Solved problem For HSS section that is a slender section

Example 6-8 McCormac Based on CM # 15 AISC-360-16

(A) Determine ϕP_n & $\frac{P_n}{\Omega}$ values

HSS $14 \times 10 \times \frac{1}{4}$

Confirm answer → Using Table 4-3

(B)

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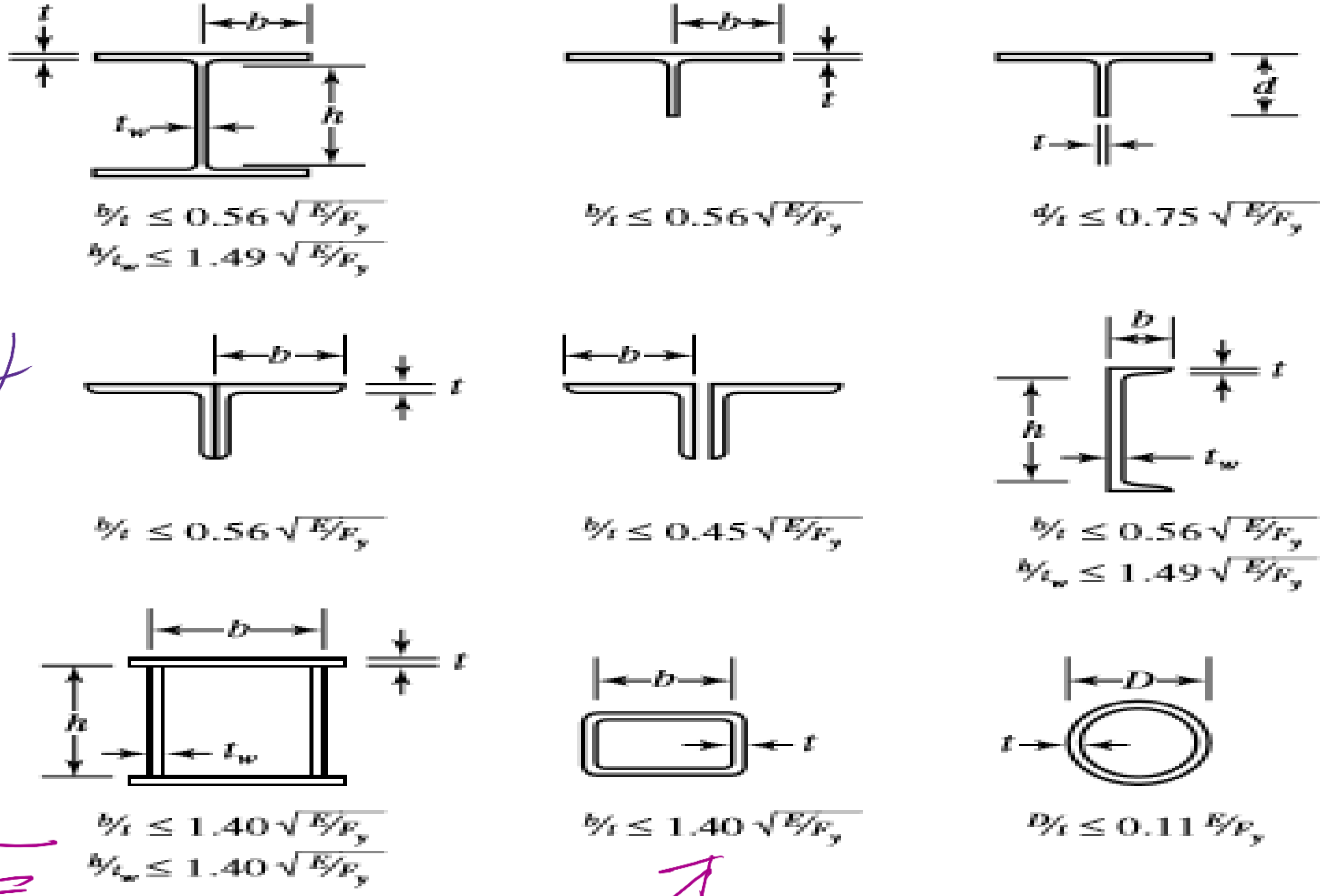
Prof. SEGUi's book

FIGURE 4.9

thickness to width requirement

H.S.S

$$\frac{b}{t} \leq 1.4\sqrt{\frac{E}{Fy}}$$



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JACK - McCormac

CM #15

AISC - 360 - 16

Example 6-8

Determine the axial compressive design strength $\phi_c P_n$ and the allowable design strength $\frac{P_n}{\Omega_c}$ of a 24-ft HSS 14 × 10 × $\frac{1}{4}$ column section. The base of the column is considered to be fixed, while the upper end is assumed to be pinned. $F_y = 50$ ksi.

$F_y = 50$ ksi

$A = 10.80$ inch²

$b/t = 39.90$

$h/t = 57.1$

$t = 0.233$

- Part - 1

$I_x = 310$ inch⁴

$t = 0.233''$

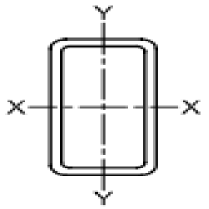
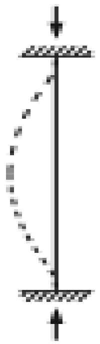
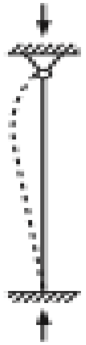
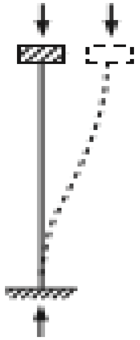
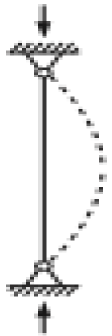
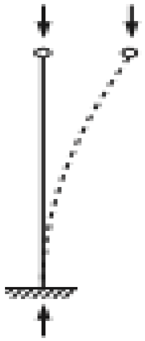
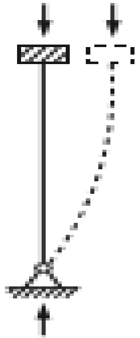


Table 1-11 (continued)
Rectangular HSS
Dimensions and Properties

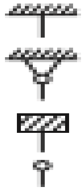
Shape	Design Wall Thickness, *t*	Nominal Wt.	Area, *A*	*b/t*	*h/t*	Axis X-X *I*	*S*	*r*	*Z*
	in.	lb/ft	in.²			in.⁴	in.³	in.	in.³
HSS16×4×5/8	0.581	76.33	21.0	3.88	24.5	539	67.3	5.06	92.9
×1/2	0.465	62.46	17.2	5.60	31.4	455	56.9	5.15	77.3
×3/8	0.349	47.90	13.2	8.46	42.8	360	45.0	5.23	60.2
×5/16	0.291	40.35	11.1	10.7	52.0	308	38.5	5.27	51.1
×1/4	0.233	32.63	8.96	14.2	65.7	253	31.6	5.31	41.7
×3/16	0.174	24.73	6.76	20.0	89.0	193	24.2	5.35	31.7
HSS14×10×5/8	0.581	93.34	25.7	14.2	21.1	687	98.2	5.17	120
×1/2	0.465	76.07	20.9	18.5	27.1	573	81.8	5.23	98.8
×3/8	0.349	58.10	16.0	25.7	37.1	447	63.9	5.29	76.3
×5/16	0.291	48.86	13.4	31.4	45.1	380	54.3	5.32	64.6
×1/4	0.233	39.43	10.8	39.9	57.1	310	44.3	5.35	52.4

Lower Fixed

TABLE C-A-7.1
APPROXIMATE VALUES OF EFFECTIVE LENGTH FACTOR, K

BUCKLED SHAPE OF COLUMN IS SHOWN BY DASHED LINE.	(a)	(b)	(c)	(d)	(e)	(f)
						
THEORETICAL K VALUE	0.5	0.7	1.0	1.0	2.0	2.0
RECOMMENDED DESIGN VALUE WHEN IDEAL CONDITIONS ARE APPROXIMATED	0.65	0.80	1.2	1.0	2.10	2.0

END CONDITION CODE



ROTATION FIXED AND TRANSLATION FIXED
ROTATION FREE AND TRANSLATION FIXED
ROTATION FIXED AND TRANSLATION FREE
ROTATION FREE AND TRANSLATION FREE

Example 6-8 → Case 2 $K=0.70$

K recommended $=0.80$

HSS 14" x 10" x 1/4"

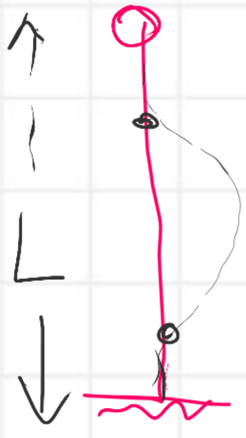
Table 1-11 (continued)
Rectangular HSS
Dimensions and Properties

HSS16-HSS12

Part -2

Shape	Axis Y-Y				Workable Flat		Torsion		Surface Area
	I	*S*	*r*	*Z*	Depth	Width	*J*	*C*	
	in.^{4}	in.^{3}	in.	in.^{3}	in.	in.	in.^{4}	in.^{3}	ft^2/ft
HSS16×4×5/8	54.1	27.0	1.60	32.5	13 3/16	–	174	60.5	3.17
×1/2	47.0	23.5	1.65	27.4	13 3/4	–	150	50.7	3.20
×3/8	38.3	19.1	1.71	21.7	14 5/16	2 5/16	120	39.7	3.23
×5/16	33.2	16.6	1.73	18.5	14 5/8	2 5/8	103	33.8	3.25
×1/4	27.7	13.8	1.76	15.2	14 7/8	2 7/8	85.2	27.6	3.27
×3/16	21.5	10.8	1.78	11.7	15 3/16	3 3/16	65.5	21.1	3.28
HSS14×10×5/8	407	81.5	3.98	95.1	11 3/16	7 3/16	832	146	3.83
×1/2	341	68.1	4.04	78.5	11 3/4	7 3/4	685	120	3.87
×3/8	267	53.4	4.09	60.7	12 5/16	8 5/16	528	91.8	3.90
×5/16	227	45.5	4.12	51.4	12 9/16	8 9/16	446	77.4	3.92
×1/4	186	37.2	4.14	41.8	12 7/8	8 7/8	362	62.6	3.93

$r_y = 4.14"$ $J_y = 186$ inch^4



$L = 24' = 24(12) = 288''$

$L_e = 0.80\,L$

$r_x = 5.35''$ & $r_y = 4.14''$

$$\frac{KL}{r_x} = \frac{L_{ex}}{r_x} = \frac{0.8(288)}{5.35} = 26.92$$

$$\frac{(KL)_y}{r_y} = \frac{L_{ey}}{r_y} = \frac{0.8(288)}{4.14} = \underbrace{55.65}$$

Select bigger value

Check whether given Column is inelastic

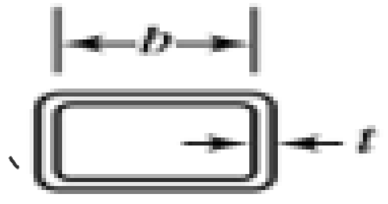
(a) check whether Column is long or inelastic

Limiting ratio $4.71\sqrt{\frac{E}{F_y}}$ $\quad E = 29000$, $F_y = 50$ ksi

$\lambda = 4.71\sqrt{\frac{29000}{50}} = 113.43$

$\left(\frac{l_e}{r_y}\right)_{max\ value} = 55.65 < 113.43$ Column is inelastic

(b) check whether slender or not



$b/t \le 1.40\sqrt{E/F_y}$

$\frac{h}{t} = 1.4\sqrt{\frac{E}{F_y}} = 33.72$

Section is slender

$\frac{b}{t} = 39.9 > 33.72$ $\quad t = 0.233''$ Table 1-11

$h/t = 57.1$

(c) $F_e = \frac{\pi^2 E}{(L/r)_y^2} = \frac{3.1415^2 (29000)}{(55.65)^2} = 92.42$ ksi

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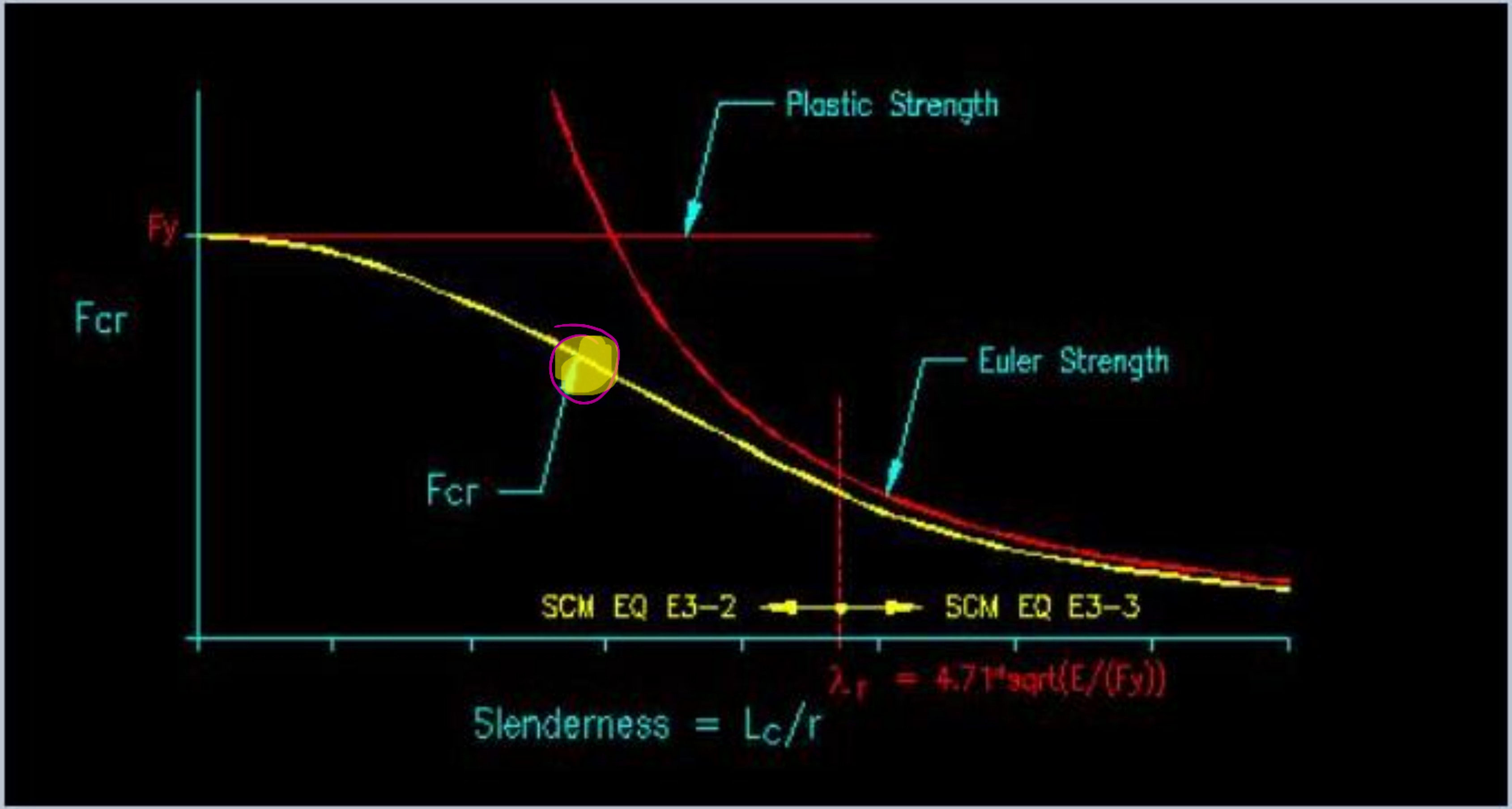
CM#15 AISC-360-16

Figure 7.3.1

F_{cr} vs. Slenderness

Click on image for larger view

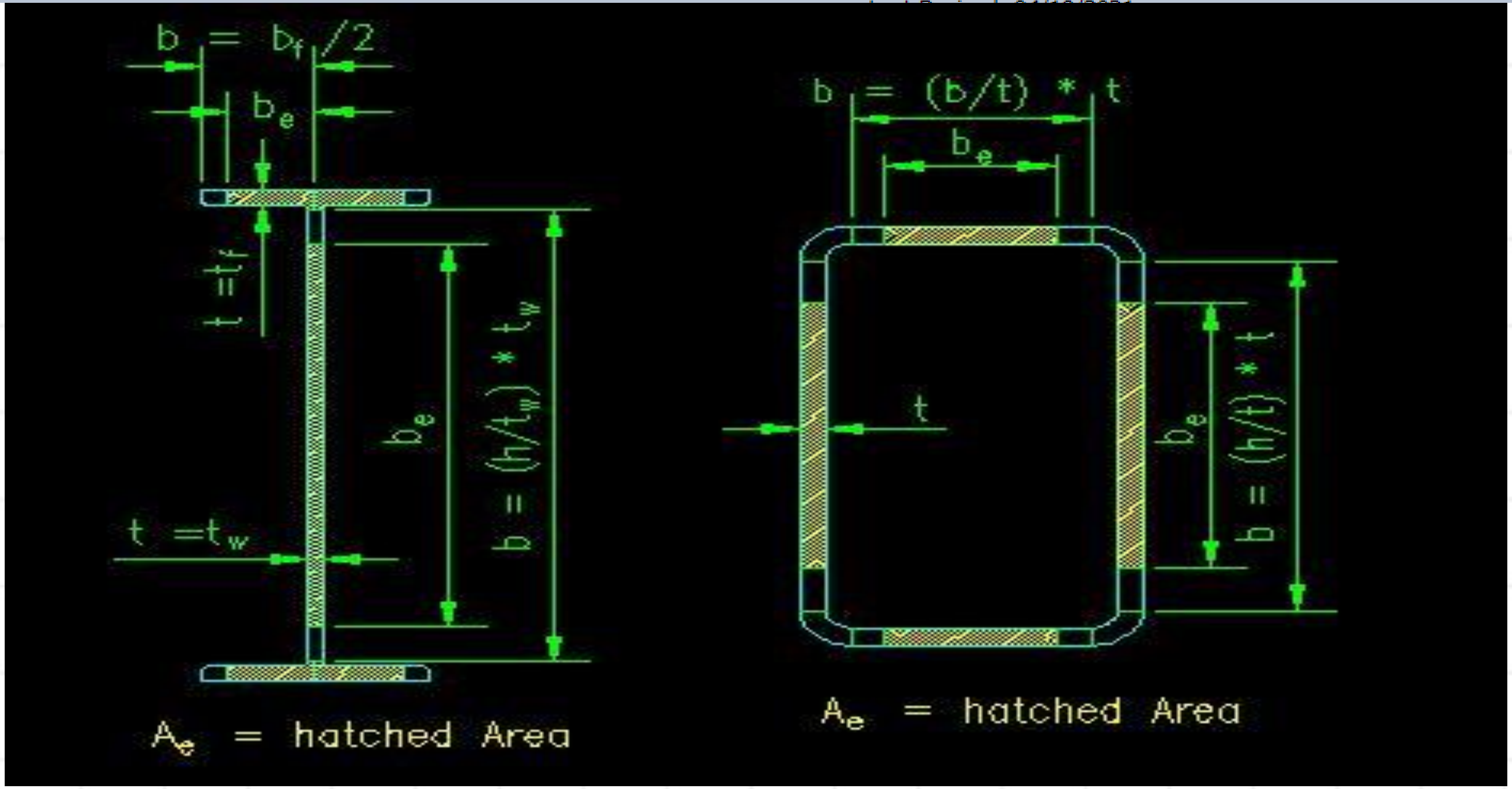


$(\frac{Le}{r})_y \Rightarrow 55.65 \uparrow$ $\|$ 113.43 $\longrightarrow$ Le/r

Chapter 7 - Concentrically Loaded Compression Members

Section 7.4

Limit State of Flexural Buckling for Slender Sections



Effective area are hatched

B.4.1d recommendation

(d) For flanges of rectangular hollow structural sections (HSS), the width, b, is the clear distance between webs less the inside corner radius on each side. For webs of rectangular HSS, h is the clear distance between the flanges less the inside corner radius on each side. If the corner radius is not known, b and h shall be taken as the corresponding outside dimension minus three times the thickness. The thickness, t, shall be taken as the design wall thickness, per Section B4.2.

$\left.\begin{array}{l} b-3t \\ h-3t \end{array}\right\}$ recommended dimensions

$F_y = 50$ ksi

$F_e = 92.42$ ksi

$h - 3t = 14 - 3(0.233) = 13.301''$

$b - 3t = 10 - 3(0.233) = 9.301''$

$(14 \times 10 \times \frac{1}{4}'')$

$\lambda^2 = \frac{F_y}{F_e}$

$F_{cr} = 0.658^{\frac{F_y}{F_e}} F_y$

$= 0.658^{\left(\frac{50}{92.42}\right)} (50) = 39.87$ ksi

Column is inelastic

1. Slender Element Members Excluding Round HSS

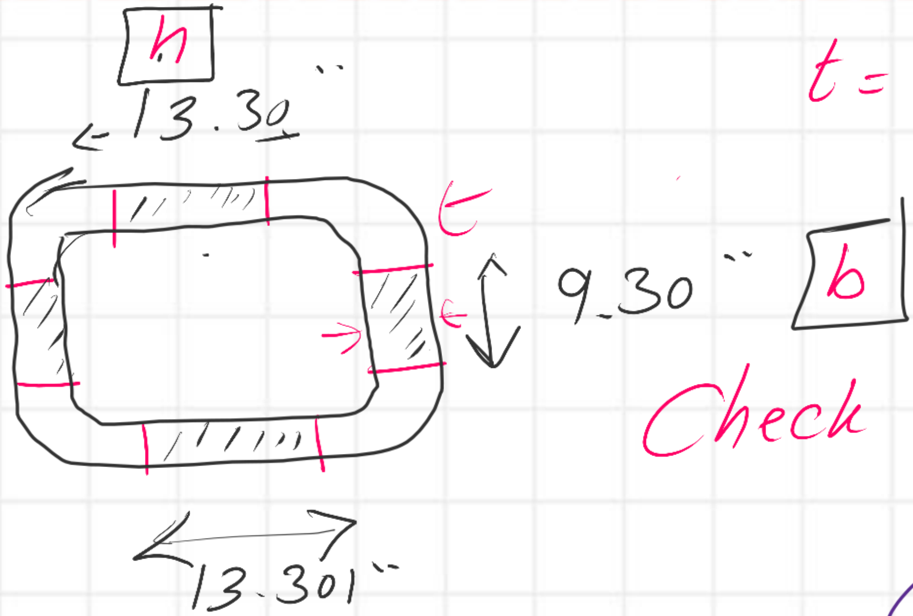
The effective width, b_e, (for tees, this is d_e; for webs, this is h_e) for slender elements is determined as follows:

(a) When $\lambda \leq \lambda_r \sqrt{\frac{F_y}{F_{cr}}}$

$$b_e = b \tag{E7-2}$$

(b) When $\lambda > \lambda_r \sqrt{\frac{F_y}{F_{cr}}}$

$$b_e = b\left(1 - c_1\sqrt{\frac{F_{el}}{F_{cr}}}\right)\sqrt{\frac{F_{el}}{F_{cr}}} \tag{E7-3}$$



$t = 0.233''$

$F_y = 50$ ksi

$F_{cr} = 39.87$ ksi

Check whether $b_e = b$ or not

when $\frac{h}{t} \leq \lambda_r \sqrt{\frac{F_y}{F_{cr}}}$

$b/t = 39.9$ while $\frac{h}{t} = 57.1$

$$\lambda_r = 1.4\sqrt{\frac{E}{F_y}} = 33.72$$

$$\lambda = \lambda_r \sqrt{\frac{F_y}{F_{cr}}} = 33.72\left(\sqrt{\frac{50}{39.87}}\right) = 37.76$$

Since $\frac{b}{t} > \lambda$ Use E-7-3 Equation

$39.9 > 37.76$

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$\lambda = b/t = 39.90 \rightarrow$

$$b_e = b\left(1 - c_1\sqrt{\frac{F_{el}}{F_{cr}}}\right)\sqrt{\frac{F_{el}}{F_{cr}}} \qquad \text{(E7-3)}$$

$b = b - 3t = 9.30$

$c_1 = 0.20$

$c_2 = 1.38$

$\lambda_r = 33.72$

$\lambda = \frac{b}{t} = 39.90$

TABLE E7.1
Effective Width Imperfection Adjustment Factors, c_1 and c_2

Case	Slender Element	c_1	c_2
(a)	Stiffened elements except walls of square and rectangular HSS	0.18	1.31
(b)	Walls of square and rectangular HSS	0.20	1.38
(c)	All other elements	0.22	1.49

where

b = width of the element (for tees this is d; for webs this is h), in. (mm)

c_1 = effective width imperfection adjustment factor determined from Table E7.1

$$c_2 = \frac{1 - \sqrt{1 - 4c_1}}{2c_1} \qquad \text{(E7-4)}$$

λ = width-to-thickness ratio for the element as defined in Section B4.1

λ_r = limiting width-to-thickness ratio as defined in Table B4.1a

$$F_{el} = \left(c_2 \frac{\lambda_r}{\lambda}\right)^2 F_y \qquad \text{(E7-5)}$$

$F_{cr} = 39.87$ ksi

$F_{el} = \left(C_2 \frac{\lambda r}{\lambda}\right)^2 F_y$

$F_{el} = (1.38)^2\left(\frac{33.72}{39.90}\right)^2(50)$

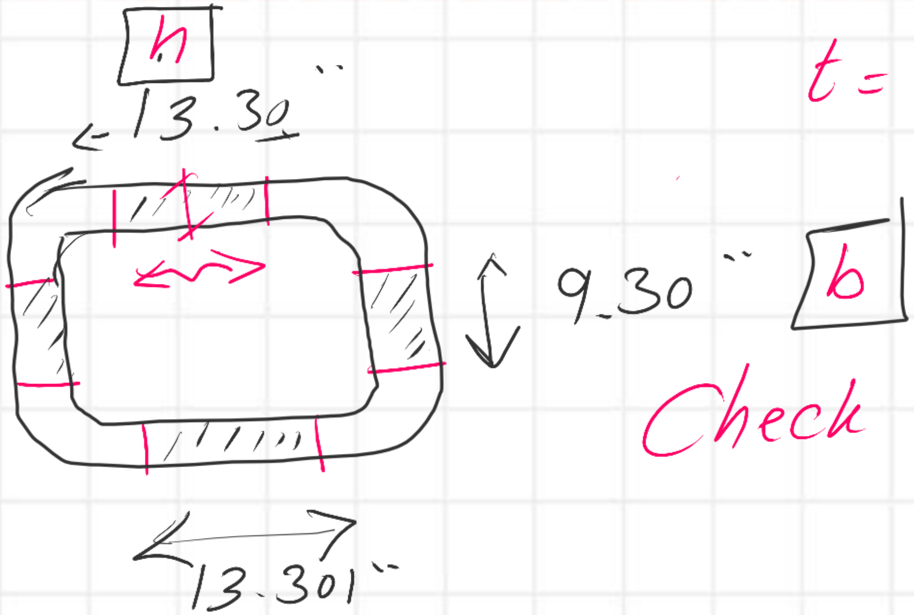
$= 68.01$ ksi

get be

$= 9.30\left(1 - 0.2\sqrt{\frac{68.01}{39.87}}\right)\sqrt{\frac{68.01}{39.87}}$

$= 6.8705(1.3061)$

$= 8.974''$



$t = 0.233''$

$F_y = 50$ ksi

$F_{cr} = 39.87$ ksi

Check whether $b_e = b$ or not

when $\frac{h}{t} \le \lambda_r \sqrt{\frac{F_y}{F_{cr}}}$

$h/t = 57.1$

$$\lambda_r = 1.4\sqrt{\frac{E}{F_y}} = 33.72$$

$$\lambda = \lambda_r \sqrt{\frac{F_y}{F_{cr}}} = 33.72\left(\sqrt{\frac{50}{39.87}}\right) = 37.76$$

Since $\frac{h}{t} > \lambda$ Use E-7-3 Equation

Prepared by Eng.Maged Kamel.

$\lambda = h/t = 57.10$

$$b_e = b\left(1 - c_1\sqrt{\frac{F_{el}}{F_{cr}}}\right)\sqrt{\frac{F_{el}}{F_{cr}}} \qquad \text{(E7-3)}$$

$h = h - 3t = 13.30$

$c_1 = 0.20$

$c_2 = 1.38$

$\lambda_r = 33.72$

$\lambda = \frac{h}{t} = 57.10$

$F_{el} = \left(C_2 \frac{\lambda_r}{\lambda}\right)^2 F_y$

$F_{el} = (1.38)^2 \left(\frac{33.72}{57.10}\right)^2 (50)$

$= 33.21$ ksi

TABLE E7.1
Effective Width Imperfection Adjustment Factors, c_1 and c_2

Case	Slender Element	c_1	c_2
(a)	Stiffened elements except walls of square and rectangular HSS	0.18	1.31
(b)	Walls of square and rectangular HSS	0.20	1.38
(c)	All other elements	0.22	1.49

get h_e

$h_e = 13.30\left(1 - 0.2\sqrt{\frac{33.21}{39.87}}\right)\sqrt{\frac{33.21}{39.87}}$

$= 9.923''$

where

b = width of the element (for tees this is d; for webs this is h), in. (mm)

c_1 = effective width imperfection adjustment factor determined from Table E7.1

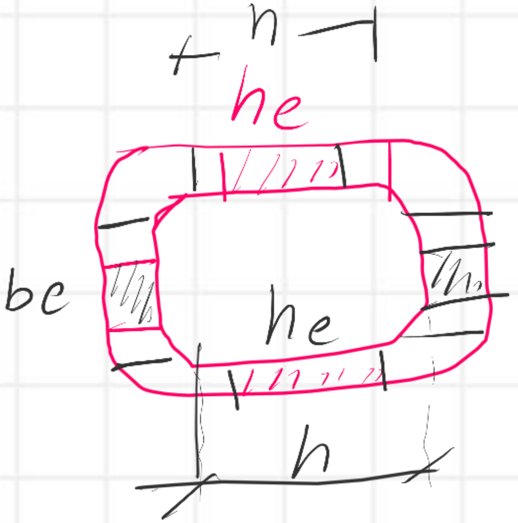
$$c_2 = \frac{1 - \sqrt{1 - 4c_1}}{2c_1} \qquad \text{(E7-4)}$$

λ = width-to-thickness ratio for the element as defined in Section B4.1

λ_r = limiting width-to-thickness ratio as defined in Table B4.1a

$$F_{el} = \left(c_2 \frac{\lambda_r}{\lambda}\right)^2 F_y \qquad \text{(E7-5)}$$

$F_{cr} = 39.87$ ksi



$h_e = 9.923''$ $\Rightarrow h = 13.30''$

$b_e = 8.974''$ $\Rightarrow b = 9.30''$

$t = 0.233$

$A_g = 10.80 \text{ inch}^2$

$$A_e = A_g - 2[bt - b_e t] - 2(ht - h_e t)$$

$$= 10.80 - 2(0.233)[9.30 - 8.974] - 2(0.233)[13.3 - 9.923]$$

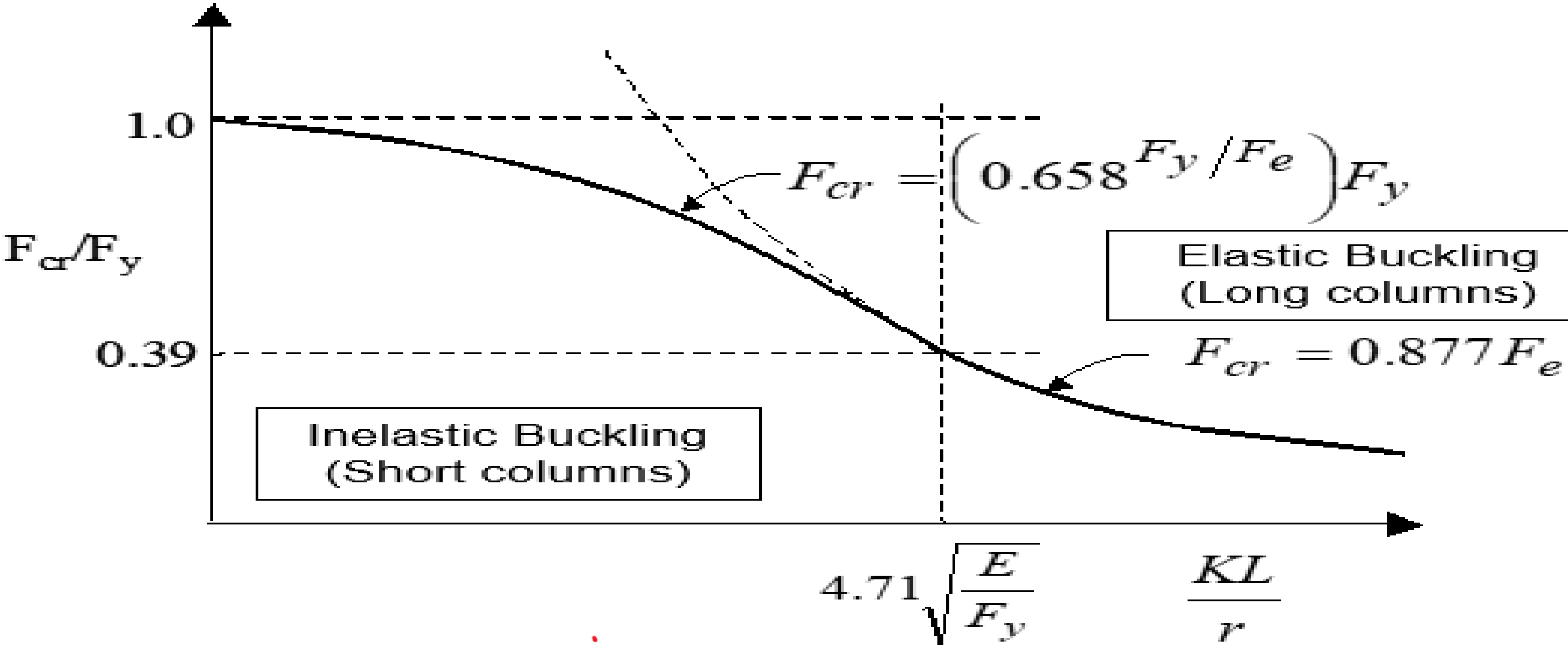
$$= 10.80 - 2(0.233)[9.30 + 13.30 - 8.974 - 9.923]$$

$$A_e = 9.074 \text{ inch}^2$$

$$P_n = F_{cr} A_e = 39.87(9.074) = 361.78 \text{ Kips}$$

$P_n = 361.78$ kips

$\phi = 0.90$ $\qquad \Omega = 1.67$



$\phi_c P_n = 0.9\,(361.78) = 325.60$ Kips

For ASD Design

$\Omega = 1.67$

$P_n = 361.78$ kips

$\frac{P_n}{\Omega} = \frac{361.78}{1.67} = 216.63$ kips

≈ 217 kips

Author's Solution

where $t = 0.233$ in, $A_g = 10.8$ in^2

$$A_e = 10.8 - 2(13.3 - 9.92)0.233 - 2(9.3 - 8.97)0.233 = 9.07 \text{ in}^2$$
$$\therefore \; P_n = F_{cr}\, A_e = 39.87 \text{ ksi}(9.07 \text{ in}^2) = 361.6 \text{ kips}$$

LRFD $\phi_c = 0.90$

$\phi_c P_n = 0.9(361.6 \text{ k}) = 325.5$ kips

ASD $\Omega_c = 1.67$

$\frac{P_n}{\Omega_c} = \frac{361.6\ k}{1.67} = 216.5$ kips

To use Table 4-3

investigated. To determine the available strength in axial compression, the table should be entered at the larger of L_{cy} and $L_{cy\ eq}$, where

$$L_{cy\ eq} = \frac{L_{cx}}{\frac{r_x}{r_y}} \qquad r_y = 4.14'' \qquad (4\text{-}1)$$

Values of the ratio r_x/r_y and other properties useful in the design of rectangular HSS compression members are listed at the bottom of Table 4-3.

We need to Find $L_{cy\ eq} = \frac{L_{cx}}{\frac{r_x}{r_y}}$

$r_x = 5.35''$ $K = 0.80$

$L_{cx} = K_x L_x = 0.8(24) = 19.20'$

$L_{cy\ eq} = \frac{L_{cx}}{\frac{r_x}{r_y}} = \frac{19.2}{\frac{5.35}{4.14}} = 14.86'$

$L_{cy} \rightarrow = 0.8(24) = 19.20'$ bigger

LRFD

Lcy = 19.20'

Table 4-3 (continued)
Available Strength in Axial Compression, kips
Rectangular HSS

F_y = 50 ksi

HSS16–HSS14

14'' x 10'' x $\frac{1}{4}$

Shape	HSS16×8×		HSS14×10×									
	1/4[c]		5/8		1/2		3/8[c]		5/16[c]		1/4[c]	
t_{des}, in.	0.233		0.581		0.465		0.349		0.291		0.233	
lb/ft	39.43		93.34		76.07		58.10		48.86		39.43	
Design	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	240	360	769	1160	626	940	462	695	360	542	251	377
6	234	352	751	1130	611	919	454	682	354	532	247	372
7	232	348	745	1120	606	911	451	678	352	528	246	370
8	229	345	737	1110	600	902	447	672	349	524	245	368
9	227	341	729	1100	594	893	444	667	346	520	243	365
10	224	336	720	1080	587	882	439	660	343	515	241	363
11	220	331	710	1070	579	870	434	653	339	510	239	360
12	217	326	699	1050	570	857	429	645	335	504	237	356
13	213	320	688	1030	561	843	424	637	331	498	235	353
14	209	314	675	1020	551	829	418	628	327	491	232	349
15	205	308	663	996	541	813	412	619	322	484	230	345
16	201	302	649	976	530	797	405	609	317	476	227	341
17	196	295	635	954	519	781	398	599	312	468	224	336
18	191	288	620	932	508	763	391	587	306	460	221	332
19	187	280	605	910	496	745	382	574	301	452	217	327
20	182	273	590	886	483	727	372	560	295	443	214	322

spect to least radius of gyration, r_y

$\phi_c P_n$ = 327 kips → Lcy = 19
= 322 Lcy = 20

⇒ $\phi_c P_n$ = 327 − 5 (0.2)
= 326 kips

ASD

Lcy = 19.20'

Table 4-3 (continued)
Available Strength in Axial Compression, kips
Rectangular HSS

F_y = 50 ksi

HSS16–HSS14

14" x 10" x 1/4"

spect to least radius of gyration, r_y

Shape	HSS16×8×		HSS14×10×									
	1/4[c]		5/8		1/2		3/8[c]		5/16[c]		1/4[c]	
t_{des}, in.	0.233		0.581		0.465		0.349		0.291		0.233	
lb/ft	39.43		93.34		76.07		58.10		48.86		39.43	
Design	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	240	360	769	1160	626	940	462	695	360	542	251	377
6	234	352	751	1130	611	919	454	682	354	532	247	372
7	232	348	745	1120	606	911	451	678	352	528	246	370
8	229	345	737	1110	600	902	447	672	349	524	245	368
9	227	341	729	1100	594	893	444	667	346	520	243	365
10	224	336	720	1080	587	882	439	660	343	515	241	363
11	220	331	710	1070	579	870	434	653	339	510	239	360
12	217	326	699	1050	570	857	429	645	335	504	237	356
13	213	320	688	1030	561	843	424	637	331	498	235	353
14	209	314	675	1020	551	829	418	628	327	491	232	349
15	205	308	663	996	541	813	412	619	322	484	230	345
16	201	302	649	976	530	797	405	609	317	476	227	341
17	196	295	635	954	519	781	398	599	312	468	224	336
18	191	288	620	932	508	763	391	587	306	460	221	332
19	187	280	605	910	496	745	382	574	301	452	217	327
20	182	273	590	886	483	727	372	560	295	443	214	322

$\frac{1}{\Omega} P_n = 217$ kips → Lcy = 19
$= 214$ Lcy = 20

$\Rightarrow \frac{1}{\Omega} P_n = 217 - 3(0.2) = 216.40$ kips

38	90.0	135	294	442	247	371	193	290	164	246	129	194
40	82.4	124	266	399	223	334	175	262	148	223	119	179
Properties												
A_g, in.^{2}	10.8		25.7		20.9		16.0		13.4		10.8	
I_x, in.^{4}	368		687		573		447		380		310	
I_y, in.^{4}	127		407		341		267		227		186	
r_y, in.	3.42		3.98		4.04		4.09		4.12		4.14	
r_x/r_y	1.70		1.30		1.29		1.29		1.29		1.29	

ASD	**LRFD**
$\Omega_c = 1.67$	$\phi_c = 0.90$

[c] Shape is slender for compression with $F_y = 50$ ksi; tabulated values have been adjusted accordingly.

Data for HSS 14"x10"x $\frac{1}{4}$"

$A_g = 10.80$ inch^2

$I_x = 310$ inch^4

$I_y = 186$ inch^4

$r_y = 4.14$"

$\frac{r_x}{r_y} = 1.29$