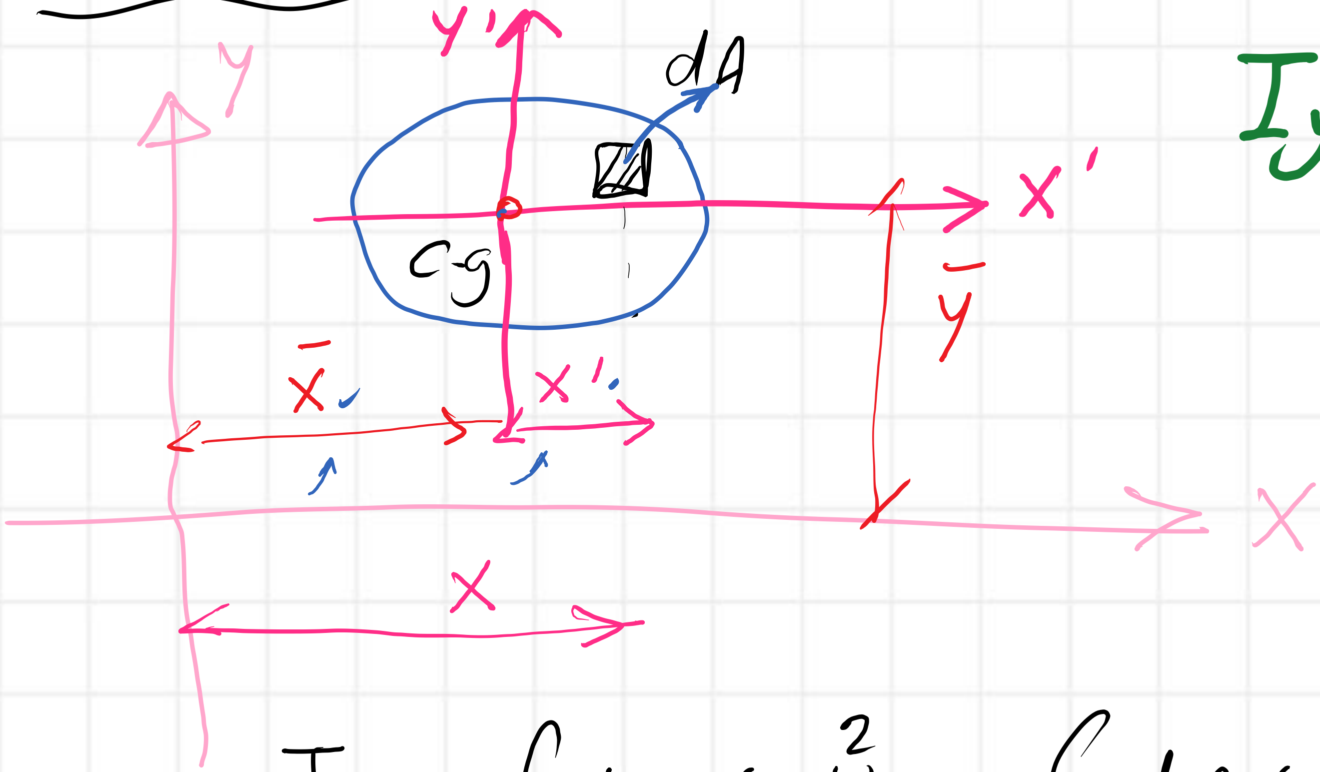
Parallel Axis theory For I_y



$$I_y = \int_A dA.(x^2)$$

$$= \int_A dA\,(x' + \bar{x})^2$$

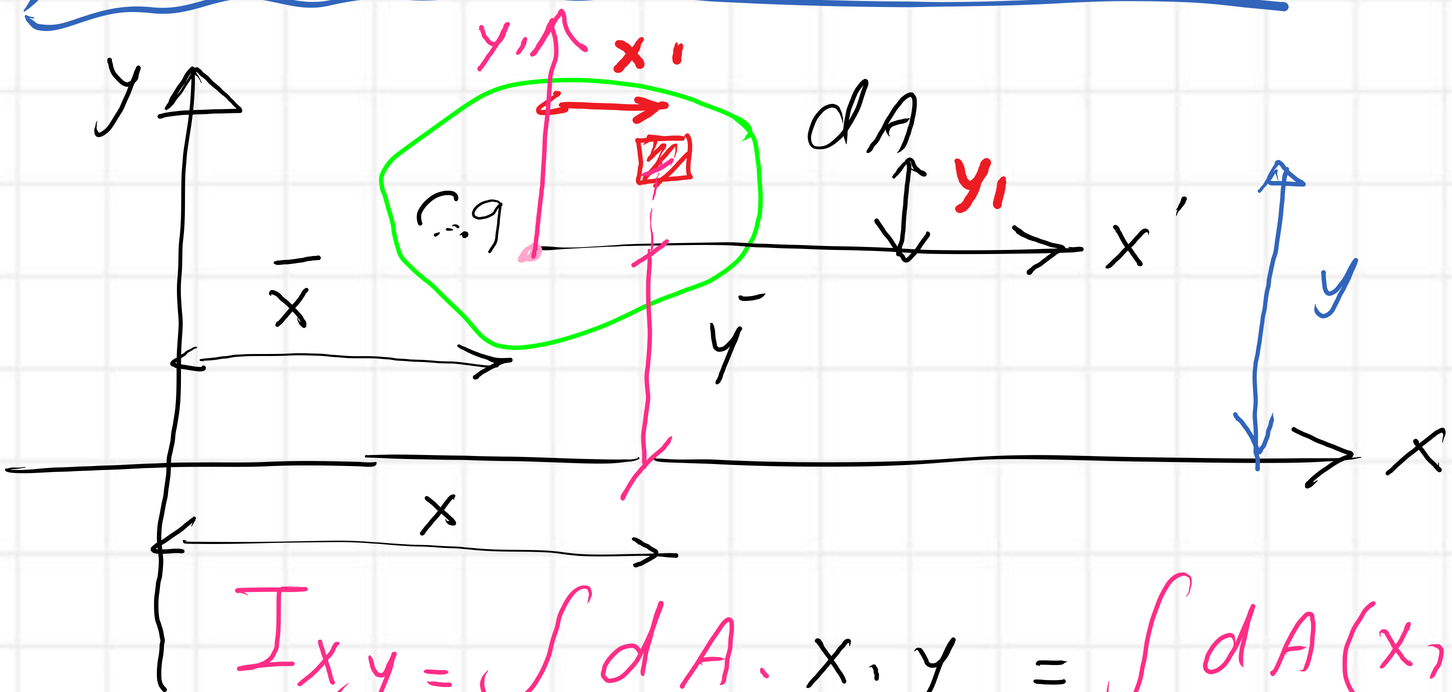
$$I_y = \int_A dA\,(x'^2) + \int_A dA(2)(\bar{x})(x') + \int_{dA} dA\,\bar{x}^2$$

$$I_y = I_{y'} + 2\bar{x}\underbrace{\int dA.x'}_{=0} + A\bar{x}^2$$

$$I_y = I_{y'} + A\bar{x}^2$$

Prepared by Eng.Maged Kamel.

Parallel moment of Inertia For I_{xy}



$$I_{xy} = \int dA \cdot x \cdot y = \int dA (x_1 + \bar{x}) \cdot (y_1 + \bar{y})$$

$$I_{xy} = \int dA \cdot (x_1 y_1 + \bar{x} y_1 + x_1 \bar{y} + \bar{x} \bar{y})$$

$$= \int_A dA \cdot (x_1 y_1) + \int_A dA (\bar{x} y_1) + \int_A dA (x_1 \bar{y}) + \int_A dA \, \bar{x} \bar{y}$$

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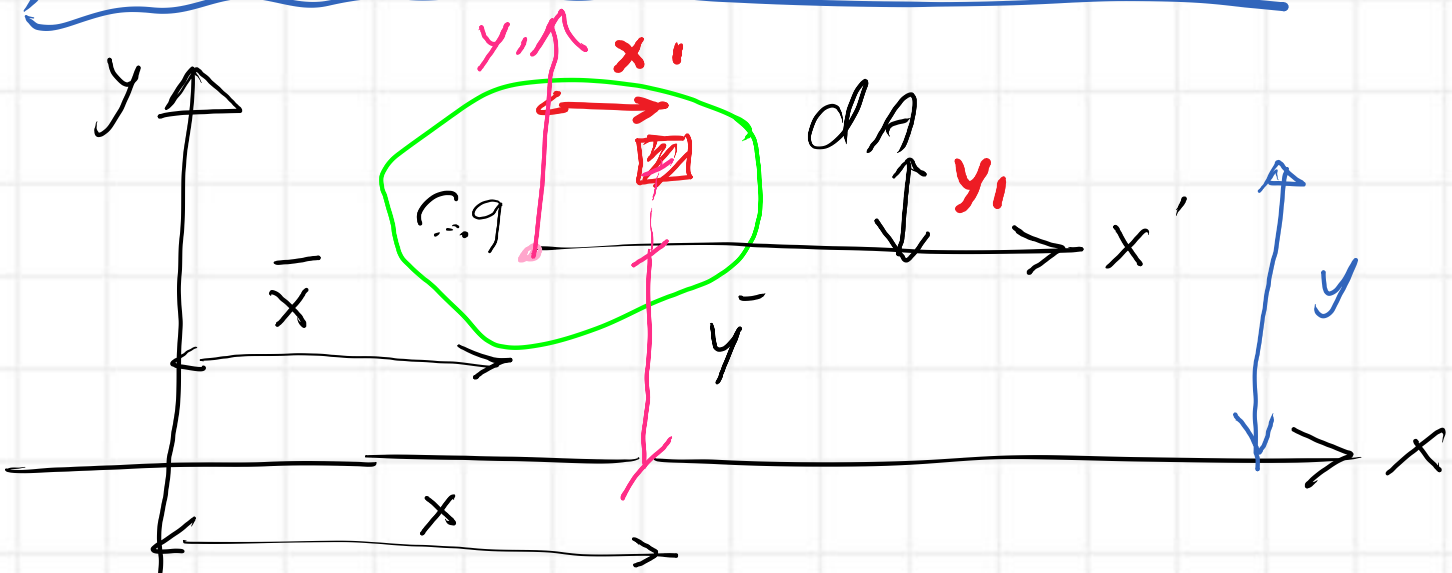
Parallel moment of Inertia for I_{xy}



$$I_{xy} = \int_A dA\cdot(x_1 y_1) + \int_A dA\,(\bar{x}\, y_1) + \int dA\,(x_1\, \bar{y}) + \int_A dA\, \bar{x}\,\bar{y}$$

$\bar{x}$, $\bar{y}$, are constants

$$I_{xy} = I_{x_1 y_1} + \bar{x}\int dA\, y_1 + \underbrace{\bar{y}\int_A dA\, x_1}_{=0} + A\,\bar{x}\,\bar{y}$$

$$I_{xy} = I_{x_1 y_1} + A\,\bar{x}\,\bar{y}$$

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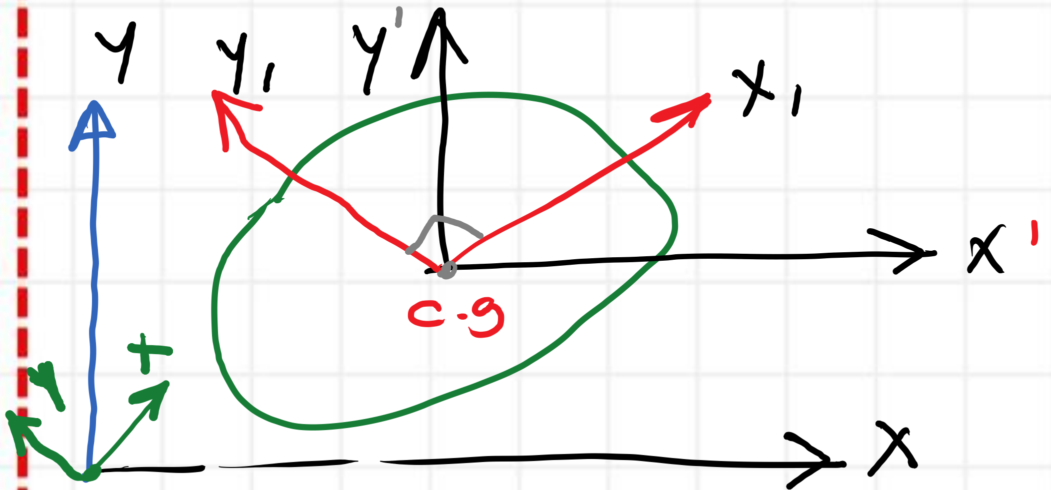
Polar Moment of Inertia

$$I_p = I_x + I_y = I_{x_1} + I_{x_2} + [A\cdot\bar{x} + A\bar{y}]$$

According to parallel axis theory

$I_{x'}$: Moment of Inertia at c.g (y' axis)

$I_{y'}$: Moment of Inertia at c.g (x' axis)



Ip is Constant value

$$I_p = I_{x_1} + I_{y_1} = I_{x'} + I_{y'}$$

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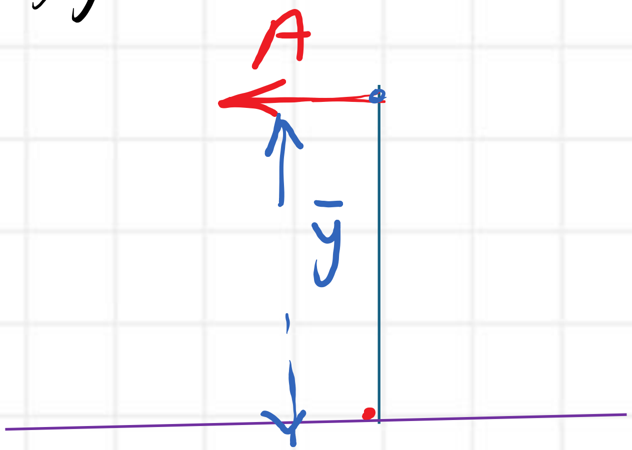
Radius of Gyration

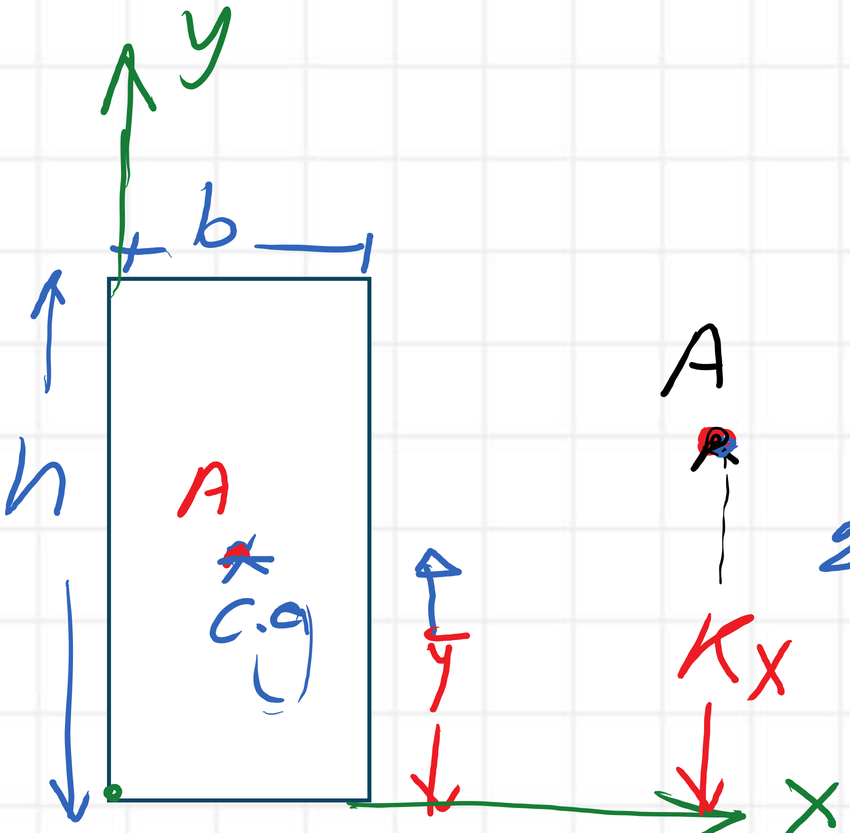
$I_x = A \cdot k_x^2 \rightarrow k_x = \sqrt{\frac{I_x}{A}}$

$I_y = A \cdot k_y^2 \rightarrow k_y = \sqrt{\frac{I_y}{A}}$

notice the difference
between
ybar
and K_x



First moment of Area $= A.\bar{y}$



2nd moment of Area

$\boxed{I_x = A K_x^2}$

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