Introduction LU Decomposition

2 x 2 Matrix - Doolittle's method For L & U matrices

How Can we Find L & U matrices elements?

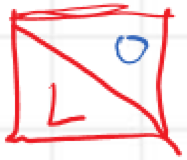
Can we have LU decomposition for a Singular Matrix?

Lu Decomposition - 2x2 Matrix.

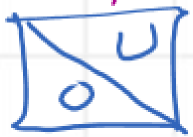
It is possible to show that any square matrix **A** can be expressed as a product of a lower triangular matrix *L* and an upper triangular matrix *U*. The procedure based on unity elements on the major diagonal of L is called the Doolittle method. The procedure based on unity elements on the major diagonal of U is called the Crout method.

A = L * U

Lower



upper



If we have for two set of equations :

2x+3y = 13
3x+4y = 18

First: Doolittle's Expression

$$A = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} L11 & 0 \\ L21 & L22 \end{pmatrix} \begin{pmatrix} U11 & U12 \\ 0 & U22 \end{pmatrix}$$

LU factorization METHOD

$$\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ L21 & 1 \end{pmatrix} \begin{pmatrix} U11 & U12 \\ 0 & U22 \end{pmatrix}$$

Unkowns
$L_{21}, U_{11}, U_{12}, U_{22}$

$L_{11} = L_{22} = 1$

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A-Multiply the two matrices and then equate to the elements of the original matrix.

$$\begin{bmatrix} 1 & 0 \\ L_{21} & 1 \end{bmatrix} \cdot \begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{bmatrix} \Rightarrow$$

We can write the following expressions

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} \\ L_{21} U_{11} & L_{21} U_{12} + U_{22} \end{bmatrix}$$

$1 * U11 = 2$ I

$1 * U12 = 3$ II

$L21 * U11 = 3$ III

$L21 * U12 + 1 * U22 = 4$ IV

From Equation I $\Rightarrow U_{11} = 2$

From Equation II $\Rightarrow U_{12} = 3$

Get L_{21} from Equation III by substitution

$L_{21}(2) = 3 \qquad L_{21} = \frac{3}{2}$

Get U_{22} value From IV

$\frac{3}{2}(3) + U_{22} = 4$

$U_{22} = -\frac{1}{2}$

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$$\begin{array}{ccc} & a_{11} & a_{12} \\ A = & \left[\begin{matrix} 2 & 3 \\ 3 & 4 \end{matrix}\right] & \end{array} = \overset{L}{\left[\begin{matrix} 1 & 0 \\ \frac{3}{2} & 1 \end{matrix}\right]} \cdot \overset{U}{\left[\begin{matrix} 2 & 3 \\ 0 & -\frac{1}{2} \end{matrix}\right]}$$

$U_{11} = 2$

$U_{12} = 3$

$L_{21} = \frac{3}{2}$

$U_{22} = -\frac{1}{2}$

$$U = \left[\begin{matrix} 2 & 3 \\ & \bigcirc \end{matrix}\right]$$

pivot

$$= A \left[\begin{matrix} \boxed{2} & 3 \\ 3 & 4 \end{matrix}\right]$$

$-3/2$

$\frac{a_{21}}{a_{11}}$

$3/2$

$$L = \left[\begin{matrix} 1 & 0 \\ \boxed{3/2} & 1 \end{matrix}\right]$$

$\to$ LU method

$L_{11}, L_{22} = 1$

$$U_{22} = a_{22} - \overset{L_{21}}{\left(\frac{3}{2}\right)} U_{12}$$

$$= 4 - \frac{3}{2}(3)$$

$U_{22} = -\frac{1}{2}$

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Case of singular matrix $\Rightarrow$ No inverse

Now Test your Knowledge For the following

Not all square matrices have an LU decomposition, and it may be necessary to permute the rows of a matrix before obtaining its LU factorization.

2×2

if $B: \begin{bmatrix} 1 & 5 \\ 2 & 10 \end{bmatrix}$ get L & U MATRICES.

Note: $|B| = 10 - 10 = 0$ → it is a singular matrix

Solution

$B = \begin{bmatrix} 1 & 5 \\ 2 & 10 \end{bmatrix}$

$\begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}$ → $L_{12} = 0$ → by definition

$L_{11} = 1$

$L_{22} = 1$

$a_{21} = 2, \quad a_{11} = 1$

while $L_{21} = \frac{a_{21}}{a_{11}} = \frac{2}{1} = 2$

$L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$= 2/1$

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Now Test your Knowledge For the following

2×2

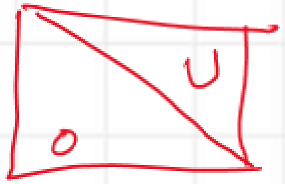
if $B: \begin{bmatrix} 1 & 5 \\ 2 & 10 \end{bmatrix}$ get L & U MATRICES.

Solution

For the upper matrix

$$\begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \rightarrow U_{21} = 0$$

by definition



$L_{21} = 2$

$a_{22} = 10$

$U_{12} = 5$

$$U_{22} = a_{22} - L_{21} \cdot U_{12}$$

$$\begin{bmatrix} 1 & 5 \\ 2 & 10 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 5 \\ 0 & U_{22} \end{bmatrix} \Rightarrow U_{22} = 10 - (2 \cdot 5) = 0$$

$$U \Rightarrow \begin{bmatrix} 1 & 5 \\ 0 & 0 \end{bmatrix}$$

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Now Test your KnowLedge For the following 2x2 matrix

2x2

iF B: $\begin{bmatrix} 1 & 5 \\ 2 & 10 \end{bmatrix}$ get L & U MATRICes.

L U

Solution

$$B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 0 & 0 \end{bmatrix} \Rightarrow$$

this is not acceptable For U diagonaL has no Zeros.