

① Local buckling for Columns

Equation Used for various types of steel sections

A given HSS Column (short Column)

Example 6-8 McCormac

CM # 14

AISC-360-10

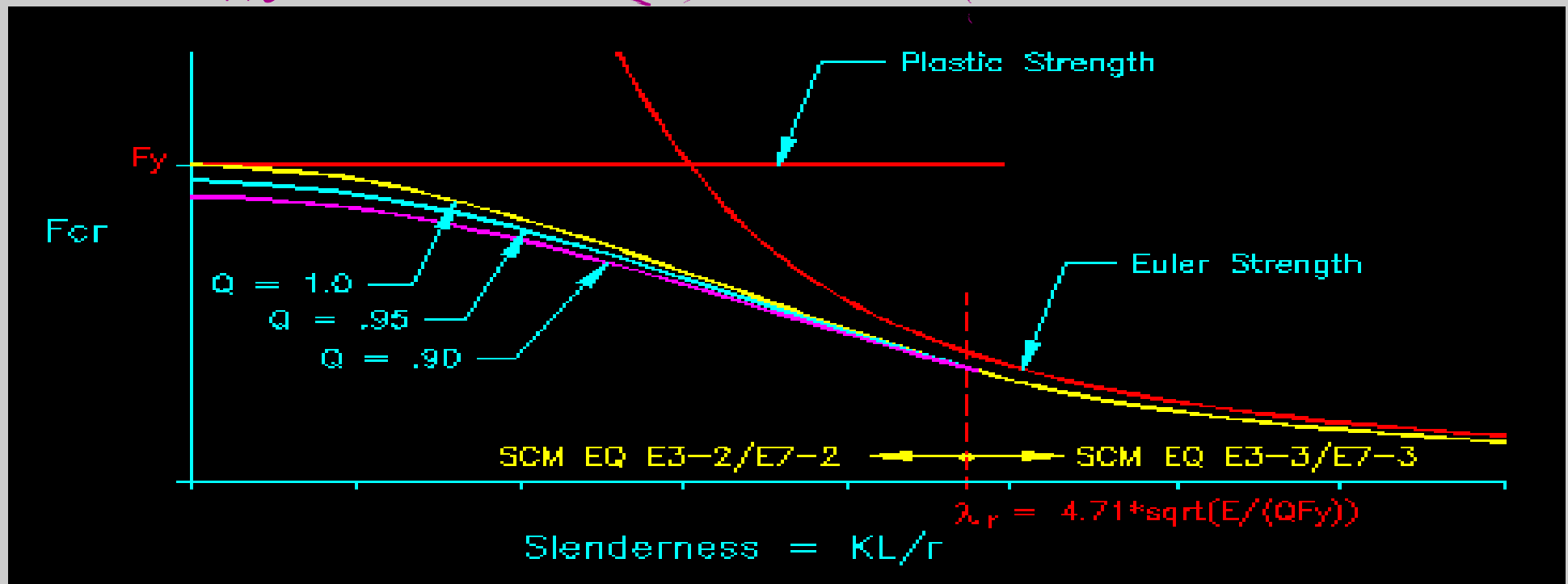
$$F_y = 46 \text{ ksi}$$

"20 10"

$$\phi = \phi_a \cdot \phi_s$$

Figure 7.4.1
 F_{cr} for Slender Sections

Click on image for larger view



E3-2 Short not slender
E7-2 Short slender

$$\lambda_r = 4.71 \sqrt{\frac{E}{\phi F_y}}$$

E3-3 Long not slender
E7-3 Long slender

Chapter E

This chapter addresses members subject to axial compression through the centroidal axis.

The chapter is organized as follows:

- E1. General Provisions
- E2. Effective Length
- E3. Flexural Buckling of Members without Slender Elements
- E4. Torsional and Flexural-Torsional Buckling of Members without Slender Elements
- E5. Single Angle Compression Members
- E6. Built-Up Members
- E7. Members with Slender Elements

$E3 \Rightarrow \text{Short or Long} \} \text{Not slender}$

←

TABLE USER NOTE E1.1
Selection Table for the Application of
Chapter E Sections

Cross Section	Without Slender Elements		With Slender Elements	
	Sections in Chapter E	Limit States	Sections in Chapter E	Limit States
 TB ←	E3 E4	FB TB	E7	LB FB TB
	E3 E4	FB FTB	E7	LB FB FTB

↓↓

$$\lambda_F > \lambda_{Fr}$$

$$\lambda_w > \lambda_{wr}$$

LB: Local buckling

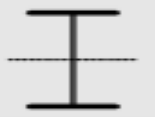
FB: Flange buckling

→ slender members
Short or long

TB Torsional buckling

Prepared by Eng. Maged Kamel.

TABLE USER NOTE E1.1
Selection Table for the Application of
Chapter E Sections

Cross Section	Without Slender Elements		With Slender Elements	
	Sections in Chapter E	Limit States	Sections in Chapter E	Limit States
	E3 E4	FB TB	E7	LB FB TB
  	E3 E4	FB FTB	E7	LB FB FTB
	E3	FB	E7	LB FB

if we have I section
 Either Long or Short
 Equations in E3
 Follow No slender

with no TB
 Equations in E 7
 Check LB Parameters
 FB

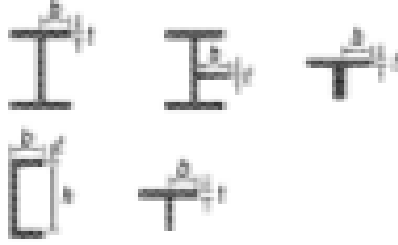
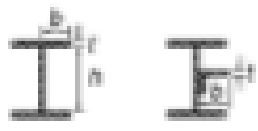
slender
Long or short
 Follow

Tables

Unstiffened table 1-4
E7- Part #1

Chapter-B

TABLE B4.1a
Width-to-Thickness Ratios: Compression Elements
Members Subject to Axial Compression

Case	Description of Element	Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratio λ_c (nonslender/slender)	Examples
1	Flanges of rolled I-shaped sections, plates projecting from rolled I-shaped sections; outstanding legs of pairs of angles connected with continuous contact, flanges of channels, and flanges of tees	b/t b/t	$0.56 \sqrt{\frac{E}{F_y}}$	
2	Flanges of built-up I-shaped sections and plates or angle legs projecting from built-up I-shaped sections	b/t b/t	$0.64 \sqrt{\frac{K_c E}{F_y}}$	
3	Legs of single angles, legs of double angles with separators, and all other unstiffened elements	b/t b/t	$0.45 \sqrt{\frac{E}{F_y}}$	
4	Stems of tees	d/t d/t	$0.75 \sqrt{\frac{E}{F_y}}$	

Rolled
I

Built
up

Angles
T

Limiting r
For no slender

Unstiffened
Flange 4 cases

$$0.56 \sqrt{\frac{E}{F_y}}$$

$$0.64 \sqrt{\frac{K_c E}{F_y}}$$

$$0.45 \sqrt{\frac{E}{F_y}}$$

$$0.75 \sqrt{\frac{E}{F_y}}$$

Limiting
 r
values

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Stiffened Elements

5 → 9

16.1 - 17

AISC

14 lbs webs

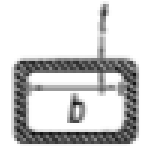
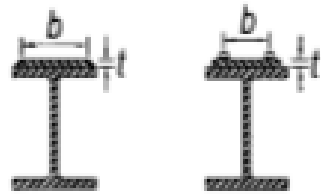
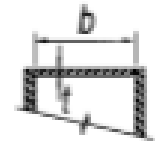
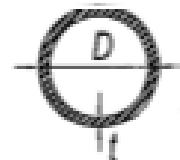
walls

Flange Cover

Other Stiffened

Round HSS

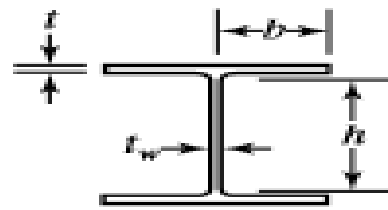
↓ Flexure

Stiffened Elements	5	Webs of doubly-symmetric I-shaped sections and channels	h/t_w	$1.49 \sqrt{\frac{E}{F_y}}$	
	6	Walls of rectangular HSS and boxes of uniform thickness	b/t	$1.40 \sqrt{\frac{E}{F_y}}$	
	7	Flange cover plates and diaphragm plates between lines of fasteners or welds	b/t	$1.40 \sqrt{\frac{E}{F_y}}$	
	8	All other stiffened elements	b/t	$1.49 \sqrt{\frac{E}{F_y}}$	
	9	Round HSS	D/t	$0.11 \frac{E}{F_y}$	

Brief description of Sections

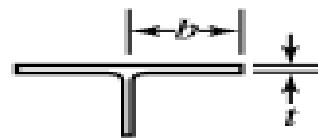
FIGURE 4.9

No
slender
Columns

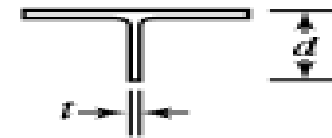


$$b/t \leq 0.56 \sqrt{E/F_y}$$

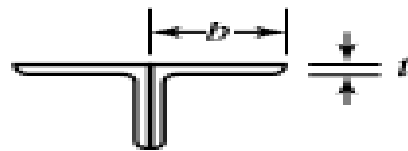
$$h/t_w \leq 1.49 \sqrt{E/F_y}$$



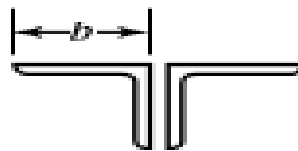
$$b/t \leq 0.56 \sqrt{E/F_y}$$



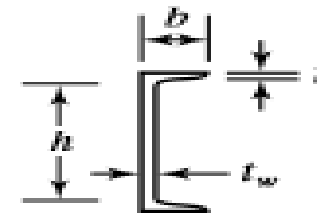
$$b/t \leq 0.75 \sqrt{E/F_y}$$



$$b/t \leq 0.56 \sqrt{E/F_y}$$

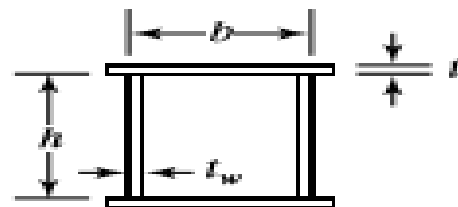


$$b/t \leq 0.45 \sqrt{E/F_y}$$



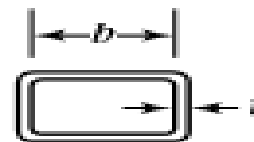
$$b/t \leq 0.56 \sqrt{E/F_y}$$

$$h/t_w \leq 1.49 \sqrt{E/F_y}$$



$$b/t \leq 1.40 \sqrt{E/F_y}$$

$$h/t_w \leq 1.40 \sqrt{E/F_y}$$



$$b/t \leq 1.40 \sqrt{E/F_y}$$



$$D/t \leq 0.11 E/F_y$$

Proof
SEGUI

CM #14 #20/0

E7. MEMBERS WITH SLENDER ELEMENTS

This section applies to slender-element compression members, as defined in Section B4.1 for elements in uniform compression.

The *nominal compressive strength*, P_n , shall be the lowest value based on the applicable *limit states of flexural buckling, torsional buckling, and flexural-torsional buckling*.

$$P_n = F_{cr} A_g \quad (\text{E7-1})$$

The critical *stress*, F_{cr} , shall be determined as follows:

(a) When $\frac{KL}{r} \leq 4.71 \sqrt{\frac{E}{QF_y}}$ $\left(\text{or } \frac{QF_y}{F_e} \leq 2.25 \right)$

$$F_{cr} = Q \left[0.658^{\frac{QF_y}{F_e}} \right] F_y \quad (\text{E7-2})$$

(b) When $\frac{KL}{r} > 4.71 \sqrt{\frac{E}{QF_y}}$ $\left(\text{or } \frac{QF_y}{F_e} > 2.25 \right)$

$$F_{cr} = 0.877 F_e \quad (\text{E7-3})$$

≤ For short
Columns

⇒ Long Columns
> than

Q For slender
Columns

where



F_e = elastic *buckling stress*, calculated using Equations E3-4 and E4-4 for doubly symmetric members, Equations E3-4 and E4-5 for singly symmetric members, and Equation E4-6 for unsymmetric members, except for single angles with $b/t \leq 20$, where F_e is calculated using Equation E3-4, ksi (MPa)

Q = net reduction factor accounting for all slender compression elements;

= 1.0 for members without slender elements, as defined in Section B4.1, for elements in uniform compression

= $Q_s Q_a$ for members with *slender-element sections*, as defined in Section B4.1, for elements in uniform compression.

User Note: For cross sections composed of only unstiffened slender elements, $Q = Q_s$ ($Q_a = 1.0$). For cross sections composed of only stiffened slender elements, $Q = Q_a$ ($Q_s = 1.0$). For cross sections composed of both stiffened and unstiffened slender elements, $Q = Q_s Q_a$. For cross sections composed of multiple unstiffened slender elements, it is conservative to use the smaller Q_s from the more slender element in determining the member strength for pure compression.

Q_a : Slender stiffened $\rightarrow$ webs, $\square$

2. Slender Stiffened Elements, Q_a

The reduction factor, Q_a , for slender *stiffened elements* is defined as follows:

$$Q_a = \frac{A_e}{A_g} \quad (\text{E7-16})$$

where

A_g = gross cross-sectional area of member, in.² (mm²)

A_e = summation of the effective areas of the cross section based on the reduced effective width, b_e , in.² (mm²)

All Sections Except HSS

The reduced effective width, b_e , is determined as follows:

(a) For uniformly compressed slender elements, with $\frac{b}{t} \geq 1.49 \sqrt{\frac{E}{f}}$, except flanges of square and rectangular sections of uniform thickness:

$$b_e = 1.92t \sqrt{\frac{E}{f}} \left[1 - \frac{0.34}{(b/t)} \sqrt{\frac{E}{f}} \right] \leq b \quad (\text{E7-17})$$

where

f is taken as F_{cr} with F_{cr} calculated based on $Q = 1.0$

where

f is taken as F_{cr} with F_{cr} calculated based on $Q = 1.0$

- (b) For flanges of square and rectangular *slender-element sections* of uniform thickness with $\frac{b}{t} \geq 1.40\sqrt{\frac{E}{f}}$: *item 7 Table B4-1*

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.38}{(b/t)}\sqrt{\frac{E}{f}}\right] \leq b \quad (\text{E7-18})$$

where

$$f = P_n/A_e$$

Case of HSS

User Note: In lieu of calculating $f = P_n/A_e$, which requires iteration, f may be taken equal to F_y . This will result in a slightly conservative estimate of *column available strength*.

Example 6-8

#2010

Determine the axial compressive design strength $\phi_c P_n$ and the allowable design strength $\frac{P_n}{\Omega_c}$ of a 24-ft HSS 14 $\times$ 10 $\times$ $\frac{1}{4}$ column section. The base of the column is considered to be fixed, while the upper end is assumed to be pinned. $F_y = 46$ ksi.

Solution Table 1-11

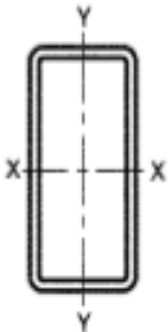
$K_x = 0.80$
 $K_y = 0.80$

Area = 10.80
inch²

$\frac{b}{t} = 39.90$ Flang part
 $\frac{h}{t} = 57.1$ web part

$I_x = 310$ inch⁴
 $r_x = 5.35$ inch



 Table 1-11 (continued) Rectangular HSS Dimensions and Properties									
Shape	Design Wall Thickness, t	Nominal Wt.	Area, A	b/t	h/t	Axis X-X			
	in.	lb/ft	in. ²			I	S	r	Z
						in. ⁴	in. ³	in.	in. ³
HSS14 $\times$ 10 $\times$ $\frac{5}{8}$	0.581	93.34	25.7	14.2	21.1	687	98.2	5.17	120
$\times \frac{1}{2}$	0.465	76.07	20.9	18.5	27.1	573	81.8	5.23	98.8
$\times \frac{3}{8}$	0.349	58.10	16.0	25.7	37.1	447	63.9	5.29	76.3
$\times \frac{5}{16}$	0.291	48.86	13.4	31.4	45.1	380	54.3	5.32	64.6
$\times \frac{1}{4}$	0.233	39.43	10.8	39.9	57.1	310	44.3	5.35	52.4

Prepared by Eng. Maged Kamel.

HSS $\Rightarrow 14'' \times 10'' \times \frac{1}{4}''$

DIMENSIONS AND PROPERTIES

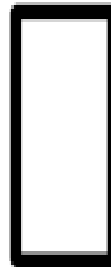
1-77

Second part
of Table
1-11

$$r_y = 4.14''$$

$$I_y = 186 \text{ inch}^4$$

Table 1-11 (continued)
Rectangular HSS
Dimensions and Properties



HSS14-HSS12

Shape	Axis Y-Y				Workable Flat		Torsion		Surface Area
	<i>I</i>	<i>S</i>	<i>r</i>	<i>Z</i>	Depth	Width	<i>J</i>	<i>C</i>	
	in. ⁴	in. ³	in.	in. ³	in.	in.	in. ⁴	in. ³	
HSS14×10× ⁵ / ₈	407	81.5	3.98	95.1	11 ³ / ₁₆	7 ³ / ₁₆	832	146	3.83
× ¹ / ₂	341	68.1	4.04	78.5	11 ³ / ₄	7 ³ / ₄	685	120	3.87
× ³ / ₈	267	53.4	4.09	60.7	12 ⁵ / ₁₆	8 ⁵ / ₁₆	528	91.8	3.90
× ¹ / ₄	227	45.5	4.12	51.4	12 ⁹ / ₁₆	8 ⁹ / ₁₆	446	77.4	3.92
× ¹ / ₄	186	37.2	4.14	41.8	12 ⁷ / ₈	8 ⁷ / ₈	362	62.6	3.93

① Check whether Column is Long or short $F_y = 46 \text{ ksi}$

$\swarrow$ $\searrow$

$\frac{b}{E} \left(\frac{KL}{r} \right)_y = \frac{(0.80)(288)}{4.14} = 55.652$

$K = 0.80$
 $L = 24' \Rightarrow 288''$
 $r_y = 4.14$
 $r_x = 5.35''$
 $r_y \text{ Governs}$
 $r_y > r_x$

$$4.71 \sqrt{\frac{E}{F_y}} \Rightarrow 4.71 \sqrt{\frac{29000}{46}} = 118.26$$

Since $\left(\frac{KL}{r_y} \right)_y < 4.71 \sqrt{\frac{E}{F_y}}$
Column is short

Find Euler stress $F_E = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 (29000)}{0.80(55.65)^2}$

$F_E = 92.42 \text{ ksi}$

$\pi = 3.14159, E = 29000 \text{ ksi}$

$L = 24' \Rightarrow 288''$

$r_y = 4.14$

$r_x = 5.35''$

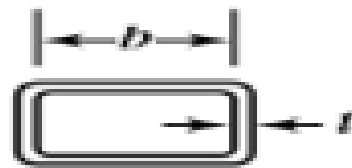
r_y Governs

Short $\rightarrow$ Slender
or non-slender

$\lambda_r = 1.40 \sqrt{\frac{E}{F_y}} = 1.40 \sqrt{\frac{29000}{46}} = 35.15$

$r_y > r_x$

$h/t = 57.10 > 35.15$
 $Q_A < 1$



$\lambda_r \leq 1.40 \sqrt{E/F_y}$

$> 1.40 \sqrt{E/F_y}$ Slender

No slender

Refer to B4.1B $\rightarrow$ Stiffened

(d) For flanges of rectangular hollow structural sections (HSS), the width, b , is the clear distance between webs less the inside corner radius on each side. For webs of rectangular HSS, h is the clear distance between the flanges less the inside corner radius on each side. If the corner radius is not known, b and h shall be taken as the corresponding outside dimension minus three times the thickness. The thickness, t , shall be taken as the *design wall thickness*, per Section B4.2.

How to
get b, h



$$\left\{ \begin{array}{l} b \Rightarrow \text{in Estimate} \\ h \Rightarrow \end{array} \right. \begin{array}{l} = b - 3tw \\ = h - 3tw \end{array}$$

outside dimensions

$$b/t = 39.9 \quad h/t = 57.10, \quad t = 0.233$$

where

f is taken as F_{cr} with F_{cr} calculated based on $Q = 1.0$

(b) For flanges of square and rectangular *slender-element sections* of uniform thick-

ness with $\frac{b}{t} \geq 1.40 \sqrt{\frac{E}{f}}$:

$$b_e = 1.92t \sqrt{\frac{E}{f}} \left[1 - \frac{0.38}{(b/t)} \sqrt{\frac{E}{f}} \right] \leq b \quad (E7-18)$$

where

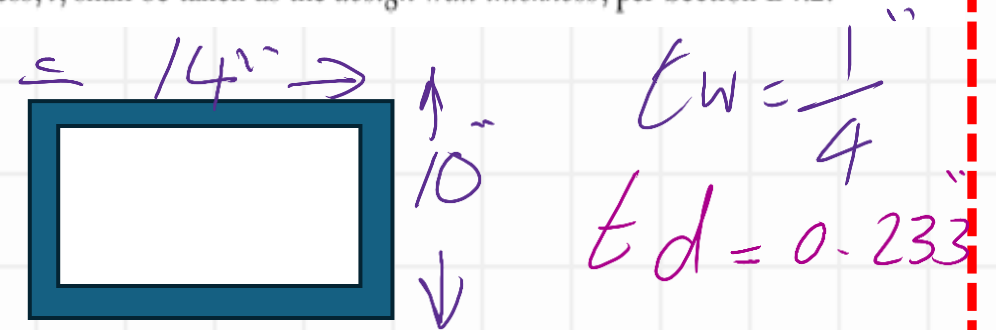
$f = P_n/A_e$

$$b/t = 39.9$$

User Note: In lieu of calculating $f = P_n/A_e$, which requires iteration, f may be taken equal to F_y . This will result in a slightly conservative estimate of *column available strength*.

$$b_e = 1.92(0.233) \sqrt{\frac{29000}{46}} \left[1 - \frac{0.38}{39.9} \sqrt{\frac{29000}{46}} \right] = 8.55" < 9.30"$$

(d) For flanges of rectangular hollow structural sections (HSS), the width, b , is the clear distance between webs less the inside corner radius on each side. For webs of rectangular HSS, h is the clear distance between the flanges less the inside corner radius on each side. If the corner radius is not known, b and h shall be taken as the corresponding outside dimension minus three times the thickness. The thickness, t , shall be taken as the *design wall thickness*, per Section B4.2.



$$h = 14 - 3(0.233) = 13.301$$

$$b = 10 - 3(0.233) = 9.301$$

$$h = 13.30" \quad b = 9.30"$$

$$t_d = 0.233$$

$$b/t = 39.9$$

$$h = 13.30", t_d = 0.233"$$

where

f is taken as F_{cr} with F_{cr} calculated based on $Q = 1.0$

(b) For flanges of square and rectangular *slender-element sections* of uniform thick-

ness with $\frac{b}{t} \geq 1.40 \sqrt{\frac{E}{f}}$:

$$h/t = 57.9 \Rightarrow$$

$$b_e = 1.92t \sqrt{\frac{E}{f}} \left[1 - \frac{0.38}{(b/t)} \sqrt{\frac{E}{f}} \right] \leq b \quad (E7-18)$$

where

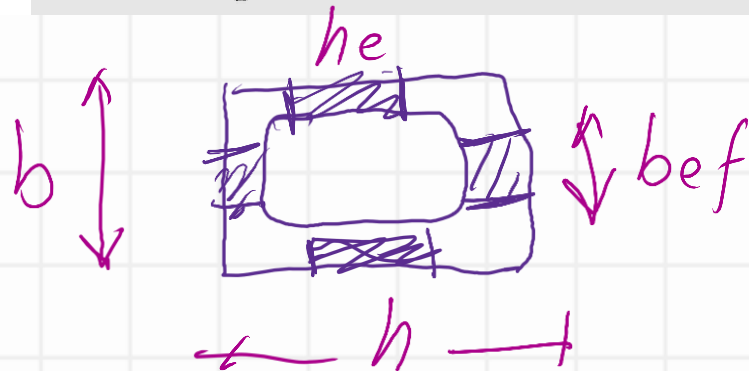
$$f = P_n/A_e$$

User Note: In lieu of calculating $f = P_n/A_e$, which requires iteration, f may be taken equal to F_y . This will result in a slightly conservative estimate of *column available strength*.

$$h_e = \frac{1.92}{(0.233)} \left(\sqrt{\frac{29000}{46}} \left[1 - \frac{0.38}{57.9} \sqrt{\frac{29000}{46}} \right] \right)$$

$$h_e = 9.36" < 13.30" \Rightarrow (h - t_d)$$

Area not used



$$\begin{aligned} & 2(b - b_e)t_d = 2[9.3 - 8.55](0.233) \\ & + 2(h - h_e)t_d = 2[13.3 - 9.36](0.233) \\ & = 0.3495 + 1.836 \Rightarrow 2.1855 \text{ inch}^2 \end{aligned}$$

$$A_g = 10.80 \text{ inch}^2$$

$$A_{\text{not used}} = 2.1855 \text{ inch}^2$$

$$A_e = 10.8 - 2.1855 = 8.61 \text{ inch}^2 \quad \left. \begin{array}{l} \phi_s = 1 \\ \phi = \phi_s \phi_A \end{array} \right\}$$

$$\phi = \phi_a = \frac{A_e}{A_g} = \frac{8.61}{10.80} = 0.7972$$

$$F_e = 92.42 \text{ ksi}$$

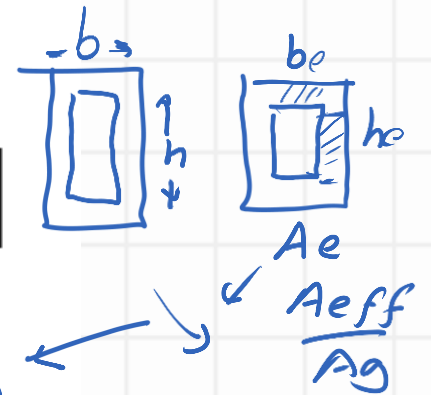
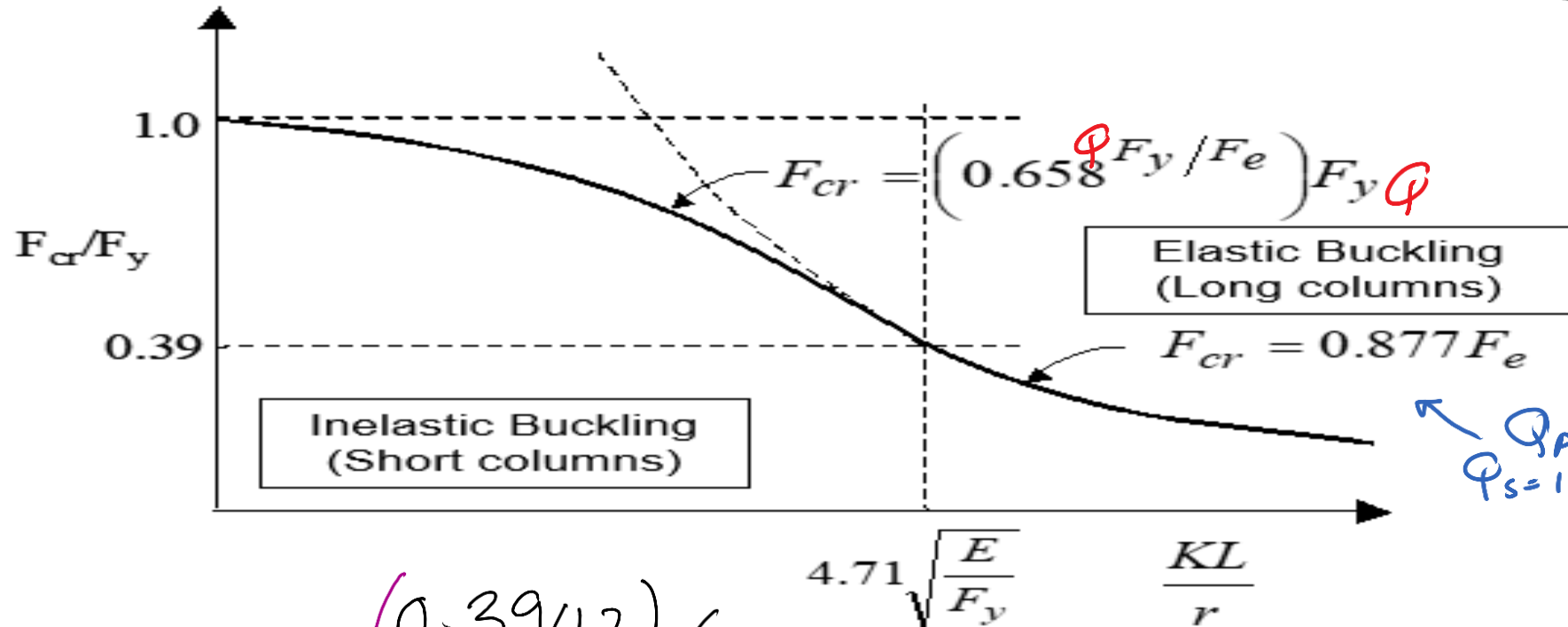
$$F_y = 46 \text{ ksi}$$

$$\left. \begin{array}{l} F_e = 92.42 \text{ ksi} \\ F_y = 46 \text{ ksi} \end{array} \right\} \phi \frac{F_y}{F_e} = 0.7972 \left(\frac{46}{92.42} \right) = 0.3968$$

$$Q = 0.7972, \quad F_{cr} = 0.658^Q F_y / F_e (Q F_y)$$

$$F_e = 92.42 \text{ ksi}$$

$$F_y = 46 \text{ ksi}$$



$$F_{cr} = 0.658^{(0.3942)} (0.7972) (46) = 31.06 \text{ ksi}$$

Prepared by Eng. Maged Kamel.

$$P_n = F_{cr} \cdot A_g$$

$$= 31.06 (10.20) = 335.45$$

$$P_n \approx 335.50 \text{ Kips}$$

$$\text{LRFD} = \phi P_n = 0.90(335.5) = 301.90$$

Kips

$$\text{ASD} = \frac{P_n}{\lambda_c} = 335.5 / 1.67 = 200.9$$

Kips

Prepared by Eng. Maged Kamel.