A simply supported beam of grade 50 steel is laterally braced at 4 ft intervals. If the beam is subjected to a uniform factored bending moment of 270 ft-kips (LRFD) or 180 ft-kips (ASD), with $C_b = 1.0$, determine (a) the lightest adequate W shape, and (b) the W shape with the minimum allowable depth.

Alan Williams - Structural Engineering Reference Manual-Professional Publications

Example 4-5

Design

ALAN Williams 8th rev.

A simply supported beam of grade 50 steel is laterally braced at 4 ft intervals. If the beam is subjected to a uniform factored bending moment of 270 ft-kips (LRFD) or 180 ft-kips (ASD), with $C_b = 1.0$, determine (a) the lightest adequate W shape, and (b) the W shape with the minimum allowable depth.

Solution

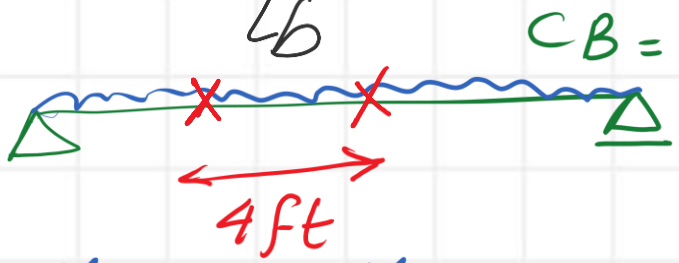
Grade 50

$M_U$: 270 Ft-kips $\longleftrightarrow$ 180 Ft-kips ASD

$C_B = 1$

$f_y = 50$ ksi

$L_b = 4$ distance between bracings




For $M_n\phi \geq M_U \rightarrow 0.90 f_y Z_x \geq 270$

$Z_x = \frac{270}{0.90(50)} \times 12 = 72$ inch$^3$ check table

Given $M_n$ Use $M_{ult} \leq \phi_b M_n$

$\phi_b = 0.90$

$\frac{M_{ult}}{\phi_b} = M_n = F_y Z_x$

$Z_x = \frac{M_n}{\phi F_y}$

only $F_y = 50$ ksi with Use Table 3-2

$Z_x$ → use → Table 3-2 → Select section W- with $L_p$ & $\phi M_p$, $L_r$ & $\phi M_r$ from Table 1-1 → get $r_y$

Find $L_p$ Equation

Check whether $L_b < L_p \rightarrow M_n = F_y Z_x$ act

## Table 3-2 (continued)
## W-Shapes
### Selection by $Z_x$

$F_y = 50$ ksi

$Z_x$

Lp Lr No Loc

Given Mult

270 Ft.kips

| Shape | $Z_x$ | $M_{px}/\Omega_b$ | $\phi_b M_{px}$ | $M_{rx}/\Omega_b$ | $\phi_b M_{rx}$ | $BF/\Omega_b$ | $\phi_b BF$ | $L_p$ | $L_r$ | $I_x$ | $V_{nx}/\Omega_v$ | $\phi_v V_{nx}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | kip-ft | kip-ft | kip-ft | kip-ft | kips | kips | | | | kips | kips |
| | in.$^3$ | ASD | LRFD | ASD | LRFD | ASD | LRFD | ft | ft | in.$^4$ | ASD | LRFD |
| **W16×40** | **73.0** | **182** | **274** | **113** | **170** | **6.67** | 10.0 | **5.55** | **15.9** | **518** | **97.6** | **146** |
| W12×50 | 71.9 | 179 | 270 | 112 | 169 | 3.97 | 5.98 | 6.92 | 23.8 | 391 | 90.3 | 135 |
| W8×67 | 70.1 | 175 | 263 | 105 | 159 | 1.75 | 2.59 | 7.49 | 47.6 | 272 | 103 | 154 |
| W14×43 | 69.6 | 174 | 261 | 109 | 164 | 4.88 | 7.28 | 6.68 | 20.0 | 428 | 83.6 | 125 |
| W10×54 | 66.6 | 166 | 250 | 105 | 158 | 2.48 | 3.75 | 9.04 | 33.6 | 303 | 74.7 | 112 |

| ASD | LRFD |
|---|---|
| $\Omega_b = 1.67$<br>$\Omega_v = 1.50$ | $\phi_b = 0.90$<br>$\phi_v = 1.00$ |

[f] Shape exceeds compact limit for flexure with $F_y = 50$ ksi.

$\phi_b M_{px}$ $\phi_b M_{rx}$ Lp Lr

Select $Z_x \geq 72$ inch$^3$ → Use W16 x 40

Lp = 5.55' & Lr = 15.90'

$\phi_b M_{px} = 274$ Ft.kips

$\phi_b M_{rx} = 170$ Ft.kips

We Can use a given $r_y$ for a section to Find $L_p$ & $L_r$ values From Table 1-1

$$L_p = 1.76\, r_y \sqrt{\frac{E}{F_y}} \Rightarrow$$

$E = 29000$ ksi

$F_y$ based on required value

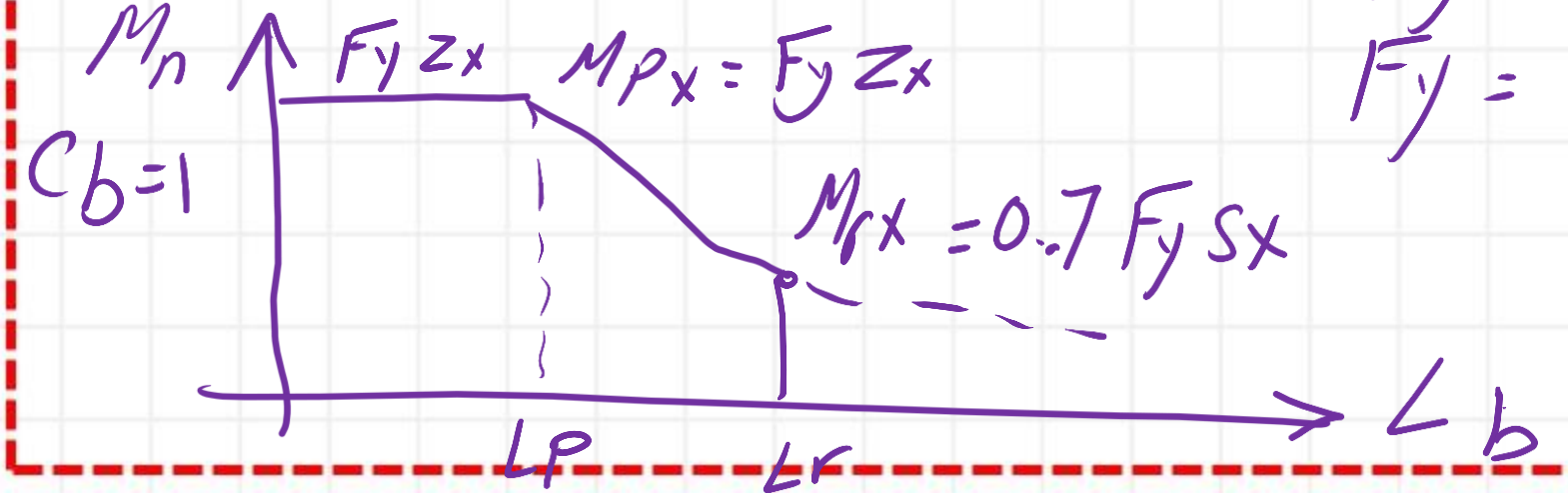
$F_y = 36$ ksi; $L_p = 49.953\, r_y$

$F_y = 42$ ksi; $L_p = 46.247\, r_y$

**$F_y = 45$ ksi; $L_p = 44.679\, r_y$**

$F_y = 50$ ksi; $L_p = 42.386\, r_y$

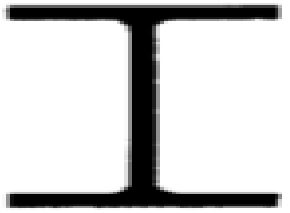
$F_y = 65$ ksi; $L_p = 37.175\, r_y$



Properties of W16x40

LRFD

## Table 1-1 (continued)
## W-Shapes
### Properties




| Nominal Wt. | Compact Section Criteria | | Axis X-X | | | | Axis Y-Y | | | | $r_{ts}$ | $h_o$ | Torsional Properties | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\frac{b_f}{2t_f}$ | $\frac{h}{t_w}$ | $I$ | $S$ | $r$ | $Z$ | $I$ | $S$ | $r$ | $Z$ | | | $J$ | $C_w$ |
| lb/ft | | | in.$^4$ | in.$^3$ | in. | in.$^3$ | in.$^4$ | in.$^3$ | in. | in.$^3$ | in. | in. | in.$^4$ | in.$^6$ |
| 50 | 5.61 | 37.4 | 659 | 81.0 | 6.68 | 92.0 | 37.2 | 10.5 | 1.59 | 16.3 | 1.89 | 15.7 | 1.52 | 2270 |
| 45 | 6.23 | 41.1 | 586 | 72.7 | 6.65 | 82.3 | 32.8 | 9.34 | 1.57 | 14.5 | 1.87 | 15.5 | 1.11 | 1990 |
| 40 | 6.93 | 46.5 | 518 | 64.7 | 6.63 | 73.0 | 28.9 | 8.25 | 1.57 | 12.7 | 1.86 | 15.5 | 0.794 | 1730 |
| 36 | 8.12 | 48.1 | 448 | 56.5 | 6.51 | 64.0 | 24.5 | 7.00 | 1.52 | 10.8 | 1.83 | 15.5 | 0.545 | 1460 |

$r_y = 1.57"$ → get $L_p = 42.386(1.57)/12 = 5.55'$

Check $\phi_b Z_x F_y = 0.90(73)(50)/12 \Rightarrow 274$ FT.kips

>> > Mult → 270 FT.kips

$L_p < L_b$ 4 ft

Get results From Table 3-2

## Table 3-2 (continued)
## W-Shapes
### Selection by $Z_x$

$F_y = 50$ ksi

$Z_x$

$M_{px}$

$L_p = 5.55'$

| Shape | $Z_x$ | $M_{px}/\Omega_b$ | $\phi_b M_{px}$ | $M_{rx}/\Omega_b$ | $\phi_b M_{rx}$ | $BF/\Omega_b$ | $\phi_b BF$ | $L_p$ | $L_r$ | $I_x$ | $V_{nx}/\Omega_v$ | $\phi_v V_{nx}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | kip-ft | kip-ft | kip-ft | kip-ft | kips | kips | | | | kips | kips |
| | in.$^3$ | ASD | LRFD | ASD | LRFD | ASD | LRFD | ft | ft | in.$^4$ | ASD | LRFD |
| **W16×40** | **73.0** | **182** | **274** | **113** | **170** | **6.67** | 10.0 | **5.55** | **15.9** | **518** | **97.6** | **146** |
| W12×50 | 71.9 | 179 | 270 | 112 | 169 | 3.97 | 5.98 | 6.92 | 23.8 | 391 | 90.3 | 135 |
| W8×67 | 70.1 | 175 | 263 | 105 | 159 | 1.75 | 2.59 | 7.49 | 47.6 | 272 | 103 | 154 |
| W14×43 | 69.6 | 174 | 261 | 109 | 164 | 4.88 | 7.28 | 6.68 | 20.0 | 428 | 83.6 | 125 |
| W10×54 | 66.6 | 166 | 250 | 105 | 158 | 2.48 | 3.75 | 9.04 | 33.6 | 303 | 74.7 | 112 |

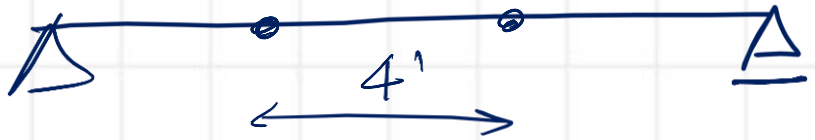
| ASD | LRFD |
|---|---|
| $\Omega_b = 1.67$<br>$\Omega_v = 1.50$ | $\phi_b = 0.90$<br>$\phi_v = 1.00$ |

[f] Shape exceeds compact limit for flexure with $F_y = 50$ ksi.

$\phi_b M_n$ From Table 3-2 $= \phi_b M_{px} = 274$ FT. kips

$L_b = 4' < L_p < 5.55'$ Same result as obtained From Table 1-1

ASD $l_b < L_p$




Given $M_T = 180$ Ft. kips

Find $Z_x$

$F_y = 50$ ksi

$\Omega_b = 1.67$

Given $M_T = 180$ Ft. kips

From

For $\frac{M_n}{\Omega_b} = M_T \Rightarrow \frac{F_y Z_x}{\Omega} = 180$ (12) $\Rightarrow$ Ft. kips

$Z_x = \frac{\Omega (180)}{F_y} = 1.67 \frac{(180)}{50} (12) = 72.144$ inch$^3$ inch. kips

$\Rightarrow$ required

Use Table 3-2 to Find W section

$Z_x$ required $= 72.14$ inch$^3$

$Z_x$ req = 72.14 inch3

ASD Design

$F_y$ = 50 ksi

Sorted Zx

## Table 3-2 (continued)
## W-Shapes
### Selection by $Z_x$

Lb = 4'
given

$Z_x$

| Shape | $Z_x$ | $M_{px}/\Omega_b$ | $\phi_b M_{px}$ | $M_{rx}/\Omega_b$ | $\phi_b M_{rx}$ | $BF/\Omega_b$ | $\phi_b BF$ | $L_p$ | $L_r$ | $I_x$ | $V_{nx}/\Omega_v$ | $\phi_v V_{nx}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | kip-ft | kip-ft | kip-ft | kip-ft | kips | kips | | | | kips | kips |
| | in.³ | ASD | LRFD | ASD | LRFD | ASD | LRFD | ft | ft | in.⁴ | ASD | LRFD |
| W16×40 | 73.0 | 182 | 274 | 113 | 170 | 6.67 | 10.0 | 5.55 | 15.9 | 518 | 97.6 | 146 |
| W12×50 | 71.9 | 179 | 270 | 112 | 169 | 3.97 | 5.98 | 6.92 | 23.8 | 391 | 90.3 | 135 |
| W8×67 | 70.1 | 175 | 263 | 105 | 159 | 1.75 | 2.59 | 7.49 | 47.6 | 272 | 103 | 154 |
| W14×43 | 69.6 | 174 | 261 | 109 | 164 | 4.88 | 7.28 | 6.68 | 20.0 | 428 | 83.6 | 125 |
| W10×54 | 66.6 | 166 | 250 | 105 | 158 | 2.48 | 3.75 | 9.04 | 33.6 | 303 | 74.7 | 112 |

| ASD | LRFD |
|---|---|
| $\Omega_b = 1.67$ | $\phi_b = 0.90$ |
| $\Omega_v = 1.50$ | $\phi_v = 1.00$ |

[f] Shape exceeds compact limit for flexure with $F_y = 50$ ksi.

ASD

Lp = 5.55'
Lr = 15.90'

Choose section W16 x 40

$Z_x = 73.00$ inch3 > 72.14 inch3

Given $M_T = 180$ FT.kips

We Can Use W16x40

ASD

Table 1-1 (continued)

# W-Shapes

## Properties

Part 2 → $r_y = 1.57$"

W16–W14

| Nominal Wt. | Compact Section Criteria | | Axis X-X | | | | Axis Y-Y | | | | $r_{ts}$ | $h_o$ | Torsional Properties | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\frac{b_f}{2t_f}$ | $\frac{h}{t_w}$ | $I$ | $S$ | $r$ | $Z$ | $I$ | $S$ | $r$ | $Z$ | | | $J$ | $C_w$ |
| lb/ft | | | in.$^4$ | in.$^3$ | in. | in.$^3$ | in.$^4$ | in.$^3$ | in. | in.$^3$ | in. | in. | in.$^4$ | in.$^6$ |
| 50 | 5.61 | 37.4 | 659 | 81.0 | 6.68 | 92.0 | 37.2 | 10.5 | 1.59 | 16.3 | 1.89 | 15.7 | 1.52 | 2270 |
| 45 | 6.23 | 41.1 | 586 | 72.7 | 6.65 | 82.3 | 32.8 | 9.34 | 1.57 | 14.5 | 1.87 | 15.5 | 1.11 | 1990 |
| 40 | 6.93 | 46.5 | 518 | 64.7 | 6.63 | 73.0 | 28.9 | 8.25 | 1.57 | 12.7 | 1.86 | 15.5 | 0.794 | 1730 |
| 36 | 8.12 | 48.1 | 448 | 56.5 | 6.51 | 64.0 | 24.5 | 7.00 | 1.52 | 10.8 | 1.83 | 15.5 | 0.545 | 1460 |

$L_p = 42.386\, r_y = 42.386(1.57) = \frac{66.546}{12} = 5.55'$

$L_b = 4' < 5.55'$ Section is Compact

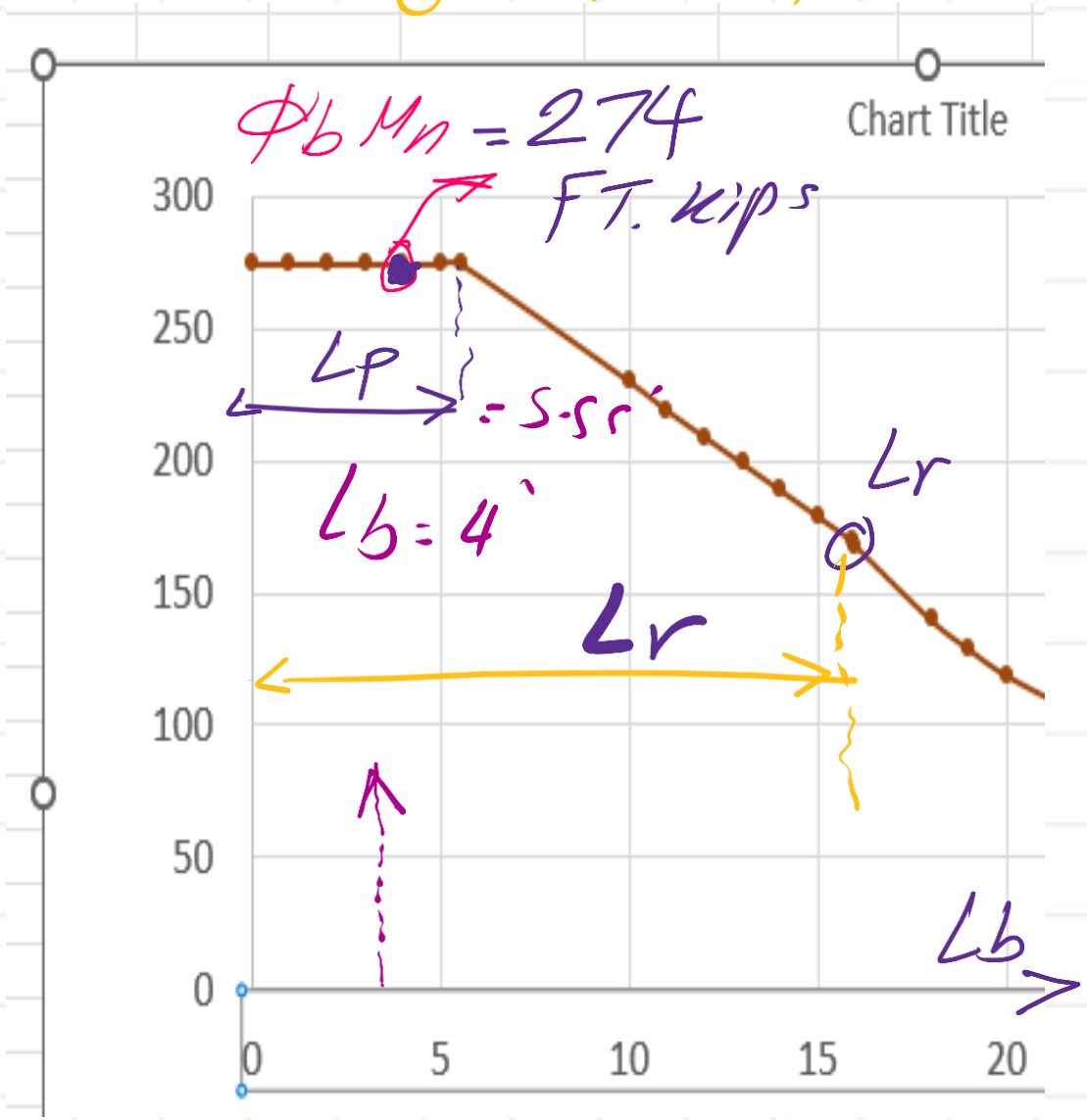
Given $\frac{M_n}{\Omega_b} = \frac{F_y Z_x}{\Omega_b} = \frac{50(73)}{1.67(12)} = 182.14$ FT.kips

$> 180$ FT.kips

# LRFD

# Excel graph - Part #1

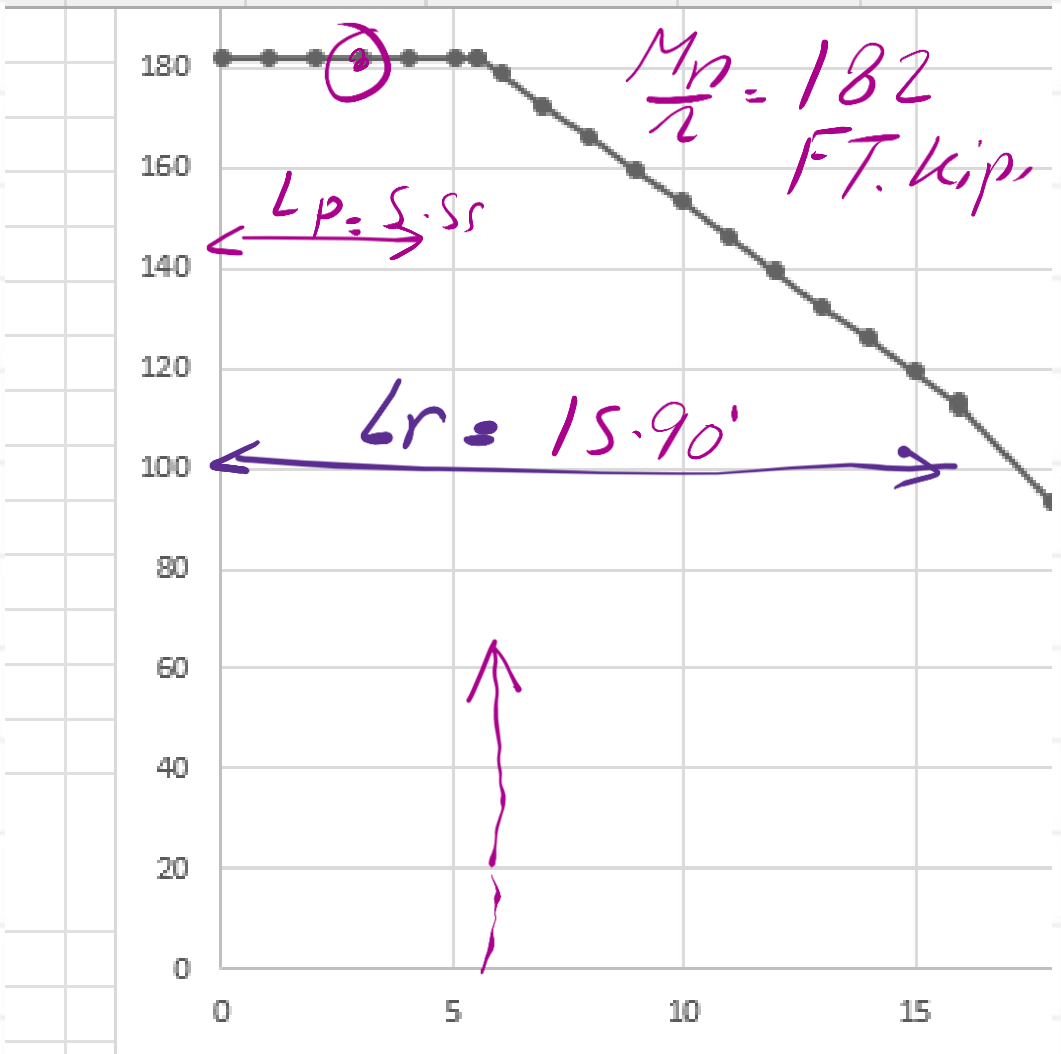
| length of bracing-Ft | Phi*Mn | Mn*(1/Ω)*1.00 |
|---|---|---|
| 0 | 274 | 182 |
| 1 | 274 | 182 |
| 2 | 274 | 182 |
| 3 | 274 | 182 |
| 4 | 274 | 182 |
| 5 | 274 | 182 |
| Lp 5.55 | 274 | 182 |
| 10 | 274 | 182 |
| 11 | 230 | 153 |
| 12 | 219 | 146 |
| 13 | 209 | 139 |
| 14 | 199 | 132 |
| 15 | 189 | 126 |
| Lr 15.895 | 179 | 119 |
| 16 | 170 | 113 |



# Excel graph #1

$M_T = 180$ Ft.kip

| length of bracing-Ft | Phi*Mn | Mn*(1/Ω)*1.00 |
|---|---|---|
| 0 | 274 | 182 |
| 1 | 274 | 182 |
| 2 | 274 | 182 |
| 3 | 274 | 182 |
| 4 | 274 | 182 |
| 5 | 274 | 182 |
| Lp 5.55 | 274 | 182 |
| 10 | 274 | 182 |
| 11 | 230 | 153 |
| 12 | 219 | 146 |
| 13 | 209 | 139 |
| 14 | 199 | 132 |
| 15 | 189 | 126 |
| Lr 15.895 | 179 | 119 |
| 16 | 170 | 113 |



# LRFD Part-2 Select W Section with min. allowable depth

$M_U = 270$ Ft.kip

$L_b = 4'$

$F_y = 50$ ksi

**Table 3-2 (continued)**

**W-Shapes**

**Selection by $Z_x$**

$Z_x$

$L_p = 9.08'$

$L_r = 36.60'$

| Shape | $Z_x$ | $M_{px}/\Omega_b$ | $\phi_b M_{px}$ | $M_{rx}/\Omega_b$ | $\phi_b M_{rx}$ | $BF/\Omega_b$ | $\phi_b BF$ | $L_p$ | $L_r$ | $I_x$ | $V_{nx}/\Omega_v$ | $\phi_v V_{nx}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | kip-ft | kip-ft | kip-ft | kip-ft | kips | kips | | | | kips | kips |
| | $in.^3$ | ASD | LRFD | ASD | LRFD | ASD | LRFD | ft | ft | $in.^4$ | ASD | LRFD |
| W10×60 | 74.6 | 186 | 280 | 116 | 175 | 2.54 | 3.82 | 9.08 | 36.6 | 341 | 85.7 | 129 |
| **W16×40** | **73.0** | **182** | **274** | **113** | **170** | **6.67** | **10.0** | **5.55** | **15.9** | **518** | **97.6** | **146** |
| W12×50 | 71.9 | 179 | 270 | 112 | 169 | 3.97 | 5.98 | 6.92 | 23.8 | 391 | 90.3 | 135 |
| W8×67 | 70.1 | 175 | 263 | 105 | 159 | 1.75 | 2.59 | 7.49 | 47.6 | 272 | 103 | 154 |
| W14×43 | 69.6 | 174 | 261 | 109 | 164 | 4.88 | 7.28 | 6.68 | 20.0 | 428 | 83.6 | 125 |
| W10×54 | 66.6 | 166 | 250 | 105 | 158 | 2.48 | 3.75 | 9.04 | 33.6 | 303 | 74.7 | 112 |

Select W10 X60 → depth 10" < 16" but has higher $Z_x = 74.60\ inch^3$ above W16X40

From Table $\phi_b M_n = 280$ Ft.kips > 270 Ft.kips

Part-2 Select W Section with min. allowable depth

$M_T = 180$ Ft.kip

$L_b = 4'$

$F_y = 50$ ksi

**Table 3-2 (continued)**

**W-Shapes**

**Selection by $Z_x$**

$Z_x$

| Shape | $Z_x$ | $M_{px}/\Omega_b$ | $\phi_b M_{px}$ | $M_{rx}/\Omega_b$ | $\phi_b M_{rx}$ | $BF/\Omega_b$ | $\phi_b BF$ | $L_p$ | $L_r$ | $I_x$ | $V_{nx}/\Omega_v$ | $\phi_v V_{nx}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | kip-ft | kip-ft | kip-ft | kip-ft | kips | kips | | | | kips | kips |
| | $in.^3$ | ASD | LRFD | ASD | LRFD | ASD | LRFD | ft | ft | $in.^4$ | ASD | LRFD |
| W10×60 | 74.6 | 186 | 280 | 116 | 175 | 2.54 | 3.82 | 9.08 | 36.6 | 341 | 85.7 | 129 |
| **W16×40** | **73.0** | **182** | **274** | **113** | **170** | **6.67** | **10.0** | **5.55** | **15.9** | **518** | **97.6** | **146** |
| W12×50 | 71.9 | 179 | 270 | 112 | 169 | 3.97 | 5.98 | 6.92 | 23.8 | 391 | 90.3 | 135 |
| W8×67 | 70.1 | 175 | 263 | 105 | 159 | 1.75 | 2.59 | 7.49 | 47.6 | 272 | 103 | 154 |
| W14×43 | 69.6 | 174 | 261 | 109 | 164 | 4.88 | 7.28 | 6.68 | 20.0 | 428 | 83.6 | 125 |
| W10×54 | 66.6 | 166 | 250 | 105 | 158 | 2.48 | 3.75 | 9.04 | 33.6 | 303 | 74.7 | 112 |

Select W10 X60 → depth 10" < 16" but has higher $Z_x = 74.60\ inch^3$ above W16X40

From Table $M_{px}/\Omega_b = 186$ Ft.kips > 180 Ft.kips

LRFD

# Excel sheet # 2

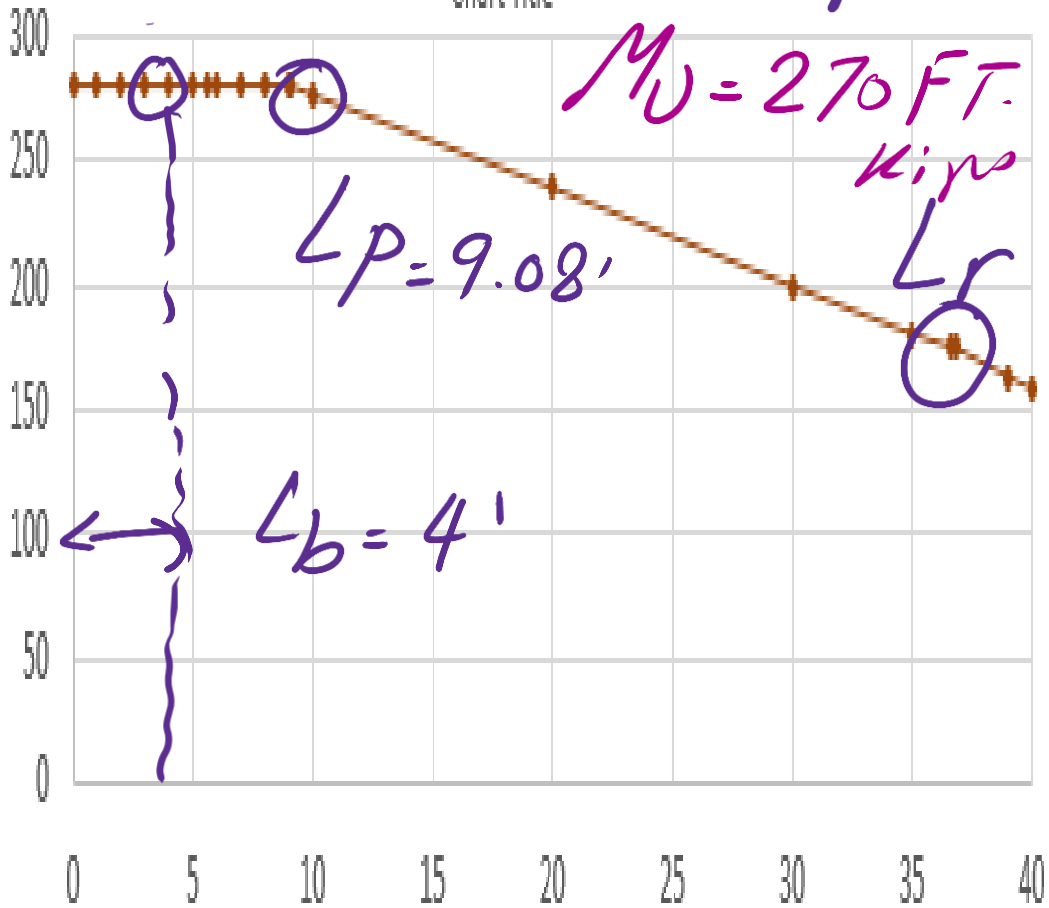
| length of bracing-Ft | Phi*Mn | Mn*(1/Ω)*1.00 |
|---|---|---|
| 0 | 280 | 186 |
| 1 | 280 | 186 |
| 2 | 280 | 186 |
| 3 | 280 | 186 |
| 4 | 280 | 186 |
| 5 | 280 | 186 |
| 5.58 | 280 | 186 |
| 6 | 280 | 186 |
| 7 | 280 | 186 |
| 8 | 280 | 186 |
| 9 | 280 | 186 |
| 9.08 | 280 | 186 |
| 10 | 276 | 184 |
| 20 | 239 | 159 |
| 30 | 200 | 133 |
| 35 | 181 | 120 |
| 36.614 | 176 | 117 |

Lb

Lp

Lr

$\phi M_n = 280$ FT. kips



# Case # 2 ASD

# W10 x 60 Section

| length of bracing-Ft |
|---|
| 0 |
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 5.58 |
| 6 |
| 7 |
| 8 |
| 9 |
| 9.08 |
| 10 |
| 20 |
| 30 |
| 35 |
| 36.614 |

| Phi*Mn | Mn*(1/Ω)*1.00 |
|---|---|
| 280 | 186 |
| 280 | 186 |
| 280 | 186 |
| 280 | 186 |
| 280 | 186 |
| 280 | 186 |
| 280 | 186 |
| 280 | 186 |
| 280 | 186 |
| 280 | 186 |
| 280 | 186 |
| 280 | 186 |
| 280 | 186 |
| 276 | 184 |
| 239 | 159 |
| 200 | 133 |
| 181 | 120 |
| 176 | 117 |