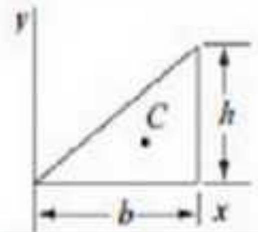
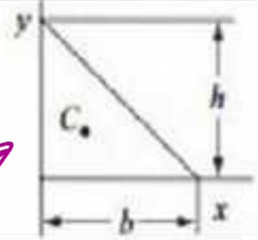
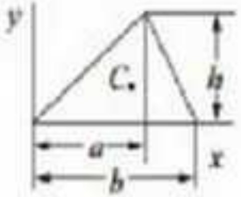
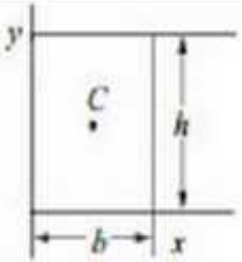
Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = bh/2$ $x_c = 2b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/4$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/2$	$I_{x_c y_c} = Abh/36 = b^2h^2/72$ $I_{xy} = Abh/4 = b^2h^2/8$
	$A = bh/2$ $x_c = b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/6$	$I_{x_c y_c} = -Abh/36 = -b^2h^2/72$ $I_{xy} = Abh/12 = b^2h^2/24$
	$A = bh/2$ $x_c = (a+b)/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = [bh(b^2 - ab + a^2)]/36$ $I_x = bh^3/12$ $I_y = [bh(b^2 + ab + a^2)]/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = (b^2 - ab + a^2)/18$ $r_x^2 = h^2/6$ $r_y^2 = (b^2 + ab + a^2)/6$	$I_{x_c y_c} = [Ah(2a-b)]/36$ $= [bh^2(2a-b)]/72$ $I_{xy} = [Ah(2a+b)]/12$ $= [bh^2(2a+b)]/24$
	$A = bh$ $x_c = b/2$ $y_c = h/2$	$I_{x_c} = bh^3/12$ $I_{y_c} = b^3h/12$ $I_x = bh^3/3$ $I_y = b^3h/3$ $J = [bh(b^2 + h^2)]/12$	$r_{x_c}^2 = h^2/12$ $r_{y_c}^2 = b^2/12$ $r_x^2 = h^2/3$ $r_y^2 = b^2/3$ $r_p^2 = (b^2 + h^2)/12$	$I_{x_c y_c} = 0$ $I_{xy} = Abh/4 = b^2h^2/4$

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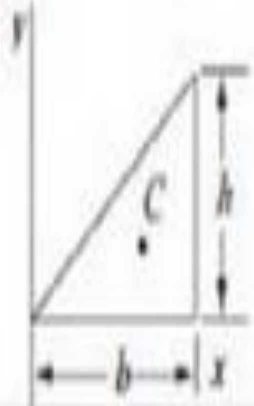
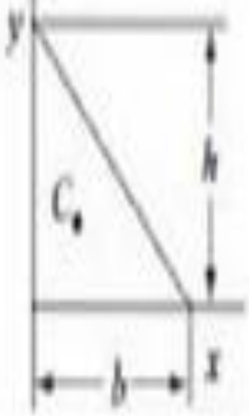
Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = bh/2$ $x_c = 2b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/4$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/2$	$I_{x_c y_c} = Abh/36 = b^2h^2/72$ $I_{xy} = Abh/4 = b^2h^2/8$
	$A = bh/2$ $x_c = b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/6$	$I_{x_c y_c} = -Abh/36 = -b^2h^2/72$ $I_{xy} = Abh/12 = b^2h^2/24$

Our Case →

$\frac{bh^3}{12}$

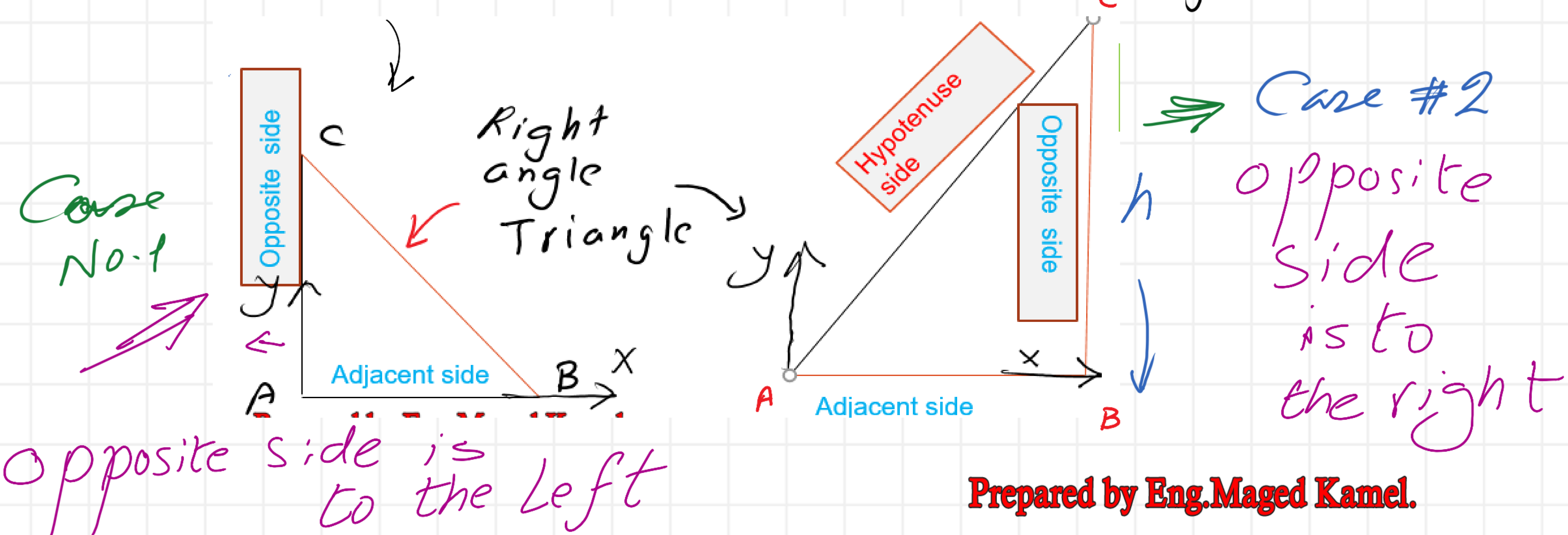
$r_x^2 = \frac{h^2}{6}$

Objective of the lecture

1- How to determine the Ix for the right- angle triangle by using:

a- Horizontal strip parallel to the external x axis. → at the adjacent base

b- Vertical strip perpendicular to the external x axis → at the adjacent base



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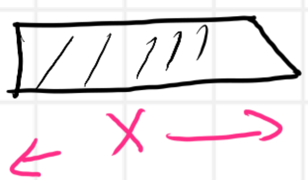
I_x For Right angle Triangle



$\rightarrow y = -\frac{h}{b}x + h$

$(y-h) = -\frac{h}{b}x \rightarrow x = -\frac{b(y-h)}{h}$

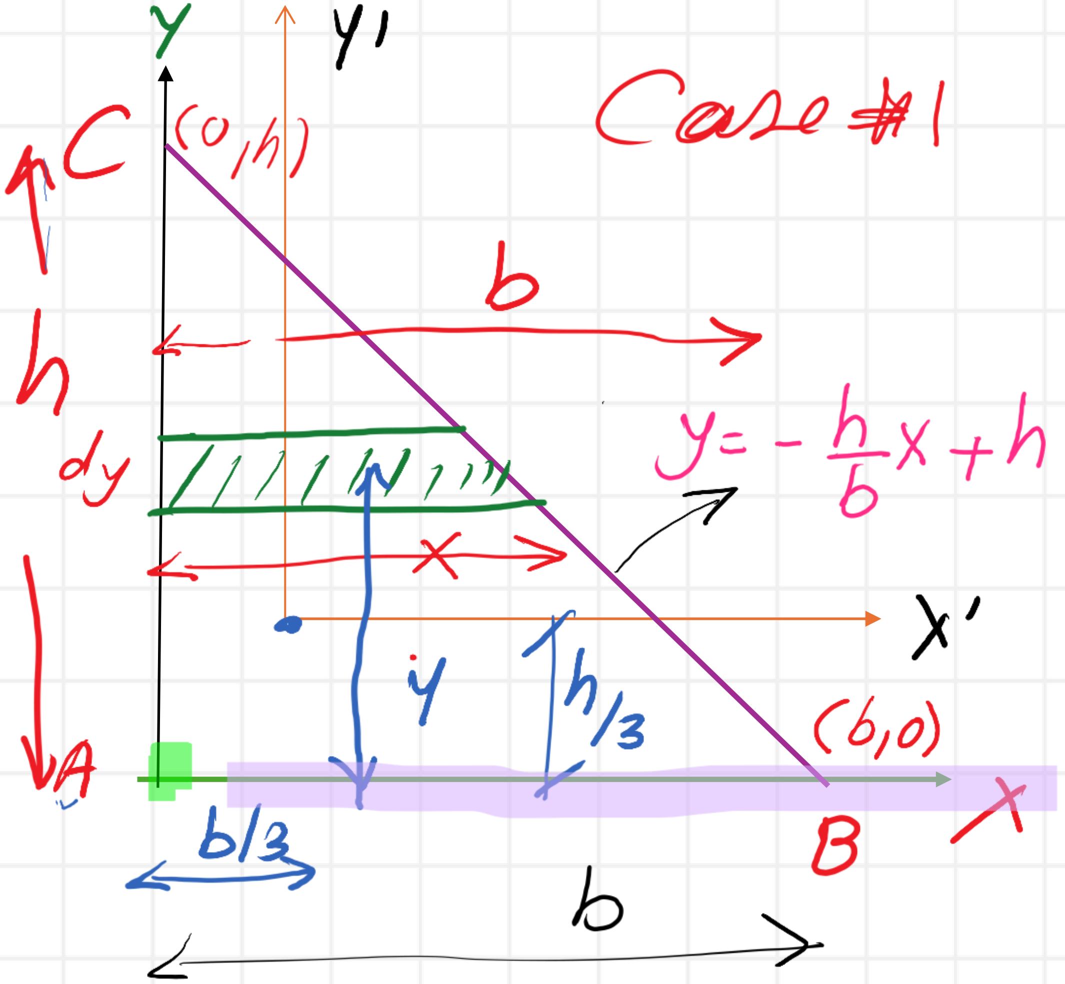
$dA = x\,dy = (b)\left(\frac{h-y}{h}\right)\cdot dy$

$I_x = \int dA \cdot y^2 = \int_0^h \frac{b}{h}(h-y)\,y^2\cdot dy$

$I_x = \frac{b}{h}\int_0^h (hy^2\,dy - y^3\,dy)$

$I_x = \frac{b}{h}\left[h\frac{y^3}{3} - \frac{y^4}{4}\right]_0^h = \frac{b}{h}\left[\frac{h^4}{3} - \frac{h^4}{4}\right] = \frac{b}{h}\left(\frac{h^4}{12}\right) = \frac{bh^3}{12}$

Case #1



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I_x for Right angle Triangle

b) using VL strip

consider as rectangular element

$$dI_x = dx(y^3)\left(\frac{1}{3}\right)$$

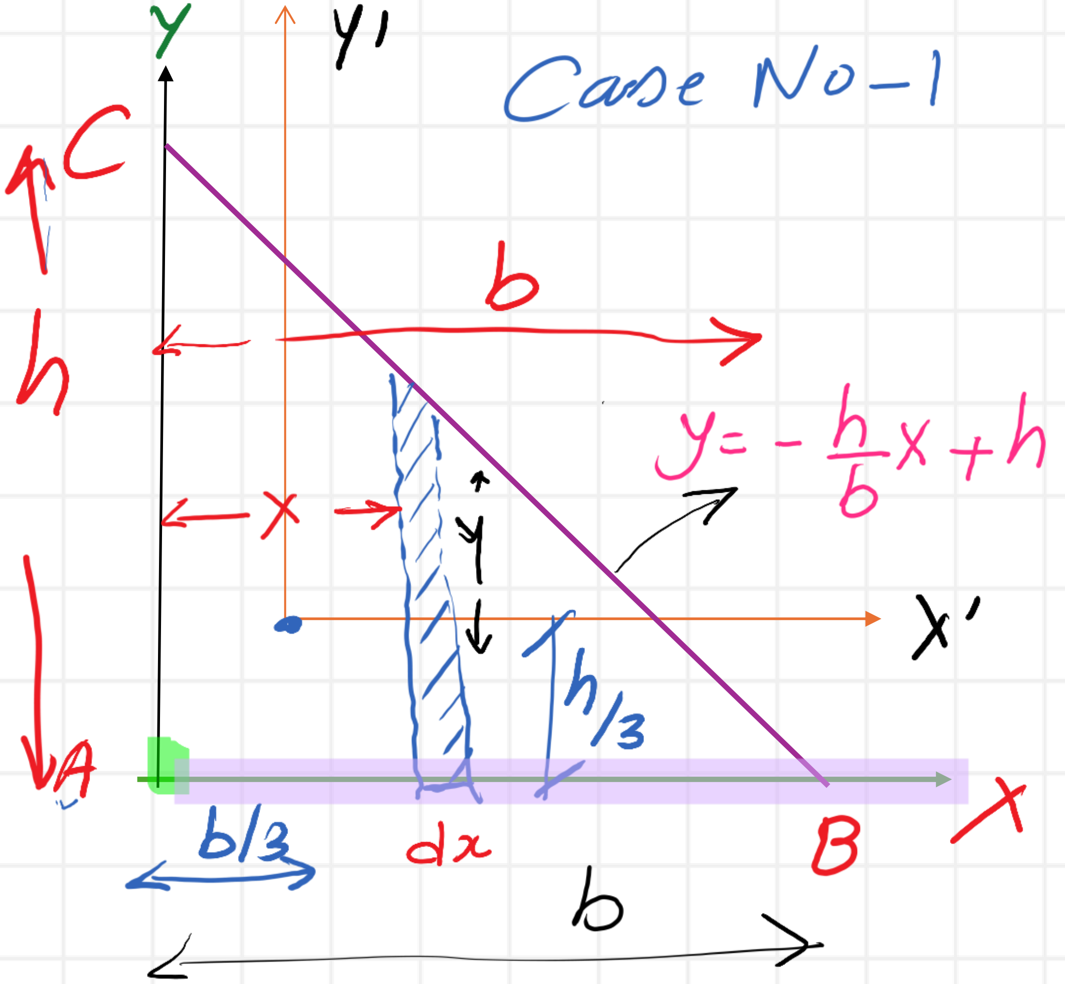
since $y = -\frac{h}{b}x + h$

$$y^3 = \left(-\frac{h}{b}x + h\right)\left(\frac{h^2x^2}{b^2} - \frac{2h^2x}{b} + h^2\right)$$

$$= -\frac{h^3x^3}{b^3} + \frac{2h^3x^2}{b^2} - \frac{h^3x}{b} + \frac{h^3x^2}{b^2} - \frac{2h^3x}{b} + h^3 =$$

$$y^3 = \left(-\frac{h^3x^3}{b^3} + 3\frac{h^3x^2}{b^2} - 3\frac{h^3x}{b} + h^3\right)$$

$$dI_x = \frac{1}{3}\left(-\frac{h^3x^3}{b^3}\,.dx + \frac{3}{b^2}h^3x^2\,.dx - \frac{3h^3}{b}x + h^3\right)$$



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I_x at base — Case # 1

$$\int dI_x = \frac{1}{3}\int_0^h \left(-\frac{h^3x^3}{b^3}\cdot dx + \frac{3}{b^2}h^3x^2\cdot dx - 3\frac{h^3}{b}x\,dx + h^3\cdot dx\right)$$

$$\int dI_x = \frac{1}{3b}\left[-\frac{h^3}{b^2}\frac{x^4}{4}\Big|_0^b + \frac{3}{b}h^3\frac{x^3}{3}\Big|_0^b - 3h^3\frac{x^2}{2}\Big|_0^b + h^3\cdot x\Big|_0^b\cdot b\right]$$

$$\int dI_x = \frac{1}{3b}\left[-\frac{h^3b^4}{4b^2} + \frac{h^3b^3}{b} - \frac{3}{2}h^3b^2 + h^3\cdot b^2\right]$$

$$\frac{1}{3b}\left[-\frac{h^3}{4}b^2 + h^3b^2 - \frac{3}{2}h^3b^2 + h^3b^2\right]$$

$$I_x = \frac{1}{3b}\left[\frac{-6+24-36+24}{24}\right]h^3\cdot b^2 = \frac{6}{3b}\left(\frac{1}{24}\right)(h^3b^2) = \frac{1}{12}(b\,h^3)$$

h

$A = \frac{1}{2}bh)$

$$k_x^2 = \frac{I_x}{A} = \frac{bh^3}{12}\left(\frac{1}{\frac{1}{2}bh}\right) = \frac{h^2}{6} \Rightarrow k_x = \frac{h}{\sqrt{6}}$$

→ radius of Gyration

x —

I_x For Right Triangle

C.g

$$I_x = \frac{bh^3}{12}$$

to get I_{xg}

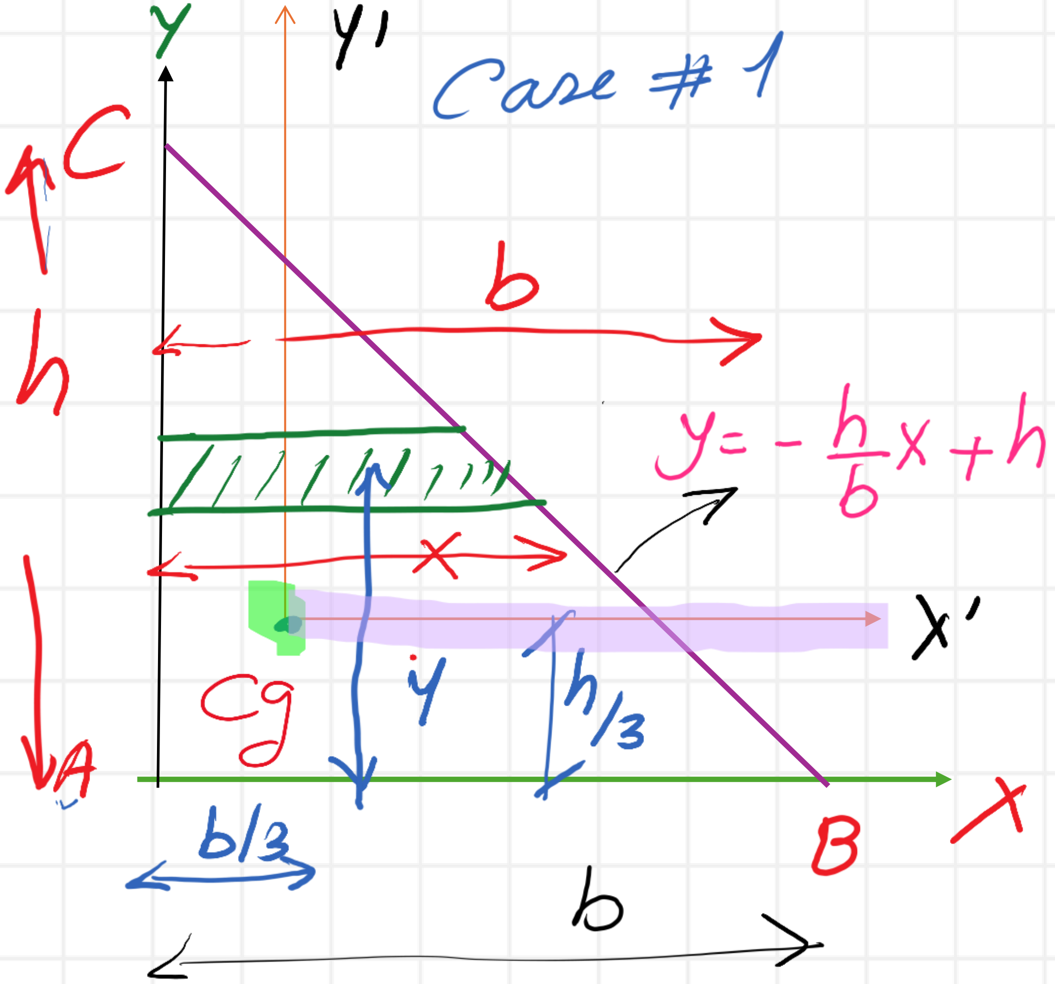
$$I_{x_{c.g}} = I_x - A \cdot \bar{y}^2$$

$$A = \frac{1}{2} b \cdot h, \quad \bar{y} = \frac{1}{3} h$$

$$I_{xG} = \frac{bh^3}{12} - \left(\frac{1}{2} bh\right)\left(\frac{h^2}{9}\right)$$

$$= bh^3 \left[\frac{1}{12} - \frac{1}{18}\right]$$

$$I_{xG} = bh^3 \left[\frac{1.5 - 1}{18}\right] = \frac{bh^3}{36}$$



$$\bar{K}_{xg}^2 = \frac{I_G}{A} = \frac{bh^3}{36} \times \frac{2}{bh} = \frac{h^2}{18}$$

$$K_{xg} = \frac{h}{3\sqrt{2}}$$

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