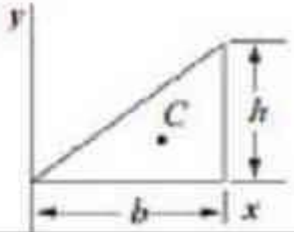
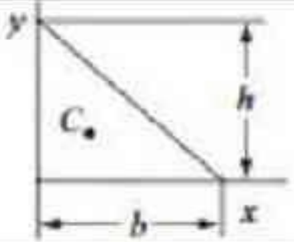
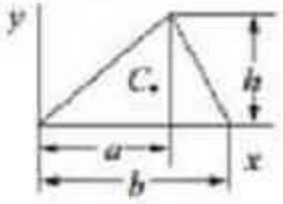
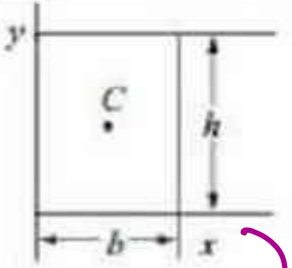
Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = bh/2$ $x_c = 2b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/4$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/2$	$I_{x_c y_c} = Abh/36 = b^2h^2/72$ $I_{xy} = Abh/4 = b^2h^2/8$
	$A = bh/2$ $x_c = b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/6$	$I_{x_c y_c} = -Abh/36 = -b^2h^2/72$ $I_{xy} = Abh/12 = b^2h^2/24$
	$A = bh/2$ $x_c = (a+b)/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = [bh(b^2 - ab + a^2)]/36$ $I_x = bh^3/12$ $I_y = [bh(b^2 + ab + a^2)]/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = (b^2 - ab + a^2)/18$ $r_x^2 = h^2/6$ $r_y^2 = (b^2 + ab + a^2)/6$	$I_{x_c y_c} = [Ah(2a - b)]/36$ $= [bh^2(2a - b)]/72$ $I_{xy} = [Ah(2a + b)]/12$ $= [bh^2(2a + b)]/24$
	$A = bh$ $x_c = b/2$ $y_c = h/2$	$I_{x_c} = bh^3/12$ $I_{y_c} = b^3h/12$ $I_x = bh^3/3$ $I_y = b^3h/3$ $J = [bh(b^2 + h^2)]/12$	$r_{x_c}^2 = h^2/12$ $r_{y_c}^2 = b^2/12$ $r_x^2 = h^2/3$ $r_y^2 = b^2/3$ $r_p^2 = (b^2 + h^2)/12$	$I_{x_c y_c} = 0$ $I_{xy} = Abh/4 = b^2h^2/4$

our Case

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The radius of Gyration

$$K_x = \sqrt{\frac{I_x}{A}}$$

$$A = bh$$

$$I_{cg} = \frac{bh^3}{12}$$

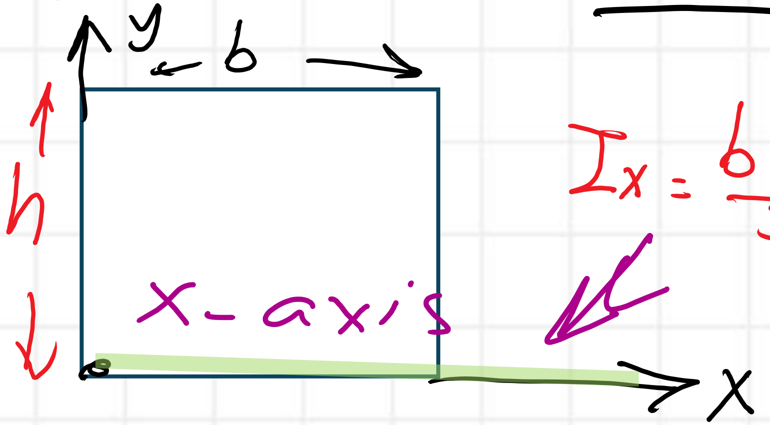
$$K_{x_G} = \left(\frac{bh^3}{12} / bh\right)^{1/2} = \left[\frac{h^2}{12}\right]^{1/2} = \frac{h}{2\sqrt{3}}$$

$$K_{x\,ecc} = \sqrt{\frac{I_x}{A}}$$

$$I_x = \frac{bh^3}{3}$$

$$K_x = \sqrt{\frac{bh^3}{3}\left(\frac{1}{bh}\right)} = \frac{h}{\sqrt{3}}$$

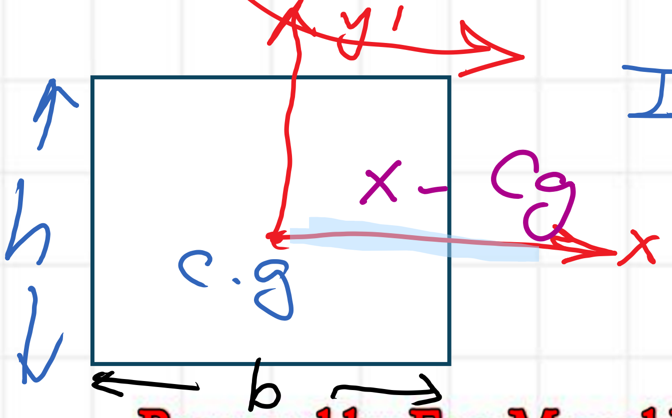
I_{xG} can be = $\dfrac{\text{parallel dim to } (x) \times (\perp\text{er distance in } y)^3}{12}$



$$I_x = \frac{bh^3}{3}$$

$$K_x = \frac{h}{\sqrt{3}}$$

External



$$I_{xG} = \frac{bh^3}{12}$$

$$K_{xG} = \frac{h}{2\sqrt{3}}$$

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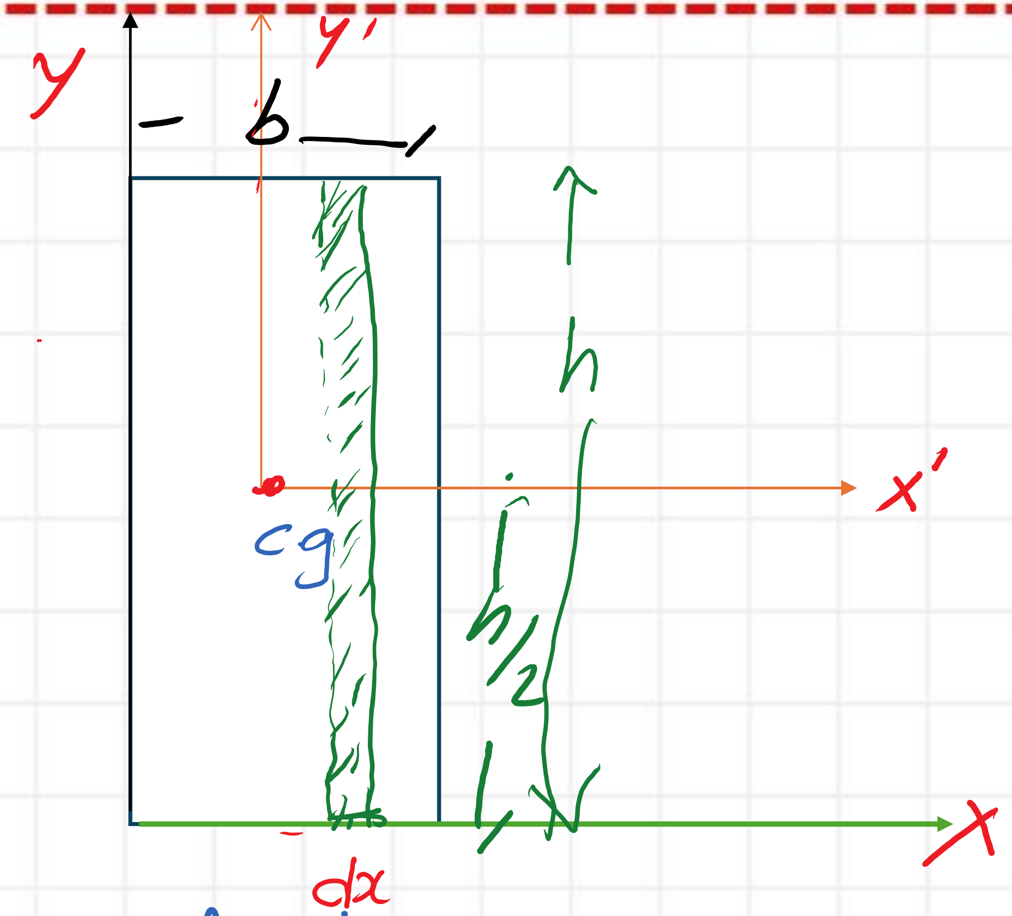
Rectangular Section (I_y)

① We use a VL. strip of width dx and height = h.

② x, y Two axes at the corner of rectangle.

③ x', y' Two axes at the c.g



$x_G = c.g = b/2$, $y_{c.g} = h/2$, $A = bh$

The area of the strip $dA = dx \cdot h$

C.g distance to y axis = x

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Radius of Gyration

$I_y = \text{Parallel Length}(\text{Perpendicular})^3/12$

$A = bh$

$K_{y\,c.g}^2 = I_{y\,c.g}/A$

$K_{yG} = \sqrt{\dfrac{I_y}{A}} = \sqrt{\dfrac{\cancel{h}b^3}{12(b\cancel{h})}} = \dfrac{b}{2\sqrt{3}}$

about y'_axis Passing by Cg

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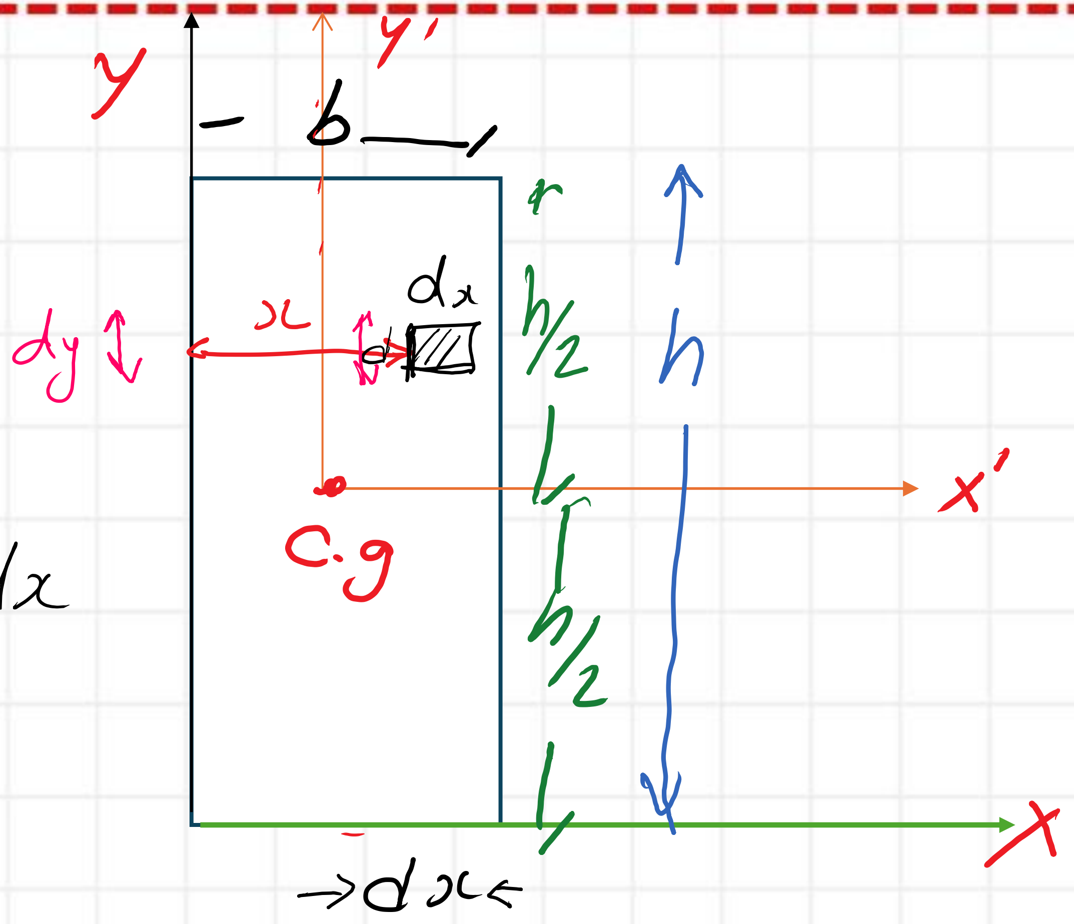
Rectangular Section (I_y)

Using infinitesimal area $\qquad dA = dx \cdot dy$

$$dI_y = dA\,(x)^2 = dx\,(x^2)\,dy$$

$$I_y = \int_0^{b}\int_0^{h} dy\,(x^2\,dx) = y\Big|_0^h \int_0^b x^2 \cdot dx$$

$$= h\left[\frac{x^3}{3}\right]_0^b = h\left(\frac{b^3}{3}\right)$$



For $I_{yG} = I_y - A\left(\frac{b}{2}\right)^2 = \frac{hb^3}{3} - \left((bh)\left(\frac{b^2}{4}\right)\right) = \frac{hb^3(4-3)}{12}$

$$I_{yG} = \frac{hb^3}{12}$$

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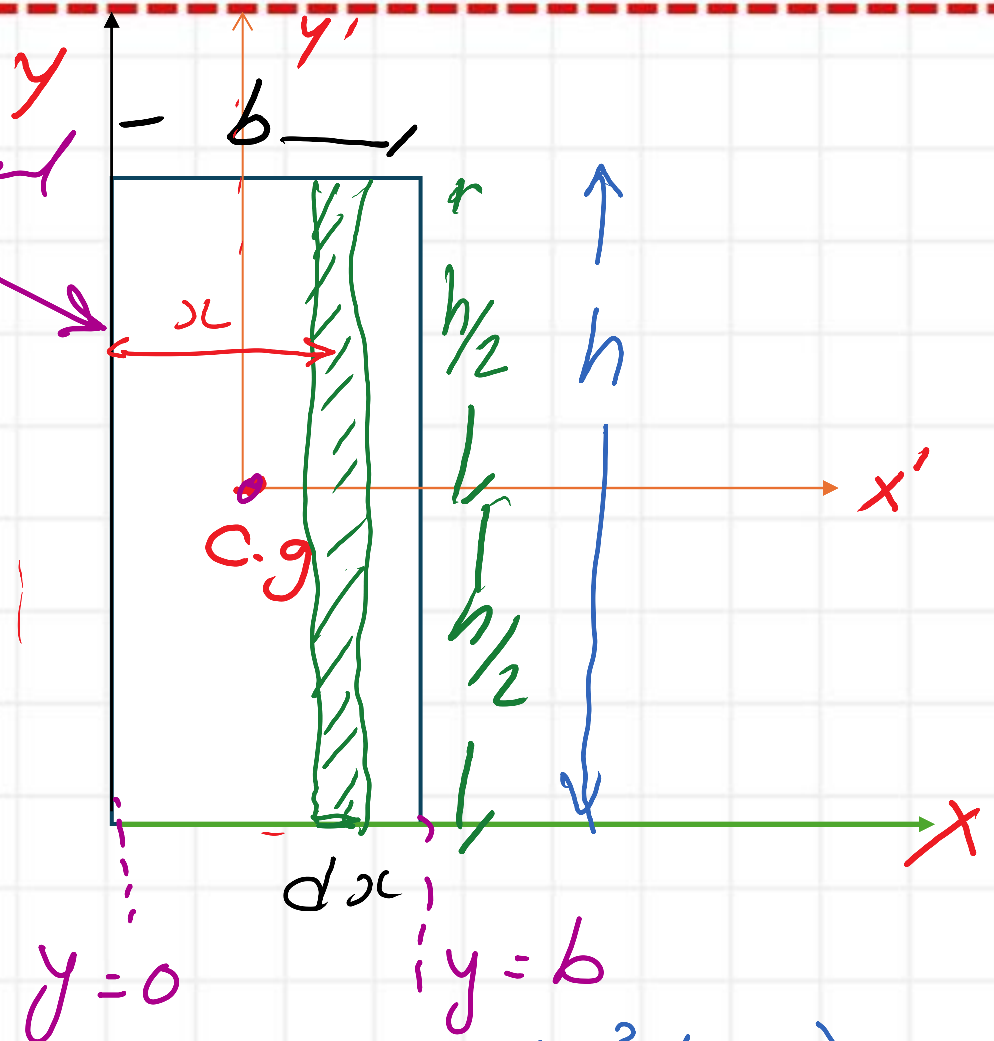
Rectangular Section (I_y)

$$I_y = \int_0^b dA \cdot \bar{x}^2 \qquad \text{VL strip}$$

$$I_y = \int_0^b (h \cdot dx)(x^2) = \int_0^b h(x^2 dx)$$

$$= h\left[\frac{x^3}{3}\right]_0^b = h\left(\frac{b^3}{3}\right)$$



For $I_{yG} = I_y - A\left(\frac{b}{2}\right)^2 = \frac{hb^3}{3} - ((bh)(\frac{b^2}{4}) = \frac{hb^3(4-3)}{12}$

$I_{yG} = \frac{hb^3}{12}$ at Cg (axis y')

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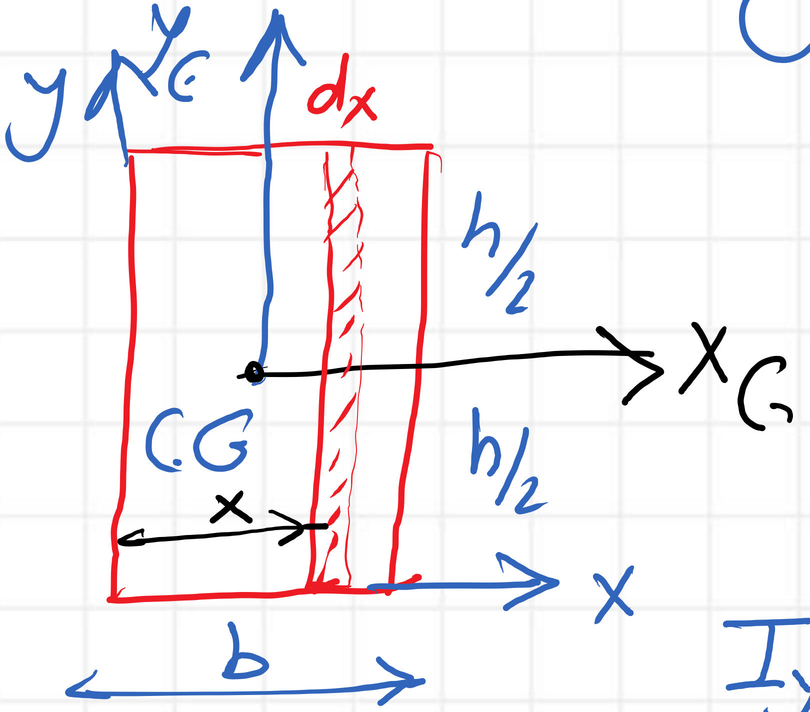
How about I_{yG}, other alternative method

The moment of Inertia For a Rectangle section at Y axis passing by the C.g Point.



First method parallel axis

$$dA = (dx \cdot h)$$

$$I_y = \int_{x=0}^{x=b} dx \cdot h (x^2) = h \left[\frac{x^3}{3} \right]_0^b$$

$$= \frac{hb^3}{3}$$

$$I_{yG} = \frac{hb^3}{3} - hb\left(\frac{b}{2}\right)^2$$

$$= hb^3/12$$

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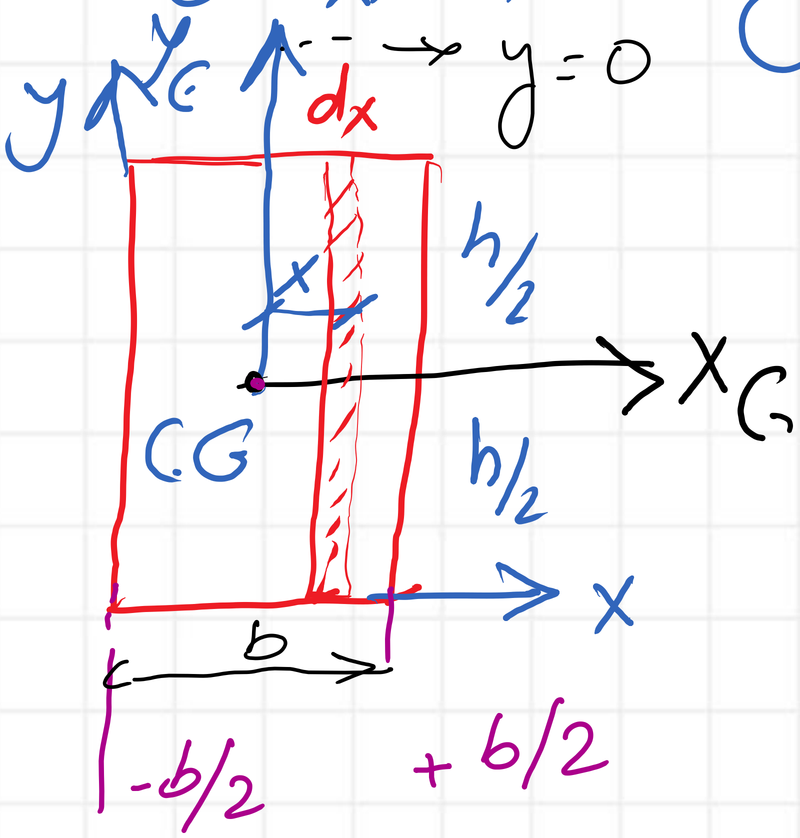
How about I_{yG}

The moment of Inertia For a Rectangle section at Y axis Passing by the C.g Point.



2nd method

→ integrate from $-b/2 \rightarrow +\frac{b}{2}$

$$dA = (dx \cdot h)$$

$$I_{yG} = \int_{-b/2}^{b/2} dx \cdot h\,(x^2) = h\left[\frac{x^3}{3}\right]_{-b/2}^{b/2}$$

$$I_{yG} = \frac{h}{3}\left[\frac{b^3}{8} - \left(-\frac{b^3}{8}\right)\right]$$

$$I_{yG} = \frac{hb^3}{12}$$

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Radius of Gyration

$I_y =$ Parallel Length (Perpendicular)$^3 / 3$

$A = bh$

$$K_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{bh^3}{3}\left(\frac{1}{bh}\right)} = \sqrt{\frac{h^2}{3}} = \frac{h}{\sqrt{3}}$$

about an External axis - y

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