Moment of Inertia

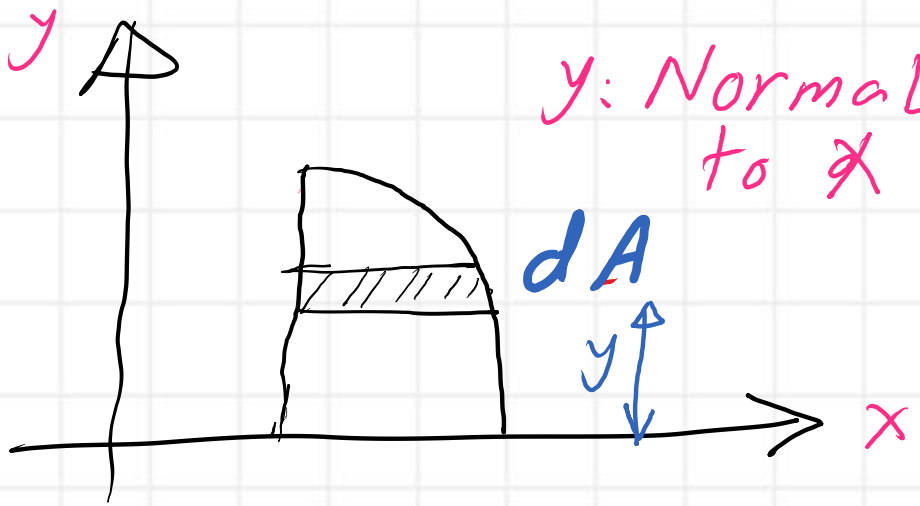
The moment of Inertia it falls into two Categories

(a) Area moment of Inertia : 2nd moment of Area

This term is used For determining, the stresses due to shear, Moment and Torsion acting on a given section [statics]

(b) Mass moment of Inertia : represents the resistance of body to angular acceleration due to Torque [Dynamics]

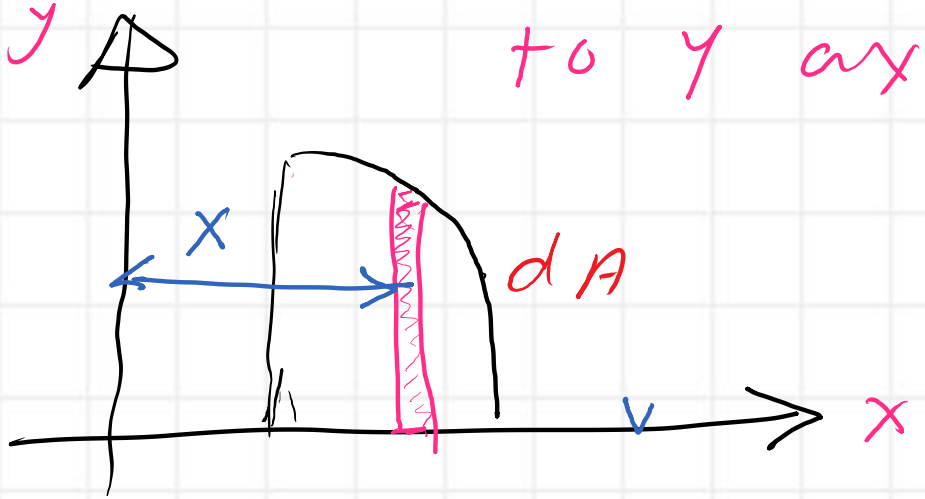
Second moment of Area



y: Normal distance to x axis

$$I_x = \int dA \cdot y^2$$

Second moment of Area about x axis



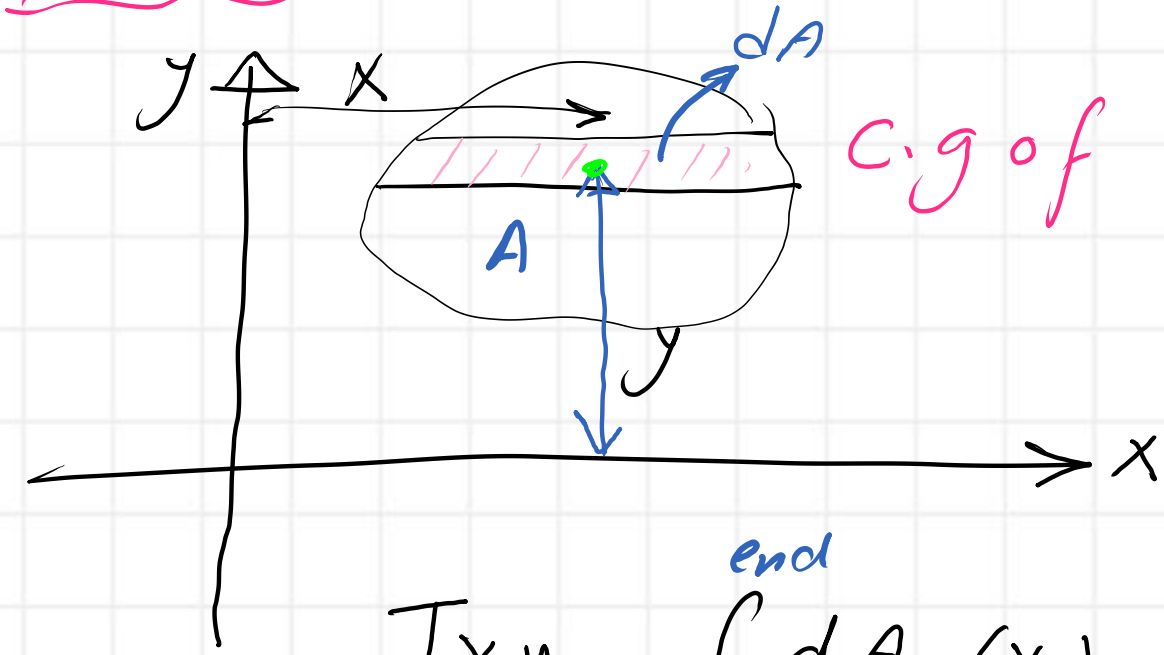
X: Normal distance to y axis

$$I_y = \int dA \cdot x^2$$

Second moment of area about y axis

$$\text{axis} = dA \cdot (\text{Verterical distance to x axis})^2$$

Product of Inertia



c.g of Area dA

$$I_{xy} = \int_{start}^{end} dA \cdot (x) \cdot (y) = \sum_{i=1}^{i=n} A_1 x_1 y_1 + A_2 x_2 y_2 + \cdots$$

Steps used to Estimate Moment of Inertia

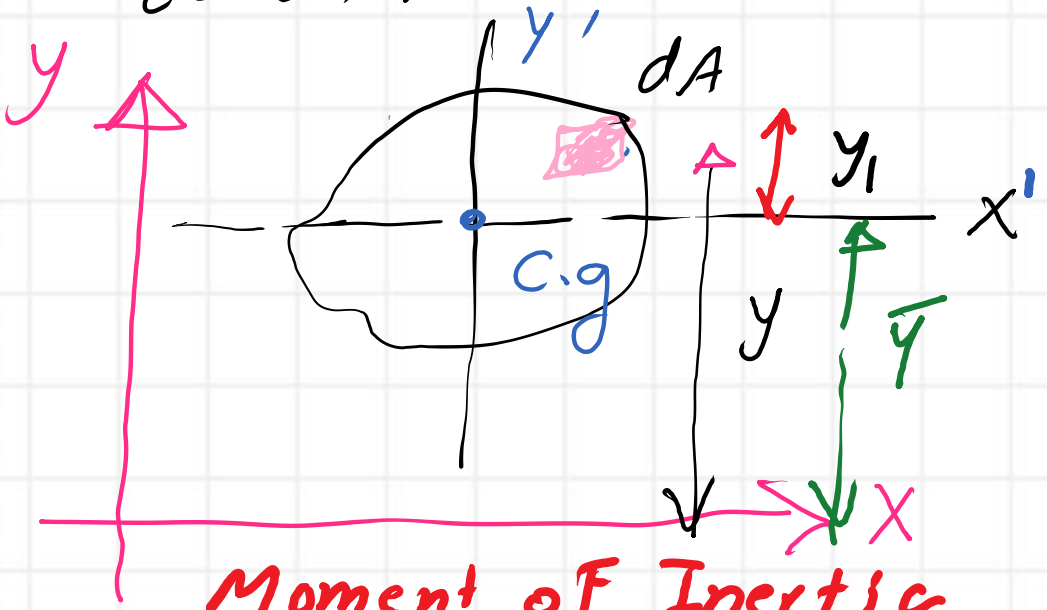
1. Draw the element.
2. Form an Expression For y → if you are calculating I_x
3. Form an Expression For x → if you are calculating For I_y

   For both x, y → make inter-relations.

   x & y : For product moment of Inertia.
4. Set the Limits.
5. Make the integration and evaluate.

Parallel axis Theory Proof For I_x

① First question to ask, where are you going to estimate The moment of Inertia ?



For the sketch shown

Let x, y Two External axes

x', y' Two axes passing by C.g

$\bar{X}$, $\bar{y}$: Distance From X, Y to C.g

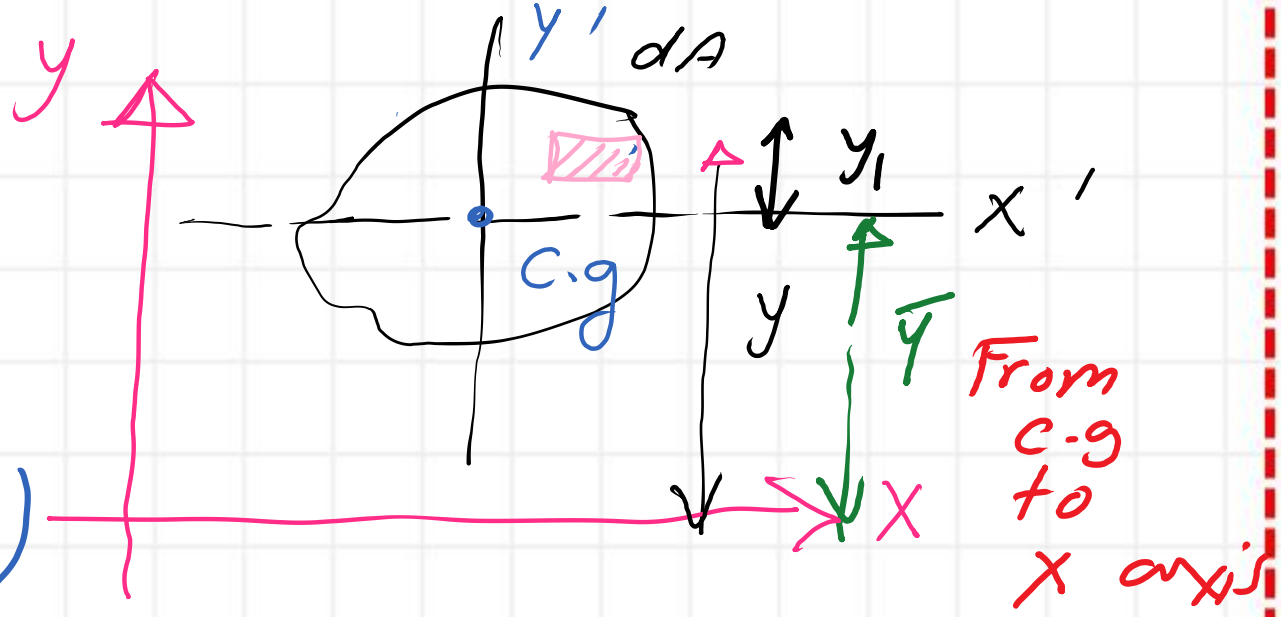
x, y: distance From dA to x, y axes

Moment of Inertia

$$I_x = \int dA \cdot (y^2)$$

$$I_y = \int dA \cdot x^2$$

Parallel axis theory For I_x



$$I_x = \int dA \cdot \bar{y}^2 = \int dA (y_1 + \bar{y})^2$$

$$I_x = \int dA (\bar{y}_1^2 + 2 y_1 \bar{y} + \bar{y}^2)$$

$$= \int_A dA \cdot y_1^2 + \int_A dA\, 2 y_1 \bar{y} + \int_A dA \cdot \bar{y}^2$$

$\bar{y}$ is constant

$$= I_{x'} + 2\bar{y}(dA \cdot y_1) + A\bar{y}^2$$

First moment of Area = 0

$$I_x = I_{x'} + A\bar{y}^2$$