Same result Can be obtained by introducig a horizontal strip of Length b, depth dy

$$I_{xy} = \int_{y=0}^{y=h} (b\,dy)\left(\frac{b}{2}\right) y$$

$$= \frac{b^2}{2}\left[\frac{y^2}{2}\right]_0^h = \frac{b^2}{4}\left[h^2-0\right]$$

$$I_{xy} = \frac{b^2h^2}{4}$$



Cg at b/2 and y

origion of axes is at Left Corner

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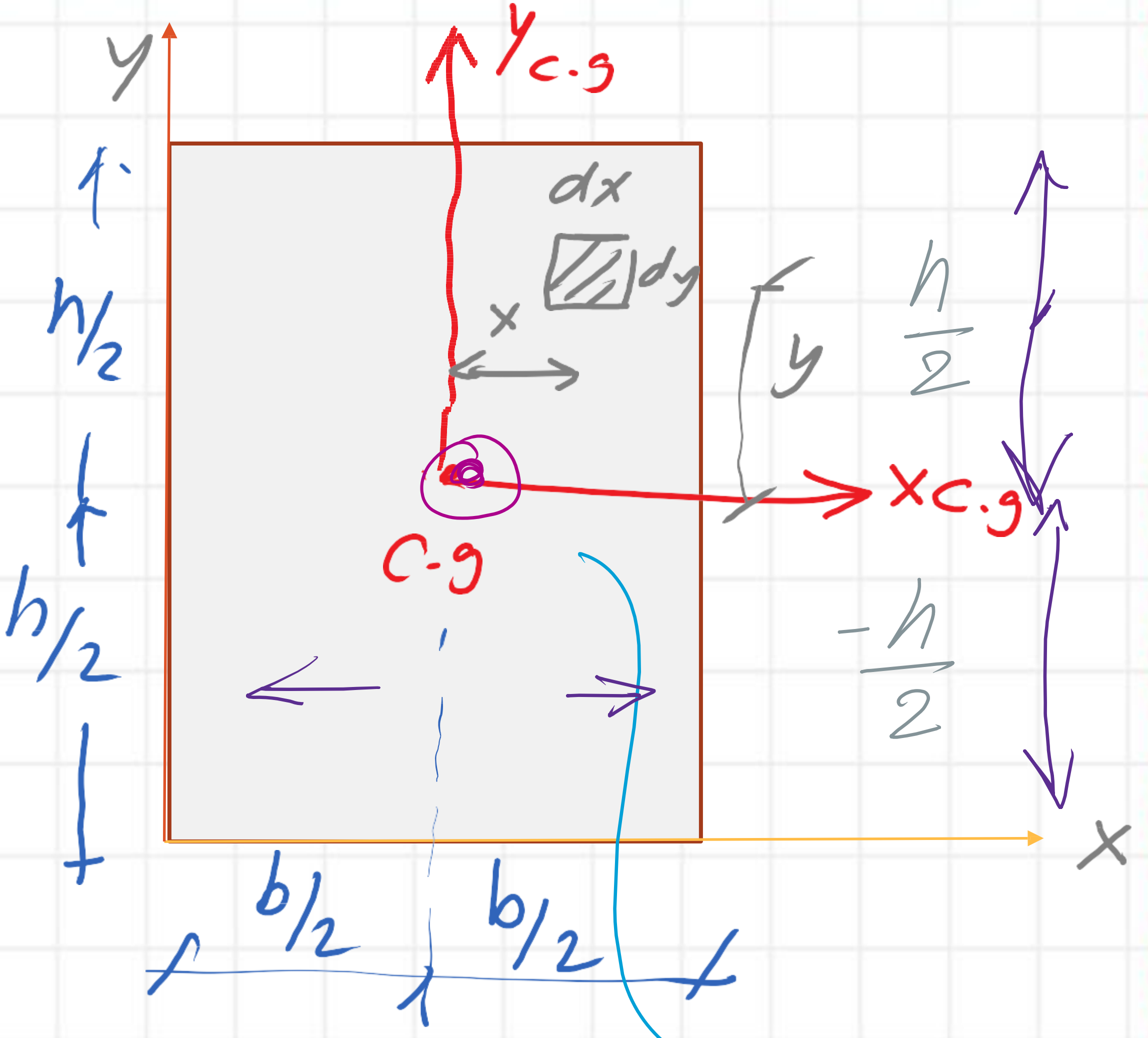
$(I_{xy})_G$ as third proof

$$I_{(xy)G} = \iint \overset{(dA)}{(dx \cdot dy)} \cdot x \cdot y$$

x, y are distance from
Y_{cg} axis to dA
X_{cg} axis to dA



$$I_{xy\,cg} = \int_{-h/2}^{h/2} y\,dy \int^{b/2} x\,dx$$

$$= \frac{y^2}{2}\Big|_{-h/2}^{h/2} \left(\frac{x^2}{2}\right)_{-b/2}^{b/2} \Rightarrow \frac{1}{4}\left[\frac{h^2}{4} - \left(\frac{h^2}{4}\right)\right]\left[\frac{b^2}{4} - \frac{b^2}{4}\right] = 0$$

Origin of axes is at Cg

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Product of Inertia I_{xy}

origion of axes at Corner

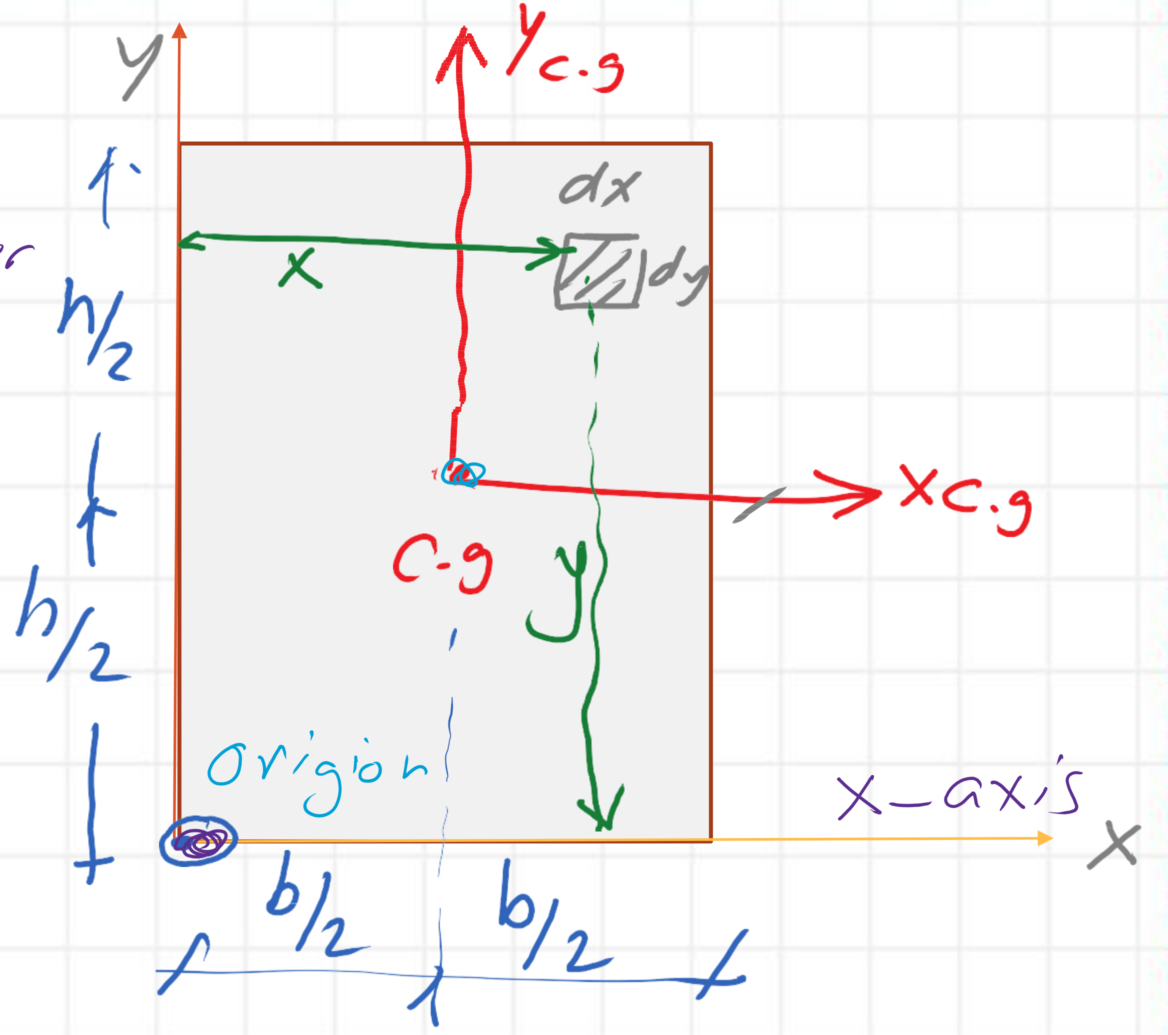
$$I_{(xy)} = \iint dx \cdot dy \cdot x \cdot y$$

$$= \int_{y=0}^{y=h} y\,dy \cdot \int_{x=0}^{x=b} x\,dx$$

$$I_{xy} = \left.\frac{y^2}{2}\right|_0^h \left[\frac{x^2}{2}\right]_0^b$$

$$I_{xy} = \frac{1}{2}(h^2-0)\left(\frac{1}{2}\right)(b^2-0) = \frac{1}{4}b^2h^2$$

at Left Corner



b : breadth
h = height

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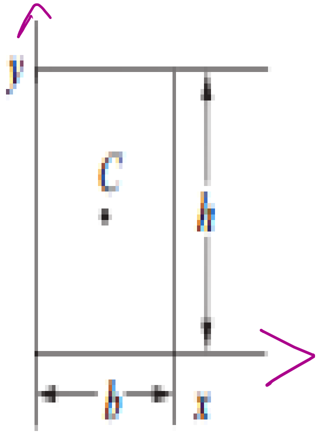
$J_{at\ corner} = \frac{bh(b^2+h^2)}{3}$ $I_{xy} = \frac{b^2h^2}{4}$ $\leftarrow$ $I_{xy}\ cg = 0$

corner =

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = bh$ $x_c = b/2$ $y_c = h/2$	$I_{x_c} = bh^3/12$ $I_{y_c} = b^3h/12$ $I_x = bh^3/3$ $I_y = b^3h/3$ $J = [bh(b^2+h^2)]/12$	$r_{x_c}^2 = h^2/12$ $\frac{I_{xc}}{A}$ $r_{y_c}^2 = b^2/12$ $\frac{I_{yc}}{A}$ $r_x^2 = h^2/3$ I_x/A $r_y^2 = b^2/3$ I_y/A $r_p^2 = (b^2+h^2)/12$ $r^2 = I_p/A$	$I_{x_cy_c} = 0$ $I \Rightarrow 0$ $I_{xy} = Abh/4 = b^2h^2/4$

$A = bh$

$x_c = \frac{b}{2}$

$y_c = \frac{h}{2}$

$I_{x_{cg}} = bh^3/12$ at cg $\Rightarrow$ $I_x = \frac{bh^3}{3}$

$I_{y_{cg}} = hb^3/12$ at c.g $\Rightarrow$ $I_y = \frac{hb^3}{3}$

$J = I_x + I_y$

$bh(b^2+h^2)$

at c.g

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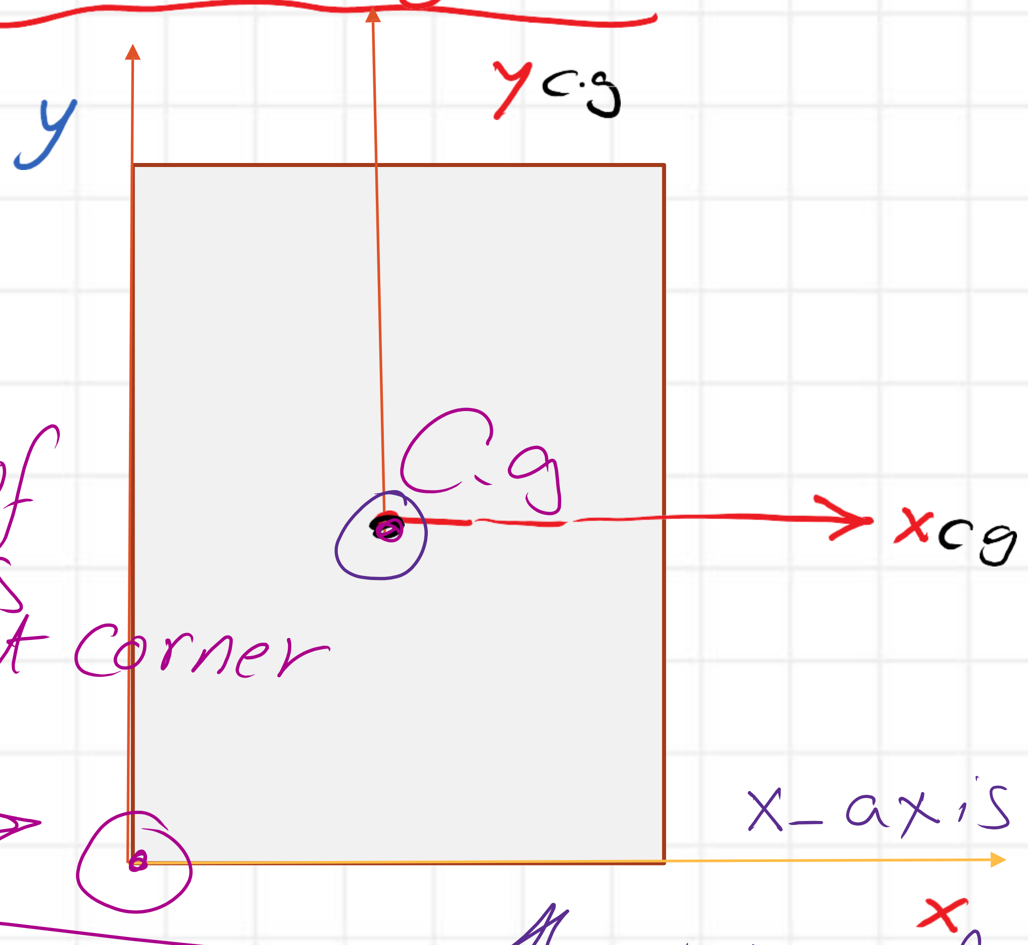
Polar moment of Inertia For a Rectangle

$$J_o = I_x + I_y$$

$$= \frac{bh^3}{3} + \frac{hb^3}{3}$$

$$J_o = \frac{bh}{3}\left[h^2 + b^2\right]$$

Origion of axes at corner



$$J_{oG} = I_{xG} + I_{yG}$$

$$= \frac{bh^3}{12} + \frac{hb^3}{12} = \frac{bh}{12}\left[h^2 + b^2\right]$$

Origion of axes at Cg

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