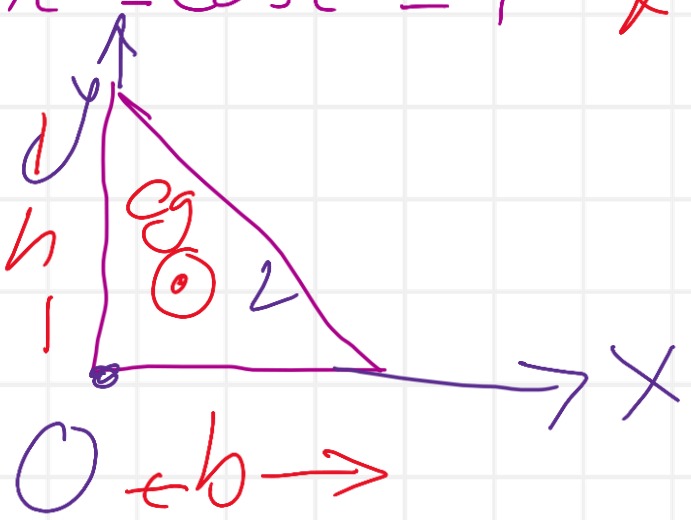
How can we derive the Expression for I_y value for the left Corner of a right-angle triangle _Case_1 & and for Cg

→ opposite to the left

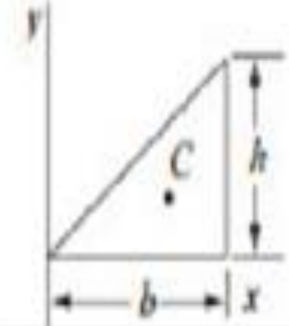
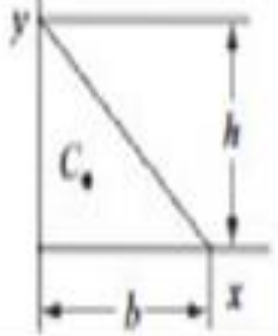
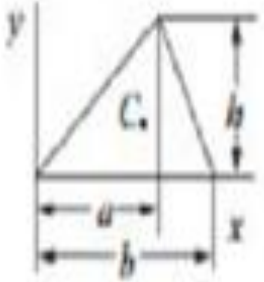


ⓐ. Use a VL strip ⟹

ⓑ. Use a HL strip

I_y Value $= hb^3/12$ or $(b^3h/12)$

Table For Ix, Iy, Ixy

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = bh/2$ $x_c = 2b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/4$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/2$	$I_{x_cy_c} = Abh/36 = b^2h^2/72$ $I_{xy} = Abh/4 = b^2h^2/8$
	$A = bh/2$ $x_c = b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/6$	$I_{x_cy_c} = -Abh/36 = -b^2h^2/72$ $I_{xy} = Abh/12 = b^2h^2/24$
	$A = bh/2$ $x_c = (a+b)/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = [bh(b^2 - ab + a^2)]/36$ $I_x = bh^3/12$ $I_y = [bh(b^2 + ab + a^2)]/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = (b^2 - ab + a^2)/18$ $r_x^2 = h^2/6$ $r_y^2 = (b^2 + ab + a^2)/6$	$I_{x_cy_c} = [Ah(2a-b)]/36$ $= [bh^2(2a-b)]/72$ $I_{xy} = [Ah(2a+b)]/12$ $= [bh^2(2a+b)]/24$

our case

I_y For a right-angle

We will use a vertical strip height $= y$, width $= dx$

From $x = 0 \rightarrow b$

① First Estimate the area we call it dA

$$dA = y \cdot dx$$

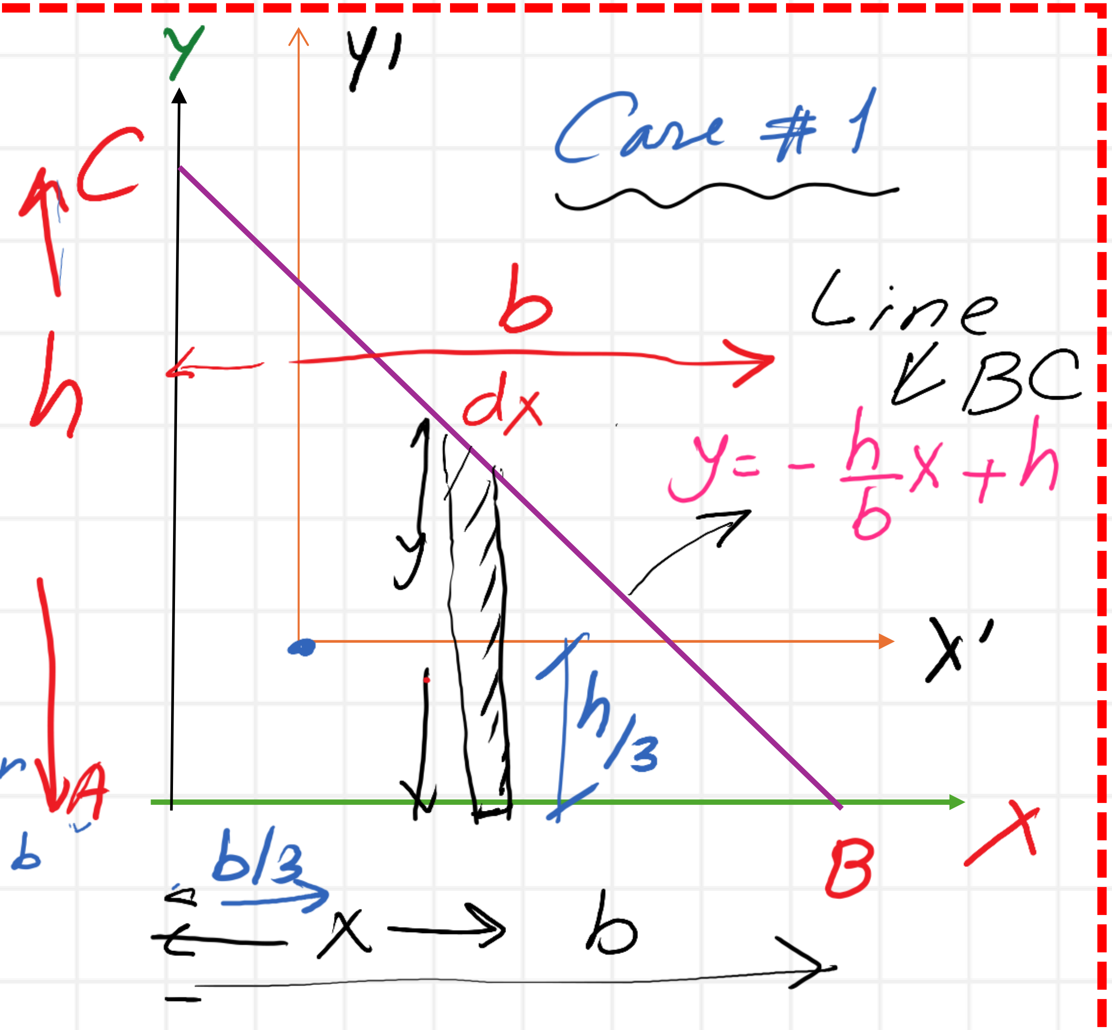
but $y = -\frac{h x}{b} + h$ From Line BC Equation

$x = 0$ $x = b$

$$dA = \left(-\frac{h}{b}x + h\right) dx$$

$$dI_y = dA(x^2) = \left(-\frac{h}{b}x + h\right)(x^2)\,dx$$

$$dI_y = -\frac{h}{b}x^3\,dx + h(x^2)\,dx$$



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I_y For Right_Triangle

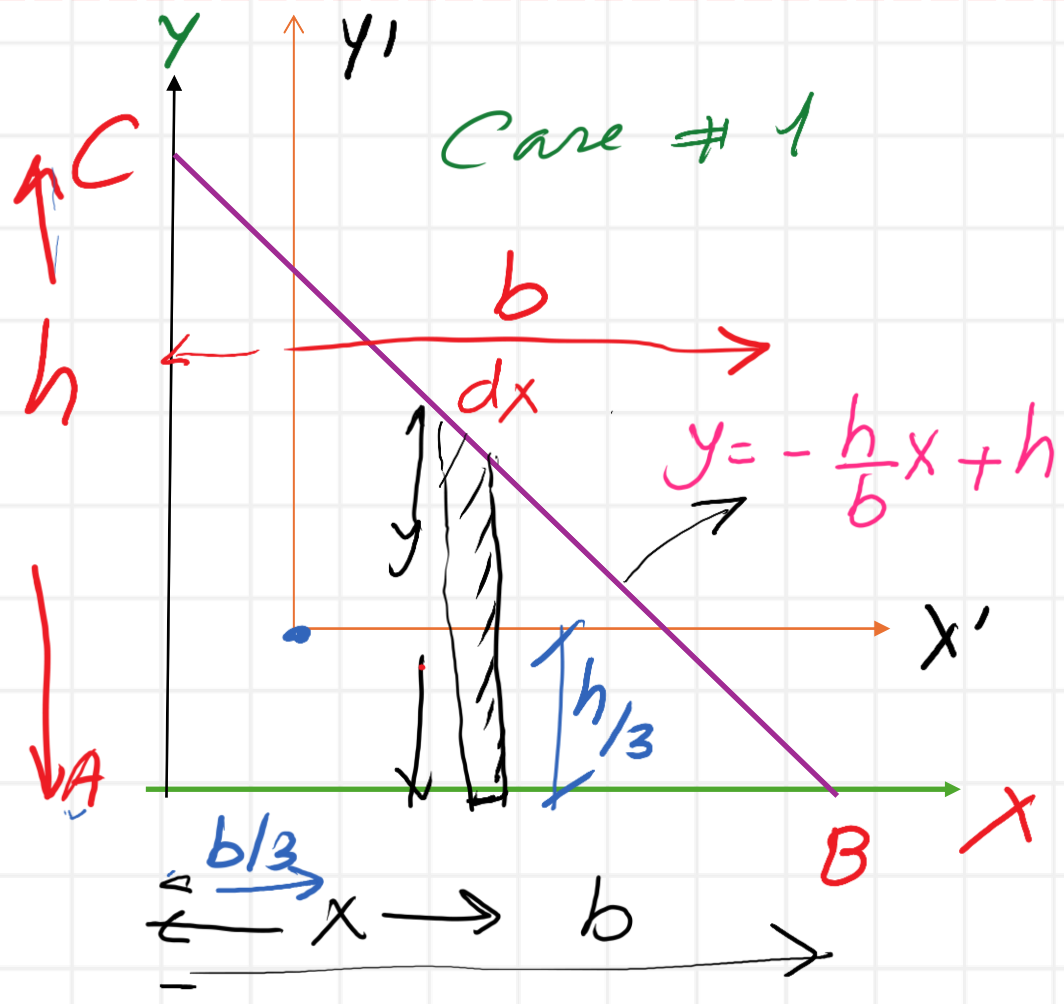
$$I_y = \int_0^b dA \cdot x^2 = \int_0^b h\left(\frac{-x^3}{b}dx + x^2 dx\right)$$

$$I_y = h\left[-\frac{x^4}{4b} + \frac{x^3}{3}\right]_0^b$$

$$I_y = h\left[\frac{-3b^4 + 4b^4}{12b}\right]$$

$$I_y = \left[\frac{h \cdot b^4}{12}\right] = \frac{hb^3}{12}$$

at Point A



$I_y = hb^3/12$

To get I_y at c.g - For y' axis

we have

$I_y = I_{yG} + A(\bar{x}^2)$

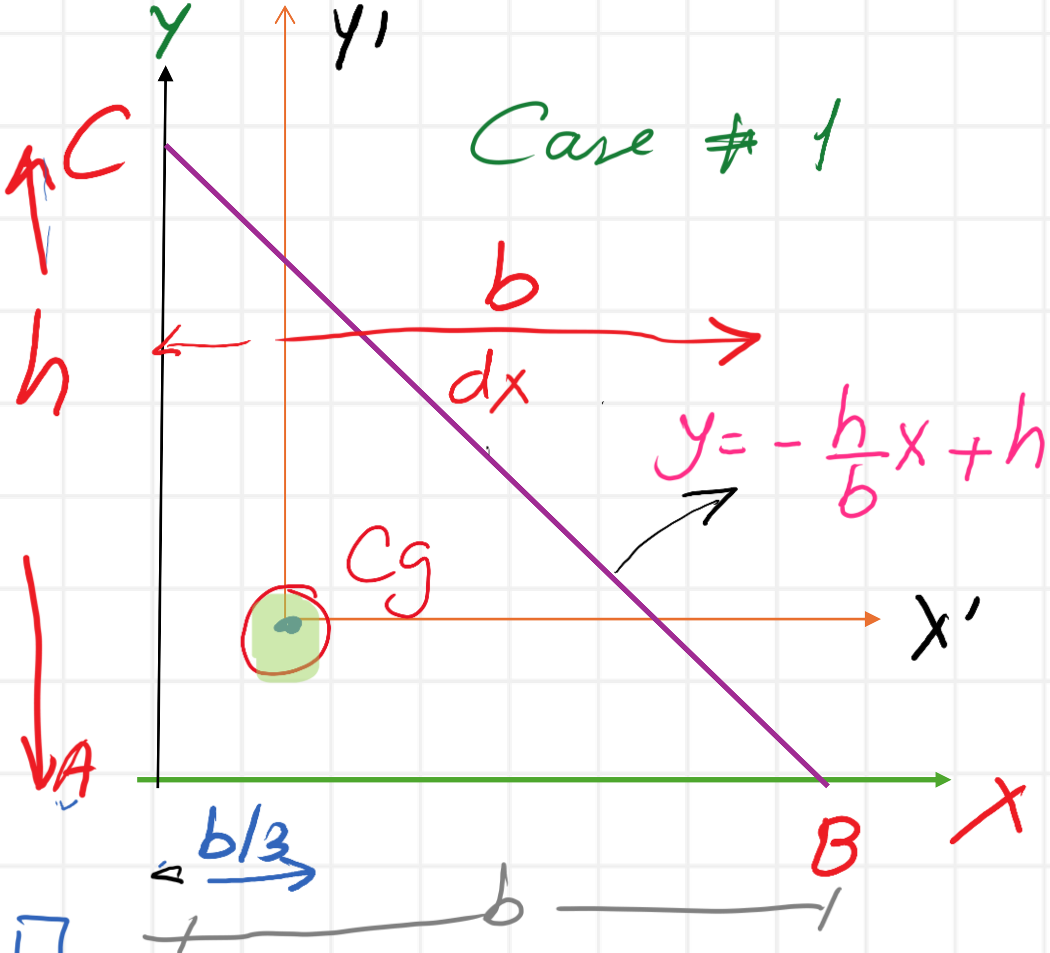
then $I_{yG} = I_y - A\bar{x}^2$

$\bar{x} = b/3 \Rightarrow \bar{x}^2 = \frac{b^2}{9}, \; A = \frac{1}{2}bh$

$I_{yG} = hb^3/12 - \left[\frac{bh}{2}\right]\left[\frac{b^2}{9}\right]$

$= hb^3\left[\frac{1}{12} - \frac{1}{18}\right] = \frac{hb^3}{18}[1.5 - 1]$

$= \frac{hb^3}{18}\left(\frac{1}{2}\right) = \frac{hb^3}{36}.$



$k_{yG}^2 = I_{yG}/A$

$= \frac{hb^3}{36} / \left(\frac{hb}{2}\right) = \frac{b^2}{18}$

$k_{yG} = \frac{b}{3\sqrt{2}}$.

Cg radius of Gyration

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Polar moment of Inertia - at A

AT Corner (x,y Intersection)

Recall refer to post-7)

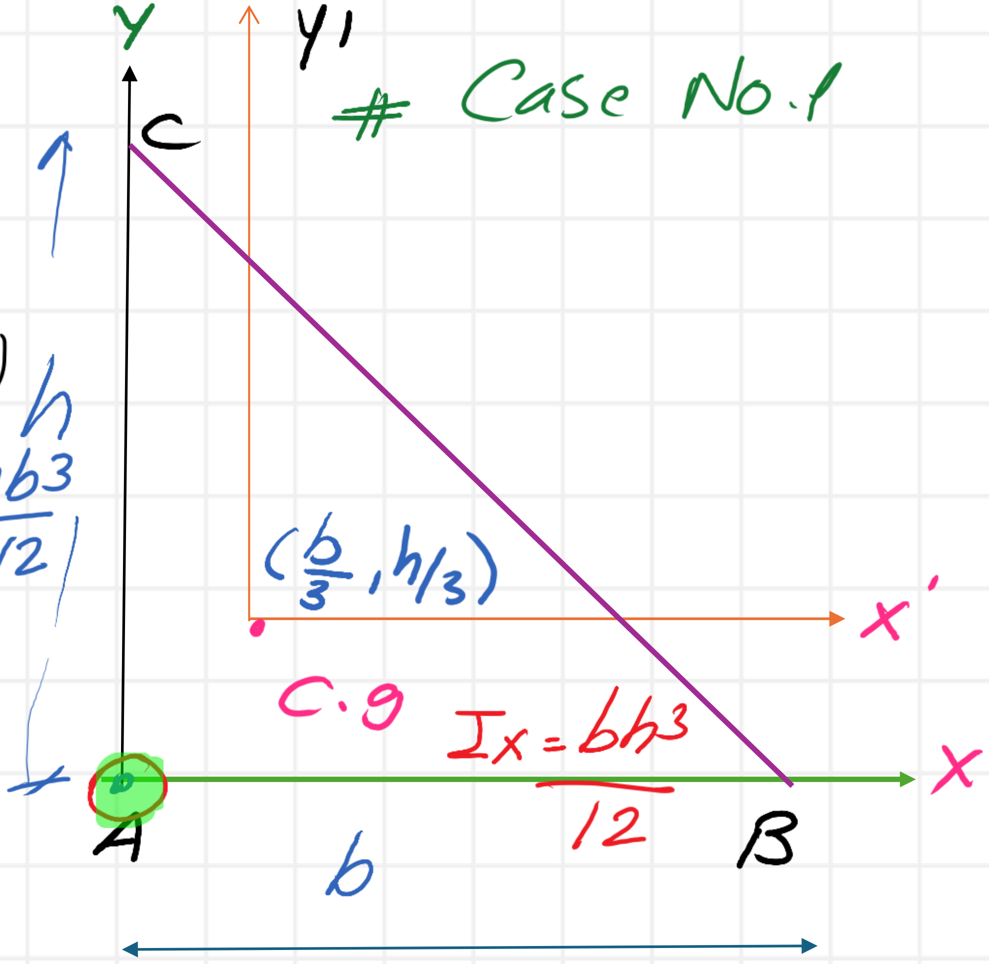
$I_x : bh^3/12$

$I_y : hb^3/12$

$$J_o = \frac{bh^3}{12} + \frac{hb^3}{12}$$

$$J_o = \frac{bh(h^2+b^2)}{12}$$

$I_y = \frac{hb^3}{12}$



while J_{oG} at Centroid

J_{oG}

$I_{xg} : bh^3/36$

$I_{yg} : hb^3/36$

add $J_{oG} = bh(h^2+b^2)/36$

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Polar radius of Gyration at c.g

$$I_{x\,c.g} = \frac{bh^3}{36}$$

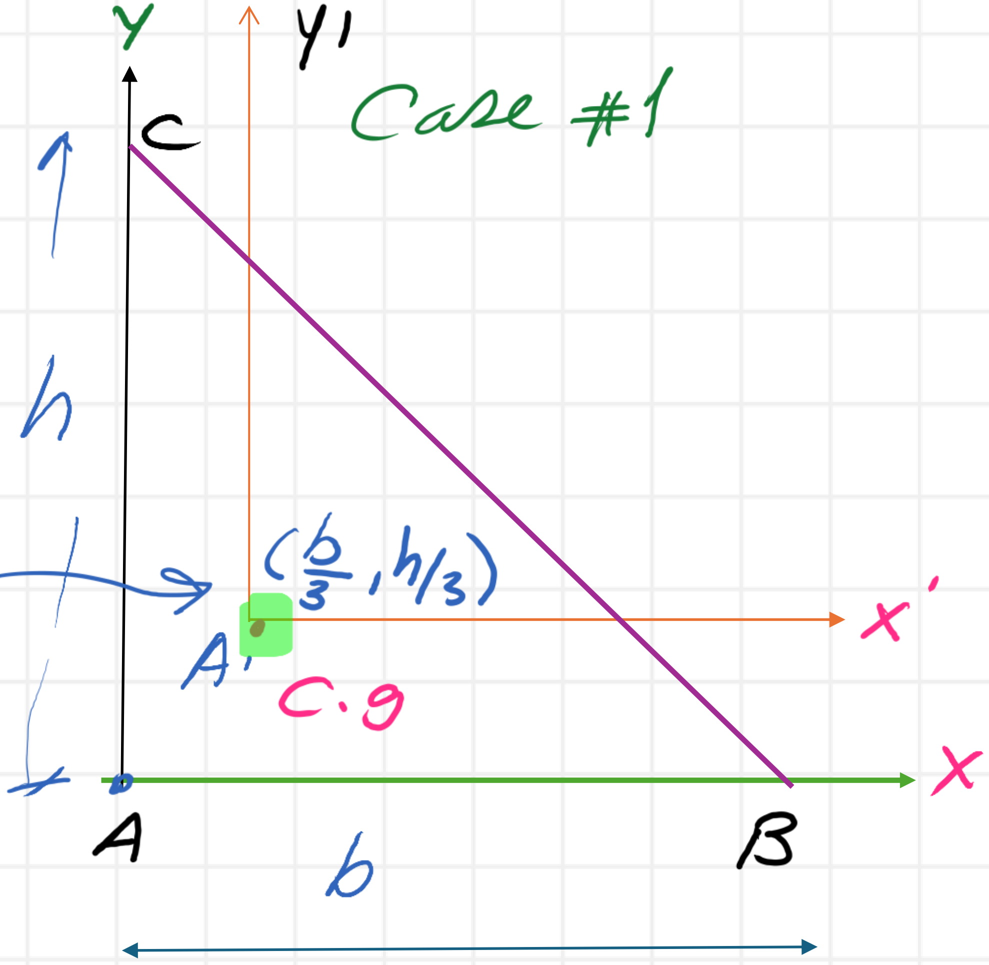
$$A = \frac{1}{2}bh$$

$$I_{y\,c.g} = \frac{hb^3}{36}$$

$$J_{o\,c.g} = \frac{bh^3}{36} + \frac{hb^3}{36} = \left(\frac{h^2+b^2}{36}\right)bh$$

$$\overline{K}^2_{o\,c.g} = \frac{J_{o\,c.g}}{A} = \frac{bh(h^2+b^2)}{36} \times \frac{1}{\frac{1}{2}bh}$$

$$K^2_{o\,c.g} = \frac{h^2+b^2}{18} \Longrightarrow \text{Polar radius of Gyration}$$



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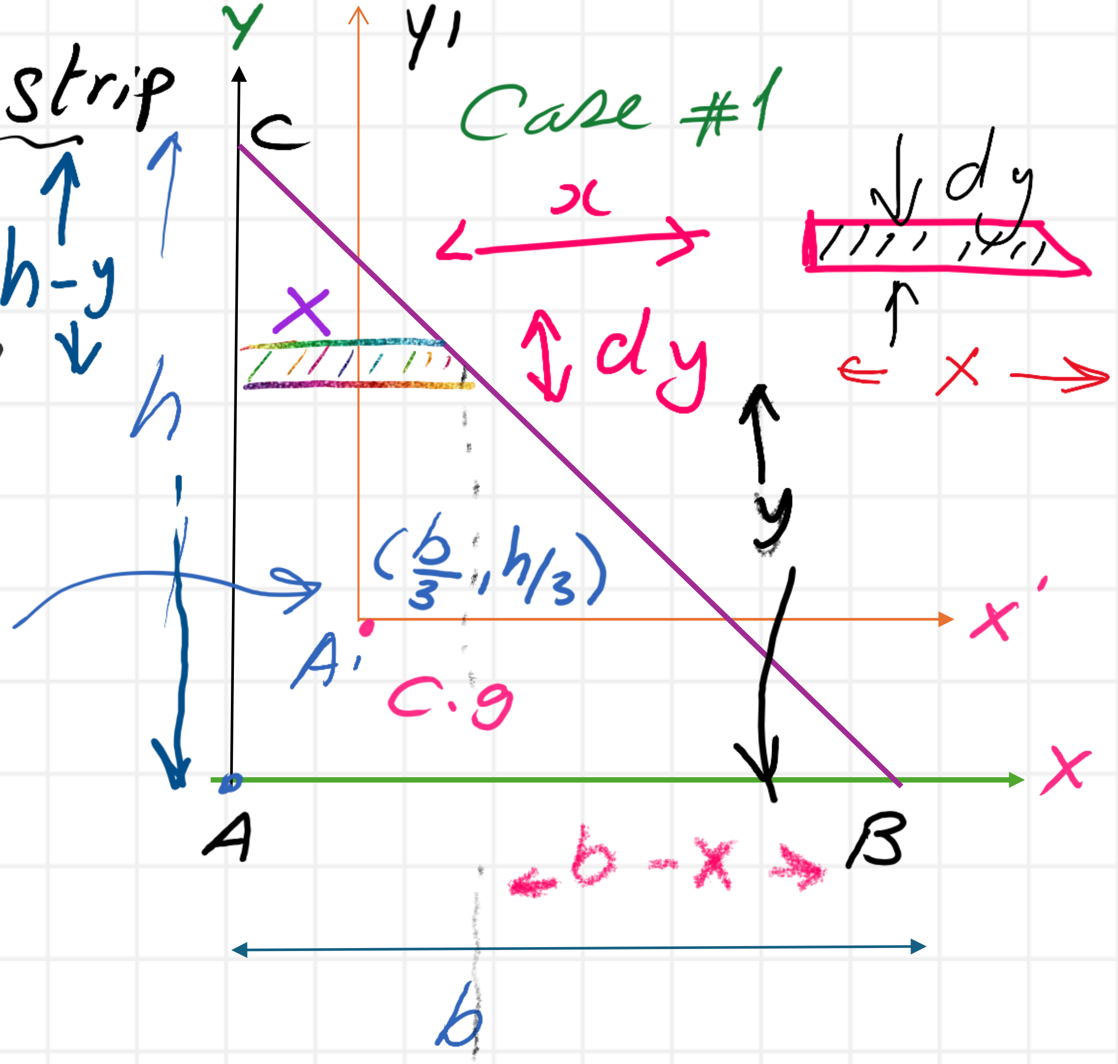
How to get I_y by using HL strip

Consider that breadth of is x and y is VL distance

The slope of Line AB is given by

$$y = mx + C$$
$$y = -\frac{h}{b}x + h$$

Check when $x = 0 \rightarrow y = h$
while for $x = b \rightarrow y = 0$
$\rightarrow x = (h-y)\frac{b}{h}$



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I_y Continued

$$x = (h-y)\frac{b}{h}$$

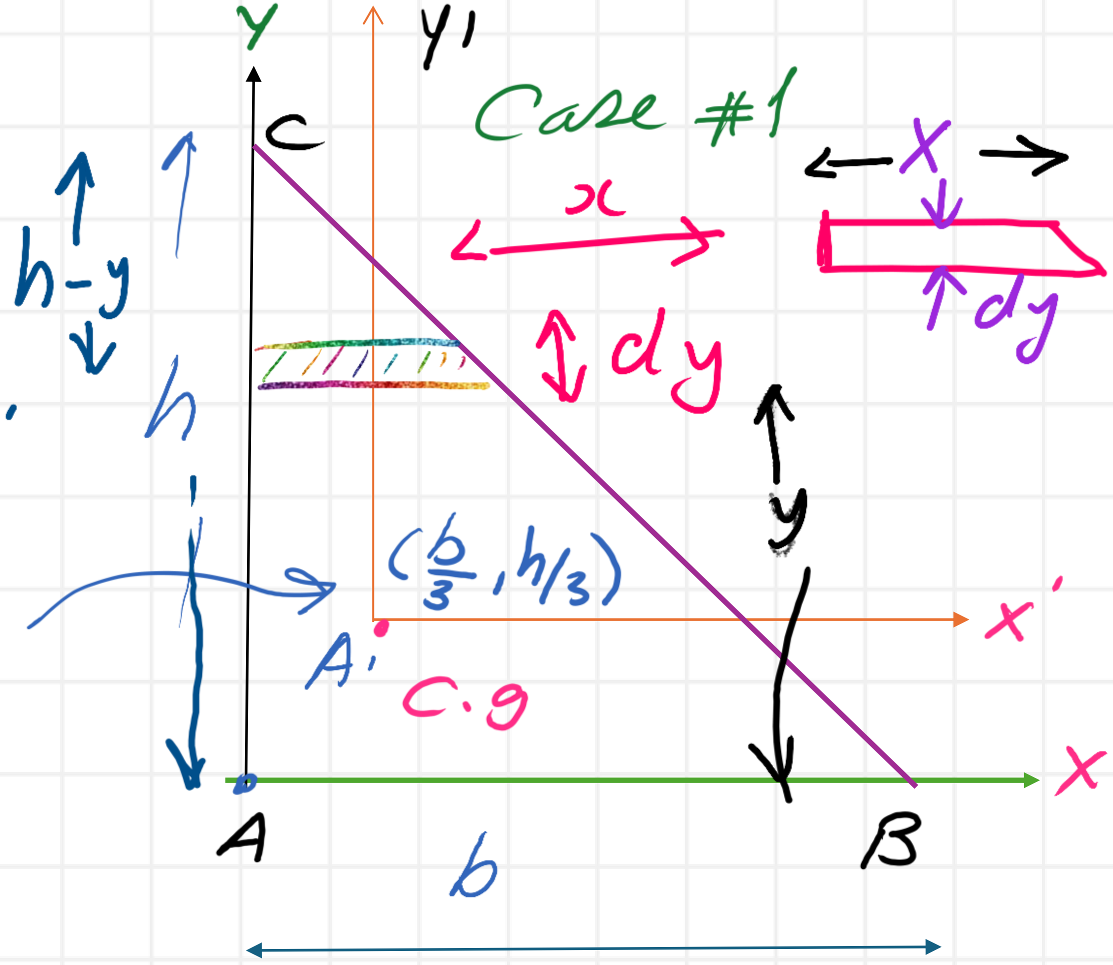
$$dI_{y'} = dy\left(\frac{x^3}{3}\right)$$

Substitute for x value

$$dI_{y'} = \frac{(h-y)^3}{3}\frac{b^3}{h^3}\,dy$$

$$dI_{y'} = \frac{b^3}{3h^3}(h-y)^3\,dy$$

$$\int dI_{y'} = \frac{b^3}{3h^3}\int_0^h (h-y)^3\,dy$$



Derive I_y by using a HL strip

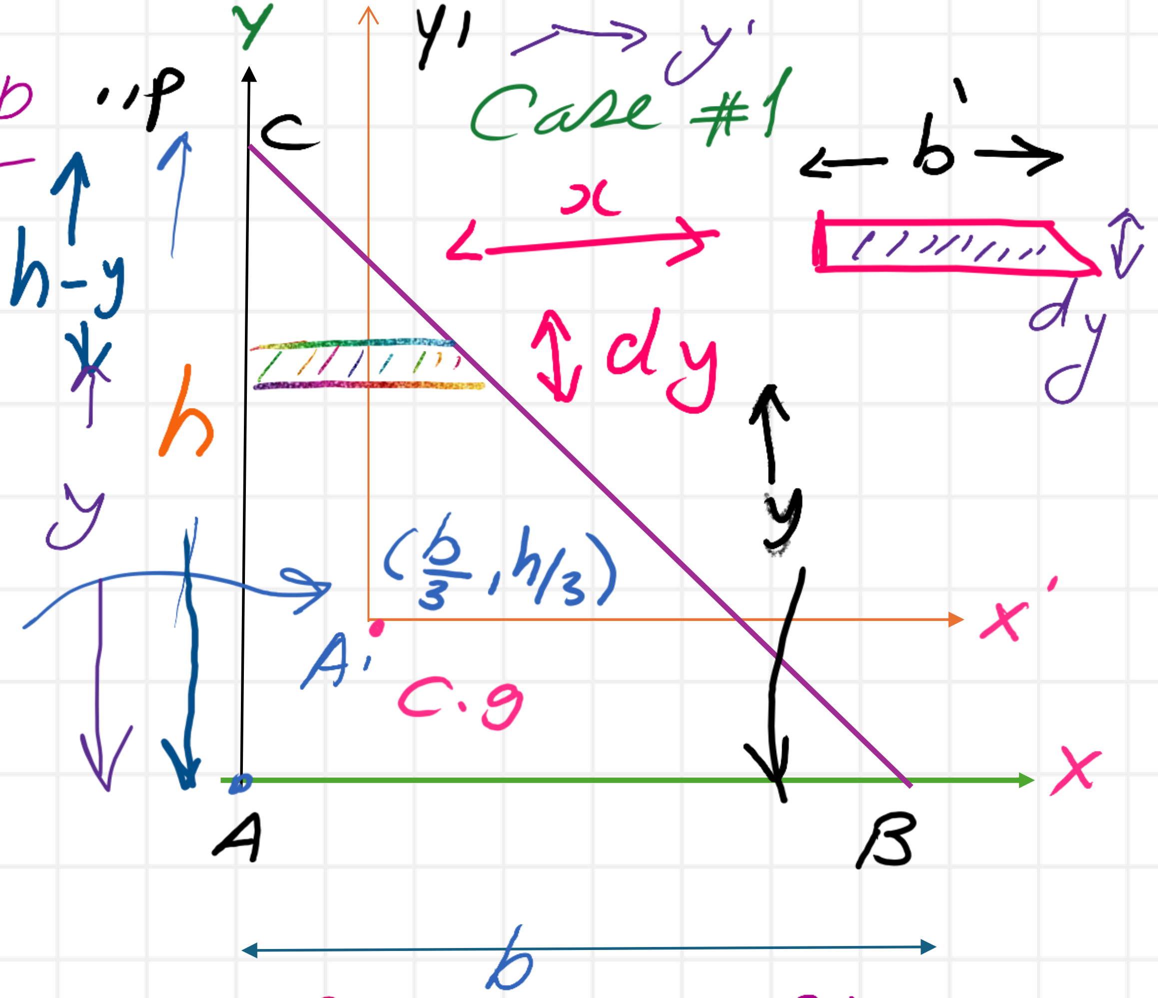
$$\int dI_{y'} = \frac{b^3}{3h^3}\int_0^h (h-y)^3\,dy$$

$$I_y = \frac{b^3}{3h^3}\int_0^h \left(h^3 - 3yh^2 + 3hy^2 - y^3\right)dy$$

$$I_y = \frac{b^3}{3h^3}\left(h^3 \cdot y\Big|_0^h - 3h^2 \cdot \frac{y^2}{2}\Big|_0^h + 3h\,\frac{y^3}{3}\Big|_0^h - \frac{y^4}{4}\Big|_0^h\right)$$

$$I_y = \frac{b^3}{3h^3}\left(h^4 - 1.50\,h^4 + h^4 - 0.25\,h^4\right) = \frac{b^3}{3h^3}\left(\frac{1}{4}\right)h^4 = \frac{b^3 h}{12}$$

finally



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